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Essays on characteristic-based portfolio optimization

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Collaboration with peers

Three out of the four research papers presented were co-authored. This section details the extent of collaboration and my specific contributions to each paper. The order of the papers follows the presentation in Table 1 in Section 1.

The first research paper is co-authored with Nusret Cakici (Fordham University), Christian Fieberg (City University of Applied Sciences Bremen), Thorsten Poddig (University of Bremen), and Adam Zaremba (Montpellier Business School; Poznan University of Economics and Business). In this collaboration, I took a leading role in acquiring, processing, and analyzing an extensive dataset. My responsibilities included programming the code to obtain the relevant data, filtering and organizing the raw data as required by the intended empirical studies, and applying statistical methods to derive meaningful insights. I worked closely with Christian Fieberg and Thorsten Poddig during this process, ensuring accuracy and consistency in our data handling techniques. While the initial idea for this research project was conceived by Christian Fieberg and Adam Zaremba, I provided critical feedback to strengthen the study's design and objectives. Nusret Cakici conducted the empirical studies, while Adam Zaremba interpreted the results and drafted the manuscript, incorporating comments and recommendations from all authors. The manuscript is under review in the Journal of Portfolio Management.

The second research is co-authored with Thorsten Poddig, Christian Fieberg, Michael Olschewsky (Hamburger Sparkasse) and Michael Falge (University of Bremen). The main idea behind this project was developed primarily by myself in consultation with Michael Falge and Thorsten Poddig. Using data provided by Christian Fieberg, I was also responsible for further data processing, conducting all empirical studies, and drafting the manuscript. The draft benefited substantially from comments and recommendations of all authors. Additionally, I presented this research project at doctoral seminars sponsored by HypoVereinsbank. This research paper was published in the International Journal of Theoretical and Applied Finance.

The third research paper is a single-author project consisting entirely of my own work. The draft has been submitted to the International Journal of Theoretical and Applied Finance.

The fourth paper is co-authored with Christian Fieberg, Thorsten Poddig, and Armin Varmaz (City University of Applied Sciences Bremen) and is currently under revise and resubmit status at the European Journal of Operational Research. The main idea was developed by Christian Fieberg and Armin Varmaz. Building upon data provided by Christian Fieberg, I was responsible for data processing and conducting all empirical studies. While the initial writing was primarily done by myself, the manuscript substantially benefited from the comments, support, and recommendations of all co-authors.

Contents

1 Overview

The current thesis comprises four research papers primarily dedicated to developing new quantitative methodologies that aim to contribute to the field of portfolio optimization, with particular emphasis on applications in practical portfolio management. Table 1 provides an overview of each paper, listing its title, authors, and the corresponding subsection that delves into its contributions in more depth. For detailed examination, the research papers can be found in the Appendix.

#	Authors	Title	Referring to
(1)	Cakici, N. Fieberg, C. Osorio, C. Poddig, T. Zaremba, A.	Picking Winners in the Factor Zoo	2.4.4
(2)	Osorio, C. Poddig, T. Fieberg, C. Olschewsky, M. Falge, M.	Market Timing in Parametric Portfolio Policies	4.3
(3)	Osorio, C.	Characteristic Timing in Parametric Portfolio Policies	4.4
(4)	Fieberg, C. Osorio, C. Poddig, T. Varmaz, A.	Enhancing Index-Tracking Performance: Leveraging Characteristic-Based Factor Models for Reduced Estimation Errors	4.5

Table 1 This table presents an overview of each paper included in this thesis, detailing its title, authors, and the corresponding subsection that provides a deeper exploration of its contributions.

Except for Paper (1), the studies listed in Table 1 introduce new portfolio optimization methodologies tailored to improve the guiding principles for effective portfolio management. To contextualize my dissertation within the relevant discourse on portfolio optimization, Section 2 starts by revisiting the seminal work by Markowitz (1952). His groundbreaking mean-variance analysis established the foundational principles of modern portfolio theory, which continue to influence contemporary research and practice in the field. Section 2 additionally explores subsequent developments within the pertinent literature to provide a comprehensive framework for understanding the historical evolution of portfolio optimization principles. Within this landscape, factor models have emerged as a key area of interest, with numerous factors proposed over time, yet without a clear consensus on how portfolio managers can select the most effective factors. As detailed in Subsubsection 2.4.4, Paper (1) attempts to provide an answer to this question by leveraging machine learning techniques to analyze the cross-section of factors and identify the key drivers of factor predictability.

While mean-variance analysis represents a widely accepted normative framework guiding theoretical explorations of optimal portfolio choice, its practical utility is constrained by two major limitations, underscoring in particular the significance of the research presented in this thesis. First, the necessity of estimating the distribution of asset returns poses substantial challenges. Given that estimation models often struggle to capture the true return distribution accurately, reliance on such models leads to suboptimal portfolio performance in practice due to the influence of estimation errors. Second, while mean-variance analysis presupposes that investors consistently seek to maximize returns and minimize risks, real-world portfolio managers are tasked with constructing portfolios that exhibit specific attributes relative to benchmarks (e.g., market indices). This has given rise to a dichotomy between passive and active portfolio management, paving the way for the emergence of relative portfolio optimization approaches, as elaborated upon in Section 3. While a passive portfolio manager focuses on replicating the benchmark at minimal costs, an active portfolio manager strives to outperform it. Papers (2) and (3) contribute to the active portfolio management domain. Paper (4) adds to the field of passive portfolio management. The portfolio optimization methodologies developed in these three research studies circumvent the estimation of the return distribution, thereby evading estimation errors and improving performance of practical portfolio strategies.

The pioneering work of Brandt et al. (2009) on characteristic-based portfolio optimization, detailed in Section 4, introduces parametric portfolio policies as a robust and efficient approach for achieving benchmark outperformance. By directly utilizing asset characteristics to compute optimal portfolio weights in a concise manner, it addresses several limitations inherent in Markowitz (1952)'s approach, particularly the errors stemming from inaccurately estimating the return distribution. However, its reliance solely on cross-sectional predictability based on asset characteristics overlooks potential benefits from time-series predictability, which could significantly enhance real-world investment strategies. Addressing this gap in the research literature, Papers (2) and (3) extend the original framework of parametric portfolio policies to incorporate predictable time variations in both aggregated market returns and the relationship between asset returns and characteristics, respectively. Given that the primary objective of parametric portfolio policies is to outperform benchmarks, this framework is not applicable to passive portfolio management strategies. Nonetheless, the core feature of modeling optimal portfolio weights as functions of asset characteristics, rather than utilizing characteristics to estimate return distributions, can also enhance the performance of tracking portfolios by circumventing estimation errors. In alignment with this principle, Paper (4) introduces a novel mixed-integer, characteristic-based tracking portfolio optimization approach that minimizes tracking errors while constraining the number of assets in the portfolio.

In summary, Section 2 delves into the foundational principles of mean-variance portfolio optimization and applications of factor models to mitigate the impact of estimation errors. Notably, Subsubsection 2.4.4 delineates the contributions made by Paper (1) to the literature on factor selection. Section 3 describes the assumptions and paradigms underlying practical portfolio management, shedding light on why it is necessary to investigate relative portfolio optimization approaches as done in this thesis. Additionally, Section 4 discusses existing research on characteristic-based portfolio optimization, with particular focus on Subsections 4.3, 4.4 and 4.5 which delve into the contributions of Papers (2), (3), and (4), respectively, introducing novel characteristic-based portfolio optimization approaches. Concluding reflections on the key facets of this thesis are presented in Section 5.

2 Modern portfolio theory

To contextualize my dissertation within the contemporary landscape of portfolio optimization, this section revisits some of the most influential related research studies, particularly the seminal work by Markowitz (1952) who revolutionized the field of portfolio optimization by introducing the concept of mean-variance analysis, laying the groundwork for what is now known as *modern portfolio theory*. Subsection 2.1 provides an overview of the key principles of mean-variance optimization. Subsection 2.2 addresses the relationship between mean-variance optimization and models of rational investor behavior, a crucial backdrop for understanding the maxims underlying the related literature. Since practical applications of modern portfolio theory are challenged by the task of estimating the true, yet unknown, distribution of asset returns, Subsection 2.3 discusses the impact of estimation errors and approaches proposed in previous literature to mitigate them. One of the most consequential strategies is the utilization of risk factor models derived from asset characteristics, as detailed in Subsection 2.4. Apart from representing a tool for robustly estimating the distribution of asset returns, risk factors are gradually substituting individual assets as the building blocks in portfolio strategies. Subsubsection 2.4.4 discusses the contributions of Paper (1), which investigates the predictability in the cross-section of factor returns, providing novel insights regarding factor selection.

2.1 Diversification and mean-variance optimization

Mean-variance optimization serves as a quantitative portfolio formation framework for rational investors, drawing upon the principle of diversification. This principle aligns with the well-known adage *"don't put all your eggs in one basket"*, emphasizing that investing in a variety of assets is less risky than concentrating investments in only a few. Preceding the advent of modern portfolio theory, earlier quantitative approaches in the academic literature neglected this diversification aspect, focusing only on maximizing the expected portfolio return. Under such a maxim, optimization theory dictates that investors allocate their entire capital to the single asset with the maximum expected return, disregarding the benefits of a diversified portfolio with assets that collectively reduce the risk. In order to capture the benefits of diversification and the trade-off between return and risk in a tractable measure of risk, Markowitz (1952) suggests using the variance[1] of the portfolio return. As discussed below, mean-variance principles can be formulated with different optimization programs.

Denoting the expected value operator and the variance operator with $\mathbb{E}$ and $\mathbb{V}$, respectively, as well as letting $\lambda > 0$, mean-variance optimization is often expressed in the literature (see, e.g., Kolm et al., 2014) as

$$\max_{w_1,\ldots,w_N} \mathbb{E}\left[\sum_{i=1}^{N} w_i r_i\right] - \lambda \mathbb{V}\left[\sum_{i=1}^{N} w_i r_i\right], \tag{1}$$

where N is the total number of assets, r_i is a random variable representing the return of asset i, and w_i denotes the proportion of capital allocated to asset i, or in other words, the portfolio weight of asset i. Accordingly, the term $\sum_{i=1}^{N} w_i r_i$ represents the portfolio return. Typically, short-selling constraints (i.e., $w_i \geq 0, \forall i$) as well as budget constraints (i.e., $\sum_{i=1}^{N} w_i = 1$) are assumed.

The optimization program outlined in Equation (1) yields the optimal portfolio that maximizes expected return while simultaneously minimizing portfolio risk, with the investor's risk-return trade-off parameter λ guiding the optimization process. Formulating mean-variance optimization as in Equation (1) facilitates the efficient determination of the entire spectrum of optimal portfolios across various risk preferences by solving the optimization program for different values of λ (see, e.g., Markowitz, 1959). However, translating concrete risk preferences of specific investors into a suitable value for λ is a challenging task (see, e.g., Holt & Laury, 2002).

In his seminal work, Markowitz (1952) presents a more practical formulation of mean-variance optimization capturing the fact that, when faced with equivalent risks, rational investors prioritize portfolios with higher returns.[2] Concretely, Markowitz (1952) proposes constraining the variance of portfolio returns to a predefined value σ^2 while maximizing the expected portfolio return.[3] This optimization program can be expressed as

[1]Since the variance quantifies the probabilistic dispersion of the portfolio returns, it is interpreted as a risk measure.

[2]Similarly, when confronted with multiple portfolios offering identical returns, rational investors favor those with lower risk.

[3]Alternatively, Markowitz (1952)'s optimization can be expressed as minimizing portfolio return vari-

$$\max_{w_1,\ldots,w_N} \mathbb{E}\left[\sum_{i=1}^{N} w_i r_i\right] \tag{2}$$

$$\text{subject to: } \mathbb{V}\left[\sum_{i=1}^{N} w_i r_i\right] = \sigma^2. \tag{3}$$

Depending on the intended application, mean-variance optimization can be implemented using the optimization program in Equation (1), which involves the additional task of determining a suitable risk-return trade-off parameter λ, or the optimization program given by Equations (2) and (3), which simply requires the portfolio manager to decide the desired portfolio return variance.[4]

Regardless of the specific formulation of the optimization program, some properties are inherent to mean-variance optimization in general. For instance, since the expected value is a linear operator, the expected portfolio return is simply the sum of the individual expected asset returns weighted by the w_i, i.e.,

$$\mathbb{E}\left[\sum_{i=1}^{N} w_i r_i\right] = \sum_{i=1}^{N} w_i \mathbb{E}\left[r_i\right]. \tag{4}$$

In particular, from Equation (4) it immediately follows that, if investors simply maximize the expected portfolio return, the asset with the highest expected return gets a weight of 1 and all other assets have zero weights, which corresponds to the rather unrealistic scenario of investor neglecting all risks in their decision making. This issue is solved by incorporating the variance of portfolio returns to the optimization program, for which it holds

$$\mathbb{V}\left[\sum_{i=1}^{N} w_i r_i\right] = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \mathrm{Cov}\left[r_i, r_j\right] = \sum_{i=1}^{N} w_i^2 \mathbb{V}\left[r_i\right] + 2\sum_{i=1}^{N}\sum_{j>i}^{N} w_i w_j \mathrm{Cov}\left[r_i, r_j\right], \tag{5}$$

where Cov denotes the covariance operator. The property depicted in Equation (5) underscores the significance of utilizing the variance of portfolio returns as a risk measure: The overall portfolio risk is not solely determined by the variances of individual assets but also by the covariances between all pairs of assets. This aspect notably captures the diversification effect. If assets in the portfolio exhibit negative covariances, the overall portfolio variance is diminished.

It is crucial to recognize that formulating the optimal portfolio selection in terms of expected portfolio return and its variance represents a special case within the broader

ance while constraining portfolio return to a predefined value μ.

[4]Sufficient conditions for exact equivalency between both optimization programs are discussed by Bodnar et al. (2013).

framework where the optimal portfolio choice depends on the distribution of asset returns. The advantage is that it reflects the acknowledgment that optimal portfolio decisions are influenced by the inherent uncertainty in the market. However, this implies that the portfolio construction process comprises two distinct stages. First, the relevant parameters of the distribution of asset returns must be estimated using historical data. Second, based on these estimates, an allocation must be determined that is optimal with respect to the investor's preferences. As emphasized by Markowitz (1952), his work primarily focuses on the latter stage by delineating the trade-off between returns and risks inherent in the decision-making process of rational investors.

2.2 Expected utility maximization

While the mean-variance approach recognizes the importance of modeling the risk-return trade-off in problems of optimal portfolio choice by rational investors, it assumes that investors' preferences can be fully captured by the first two moments of the return distribution and that investors exhibit quadratic utility. However, in practice, investors may have different risk preferences and exhibit nonlinear utility functions that better reflect their attitudes toward risk and return.[5] To allow for a more nuanced consideration of risk-return trade-offs for more general investor types, Markowitz (1959) advocates applying the principles of utility theory, positing that rational investors should seek to maximize their expected utility. Denoting with u the utility function of the investor, the optimal portfolio is hence the one that solves the optimization program[6]

$$\max_{w_1,\dots,w_N} \ \mathbb{E}\left[u\left(\sum_{i=1}^{N} w_i r_i\right)\right]. \tag{6}$$

Markowitz (1959) states that mean-variance optimization can be seen as a expected utility maximization in the case of quadratic utility $u_q(r) = r - \lambda r^2$ with $\lambda > 0$ denoting the risk aversion parameter. However, the literature often lacks precision on this point, with many authors providing verbal formulations without delving into the exact formal equivalencies between mean-variance optimization and expected utility maximization under quadratic utility. In general, these two optimization programs do not share the exact same solution. To see this, notice that maximizing expected quadratic utility leads to

[5] Other utility functions are, for example, the family of hyperbolic absolute risk aversion (HARA) utility functions, including the logarithmic and the power or constant relative risk aversion (CRRA) utilities.

[6] Equation (6) is a single-period optimization. A multi-period optimization requires modeling returns as stochastic processes $(r_{i,t+1})_t$ and weights as sequences $(w_{i,t})_t$, both with time index t.

$$\max_{w_1,\ldots,w_N} \ \mathbb{E}\left[u_q\left(\sum_{i=1}^{N} w_i r_i\right)\right] = \max_{w_1,\ldots,w_N} \ \mathbb{E}\left[\sum_{i=1}^{N} w_i r_i\right] - \lambda \mathbb{E}\left[\left(\sum_{i=1}^{N} w_i r_i\right)^2\right]. \tag{7}$$

Since for any random variable x it holds $\mathbb{V}[x] = \mathbb{E}[x^2] - \mathbb{E}[x]^2$, the optimization program in Equation (7) is equivalent to

$$\max_{w_1,\ldots,w_N} \ \mathbb{E}\left[\sum_{i=1}^{N} w_i r_i\right] - \lambda \mathbb{V}\left[\sum_{i=1}^{N} w_i r_i\right] - \lambda \mathbb{E}\left[\sum_{i=1}^{N} w_i r_i\right]^2. \tag{8}$$

Equation (8) simultaneously maximizes the expected portfolio return while minimizing the scaled variance of portfolio returns, alongside minimizing the scaled squared value of the expected portfolio return. This objective bears a strong resemblance to Equation (1). Hence, a less experienced reader might fail to recognize that the common assertion in the literature, suggesting that mean-variance optimization equates to maximizing expected utility under quadratic utility, is not always formally accurate. However, formal derivations by Bodnar et al. (2013) demonstrate that mean-variance optimization is indeed equivalent to the optimization program in Equation (8) under certain special conditions.

The relationship between mean-variance optimization and expected utility maximization under quadratic utility poses a fundamental question: is mean-variance optimization truly adequate for modeling rational investors? This question arises as quadratic utility function suffers from problematic preference specifications which are not consistent with the observed behavior of investors. First, quadratic utility is not monotonically increasing, wrongly suggesting that investors perceive utility loss for returns laying above a certain maximum.[7] Second, as demonstrated by Arrow (1965) and Pratt (1964), the absolute risk aversion of the quadratic utility function grows for increasing returns, leading to reduced risk-taking with greater wealth, contrary to common experience.

Despite these inconsistencies, Markowitz (1959) reconciles mean-variance optimization with models of rational investors by showing that mean-variance optimization is a second-order approximation of expected utility maximization for any arbitrary utility function u. This can be seen by computing the Taylor series of u around a value μ:[8]

$$u(r) = u(\mu) + u'(\mu)(r-\mu) + \frac{1}{2}u''(\mu)(r-\mu)^2 + \ldots. \tag{9}$$

Accordingly, for the expected utility it holds

[7]This aspect is also discussed by Markowitz (1952) who emphasizes that constraining the variance of the portfolio return penalizes both negative deviations from the expected portfolio return as well as positive deviations which are rather beneficial for investors. However, applying the variance as risk measure (instead of, e.g., the semi-variance which only accounts for negative deviations from the mean) allows for analytic and computational tractability.

[8]Alternatively, one can compute the Taylor series around 0 and proceed similarly.

$$\mathbb{E}\left[u(r)\right] = u(\mu) + u'(\mu)\mathbb{E}\left[(r-\mu)\right] + \frac{1}{2}u''(\mu)\mathbb{E}\left[(r-\mu)^2\right] + \dots. \tag{10}$$

Substituting $r = \sum_{i=1}^{N} w_i r_i$ and $\mu = \mathbb{E}\left[\sum_{i=1}^{N} w_i r_i\right]$ in Equation (10), it immediately follows

$$\mathbb{E}\left[u\left(\sum_{i=1}^{N} w_i r_i\right)\right] = u\left(\mathbb{E}\left[\sum_{i=1}^{N} w_i r_i\right]\right) + \frac{1}{2}u''\left(\mathbb{E}\left[\sum_{i=1}^{N} w_i r_i\right]\right)\mathbb{V}\left[\sum_{i=1}^{N} w_i r_i\right] + \dots. \tag{11}$$

Equation (11) can illustratively shed light on why mean-variance optimization approximates expected utility maximization. Since an appropriate utility function u is monotonically increasing, maximizing the expected portfolio return maximizes the first term in the right hand side of Equation (11). And since an appropriate utility function u is also concave, displaying a negative second derivative, the sign of the variance term is negative. This particularly implies that minimizing the variance increases the second term in Equation (11). Thus, maximizing expected portfolio return and minimizing portfolio return variance contributes to increasing expected utility, serving as an approximation for the maximization of the expected utility.

As shown by studies such as Levy & Markowitz (1979) and Kroll et al. (1984), mean-variance approximations of expected utility maximization problems perform very well in empirical applications. A detailed analysis of selected literature studies on mean-variance approximation of maximum expected utility can be found in Markowitz (2014). In summary, the documented evidence supports the suitability of mean-variance analysis for the second stage in problems of optimal portfolio choice by rational investors.

2.3 Estimation errors

Despite the success of mean-variance optimization as a conceptual framework for optimal portfolio choice, its real-life efficacy is notably affected by the fundamental limitation that the true distribution of asset returns, required by the objective function in the second stage of the portfolio construction process, remains unknown and can only be estimated. A substantial body of research (see, e.g., Michaud, 1989; Broadie, 1993; Chopra et al., 1993; Mynbayeva et al., 2022) extensively documents that the step of estimating the distribution of asset returns is inevitably compromised by errors due to the limitations of data samples, significantly undermining the performance of any two-step approach that optimizes based on error-prone estimates.

To compound the challenges, the literature highlights the acute sensitivity of mean-variance optimization to even minor fluctuations in estimated input variables. For instance, Best & Grauer (1991) shows through simulation studies that optimal portfolio weights can vary extremely with small changes in the estimated expected asset returns, a concern exacerbated by greater difficulties in estimating expected returns compared to variances and covariances (Merton, 1980). This heightened sensitivity can be attributed to the fact that errors in input estimates are magnified within the optimization process. This has earned mean-variance optimization the moniker of an "error-maximizer" (Michaud, 1989).

Addressing the issue of error-maximization, numerous studies explore methodological enhancements aiming to refine the optimization in the second stage of the mean-variance framework. One prominent approach is robust optimization, pioneered by Ben-Tal & Nemirovski (1998), Ben-Tal & Nemirovski (1999), El Ghaoui & Lebret (1997) and El Ghaoui et al. (1998), which models portfolio performance under uncertainty with the goal of optimizing the worst-case scenario. This requires carefully chosen uncertainty sets. However, the selection of suitable uncertainty sets remains a contentious issue among researchers. For example, Tütüncü & Koenig (2004) propose box uncertainty sets, while Bertsimas & Sim (2004) suggest the use of polyhedral uncertainty sets. An alternative, more straightforward strategy to alleviate the error-maximization property involves constraining the portfolio weights to prevent extreme allocations and enhance diversification against estimation errors (see, e.g., Frost & Savarino, 1988; Jagannathan & Ma, 2003; DeMiguel, Garlappi, & Uppal, 2009). However, while such constraints may help stabilize the optimization process, they also inherently limit portfolio performance, potentially resulting in suboptimal solutions (Zhao et al., 2019).

Approaches in the second stage of the mean-variance framework can only aim to mitigate the error-maximization property. Nevertheless, irrespective of the improvements in the treatment of error-prone input variables during optimization, the resulting optimal decisions will always exhibit errors due to misleading information. Therefore, it might be preferable to prioritize enhancing the initial stage of estimating input variables for portfolio optimization.

A prevalent strategy in the literature for refining input estimates within the mean-variance framework is shrinkage estimation (Klein & Bawa, 1976; Jorion, 1986; Frost & Savarino, 1986; Black & Litterman, 1990; Chopra & Ziemba, 1993; Ledoit & Wolf, 2003, 2004; Kan & Zhou, 2007; DeMiguel, Garlappi, Nogales, & Uppal, 2009; Frahm & Memmel, 2010; Bodnar et al., 2018).[9] Shrinkage essentially involves combining a sample estimate with another estimator, such as a constant value, to reduce variance. However, this introduces bias, which may be unacceptable depending on the application. Additionally, determining the optimal level of shrinkage entails subjective judgment and is prone to errors (DeMiguel et al., 2013). Moreover, shrinkage estimators typically require complex optimization algorithms, limiting their practical value for portfolio managers.

A more widely embraced approach among both researchers and practitioners for re-

[9]While some scholars explore explicit shrinkage techniques employing James-Stein estimation, others opt for Bayesian estimation, implicitly entailing shrinkage.

fining estimations in the initial stage of the mean-variance framework is the utilization of factor models. These models are known for their ease of use and straightforward economic interpretations. Of particular interest is their ability to capitalize on economic theories and empirical evidence of the behavior of markets to better estimate the distribution of asset returns, rather than relying solely on statistical techniques. Thus, the application of factor models to estimate input parameters for mean-variance analysis has become a notable area of interest in the field of portfolio management. The subsequent subsection provides a detailed exploration of factor models in this context.

2.4 Factor models

2.4.1 Efficient and robust estimations for mean-variance optimization

Extending the work of Markowitz (1952), Sharpe (1963) introduces the use of factor models with the primary goal of efficiently implementing mean-variance optimization. This approach consists in decomposing returns into a common component correlated to the returns of all assets in the market and an uncorrelated idiosyncratic component (i.e., into systematic and unsystematic returns).

Sharpe (1963) initially develops this framework based on a factor model[10] that can be written as

$$r_i = \alpha_i + \beta_i F + \varepsilon_i, \tag{12}$$

where r_i denotes a random variable for the return of asset i and F a random variable for a factor correlated to the returns of all assets in a systematic fashion. The specific relationship between F and the return of an individual asset i is given by its corresponding factor loading β_i. The random residual ε_i captures the unsystematic return component of asset i which is uncorrelated with F. It is assumed that each residual ε_i has a mean of zero and is uncorrelated to the residual ε_j of every other asset $j \neq i$. The coefficient α_i describes a nonrandom idiosyncratic return component of asset i with nonzero mean (i.e., not captured in the zero-mean residual term).

Note that α_i and β_i are not observable and must be estimated if a factor model is to be applied for portfolio optimization. The standard approach to estimate these coefficients is via ordinary least square estimation of the linear regression model

$$r_{i,t+1} = \alpha_i + \beta_i F_{t+1} + \varepsilon_{i,t+1} \tag{13}$$

using a sample of length T, where $r_{i,t+1}$, F_{t+1} and $\varepsilon_{i,t+1}$ denote realized values in the period from t to $t+1$. The main motivation of Sharpe (1963) in applying a factor model is that it allows to efficiently obtain the inputs required for mean-variance optimization.

[10]He refers to this model as "diagonal model".

For instance, since the residuals ε_i are assumed to have means of zero, for the expected value of r_i it holds

$$\mathbb{E}[r_i] = \mathbb{E}[\alpha_i + \beta_i F + \varepsilon_i] = \alpha_i + \beta_i \mathbb{E}[F]. \tag{14}$$

Since the residuals ε_i are also uncorrelated with the factor F, it follows for the variance of r_i that

$$\mathbb{V}[r_i] = \mathbb{V}[\alpha_i + \beta_i F + \varepsilon_i] = \beta_i^2 \mathbb{V}[F] + \mathbb{V}[\varepsilon_i]. \tag{15}$$

And since the residuals ε_i and ε_j are assumed uncorrelated for all $i \neq j$, it also holds

$$\mathrm{Cov}[r_i, r_j] = \mathrm{Cov}[\alpha_i + \beta_i F + \varepsilon_i, \alpha_j + \beta_j F + \varepsilon_j] = \beta_i \beta_j \mathbb{V}[F]. \tag{16}$$

Thus, adopting such a factor model reduces the number of parameters that necessitate estimation. Original mean-variance optimization requires estimates for N expected values, N variances and $N(N-1)/2$ covariances. In contrast, the approach pioneered by Sharpe (1963) necessitates estimating only the expected value and variance of the factor F, alongside α_i, β_i, and $\mathbb{V}[\varepsilon_i]$, resulting in a considerably smaller count of parameters that require estimation: $3N + 2 \ll 2N + N(N-1)/2$.

Although Sharpe (1963) introduces the framework of factor models in the special case of one-factor models, Equation (12) can be easily extended to a multifactor model expressed in the form of

$$r_i - r_f = \alpha_i + \sum_{m=1}^{M} \beta_{m,i} F_m + \varepsilon_i, \tag{17}$$

where F_m denotes the random variable of the m-th risk factor, $\beta_{m,i}$ the sensitivity of asset i on factor m, and M the total number of factors. In this case, assuming that the factors are uncorrelated, Equations (14), (15) and (16) become

$$\mathbb{E}[r_i] = \alpha_i + \sum_{m=1}^{M} \beta_{m,i} \mathbb{E}[F_m] \tag{18}$$

$$\mathbb{V}[r_i] = \sum_{m=1}^{M} \beta_{m,i}^2 \mathbb{V}[F_m] + \mathbb{V}[\varepsilon_i] \tag{19}$$

$$\mathrm{Cov}[r_i, r_j] = \sum_{m=1}^{M} \beta_{m,i} \beta_{m,j} \mathbb{V}[F_m], \tag{20}$$

showcasing that also in a higher dimensional case the inputs of the mean-variance analysis can be efficiently obtained from estimated moments of the factors alongside the estimated factor loadings.

Given the limited computational power available at the time of Markowitz (1952), this increased efficiency represented a key advancement at the moment of its conception. Despite the substantial increase in computing power since then, thereby mitigating concerns over estimating numerous parameters, factor models retain a profound importance in modern portfolio theory. With fewer parameters to estimate under factor models, there is also less susceptibility to estimation errors, enhancing portfolio strategies' robustness. It is important to note that this assertion holds true only under the assumption that a suitable factor model exists capable of capturing the true distribution of asset returns. As pointed out by Cochrane (2011), this poses a formidable challenge, one that the asset pricing literature has grappled with over the past six decades, which is to answer the question: *"why do prices move?"*.

2.4.2 Capital Asset Pricing Model (CAPM)

The Capital Asset Pricing Model (CAPM), developed independently by Sharpe (1964), Lintner (1965) and Mossin (1966), is the first model to attempt to provide an answer to the question of why prices move and remains one of the most prominent models for asset pricing. Assuming in particular that investors are rational, risk-averse and maximize their utility,[11] the CAPM states that the expected excess return of any asset i is fully determined by the expected excess market return in the form of

$$\mathbb{E}[r_i] - r_f = \beta_i \left(\mathbb{E}[r_{Mkt}] - r_f \right), \tag{21}$$

where r_f denotes the risk-free return rate, r_{Mkt} the market return and β_i the sensitivity of the expected return of asset i with respect to changes in the expected excess market return. The market return is theoretically the return of a portfolio holding all assets in the market. However, in practical research, scholars must estimate the market risk factor by focusing on a feasible subset of market assets and selecting a weighting scheme. Most empirical studies use a relative market capitalization weighting for assets in the market portfolio, known as a *value-weighted portfolio*. Alternatively, some studies opt for equal weighting of assets, referred to as an *equal-weighted portfolio*.

If further assumptions are added, the CAPM can be interpreted as a one-factor model positing that the excess market return is the single risk factor required to explain the asset returns,[12] which can be written as

[11]The CAPM additionally presupposes that investors diversify across a range of investments, cannot influence prices, trade without transaction or taxation costs, can lend and borrow under a universal risk-free rate of interest without limits, deal with infinitesimally divisible investments, have homogeneous expectations regarding future returns and risks and possess the same information at all times.

[12]The CAPM is a model of equilibrium of expected returns and not directly a risk factor model. In the CAPM, the only assumption regarding the idiosyncratic returns is that they have a mean of zero. In risk factor models, the idiosyncratic return of each asset is additionally assumed to be uncorrelated to

$$r_{i,t+1} - r_{f,t+1} = \alpha_i + \beta_i(r_{Mkt,t+1} - r_{f,t+1}) + \varepsilon_{i,t+1}. \tag{22}$$

After decades of research, a plethora of empirical tests consistently find that the CAPM fails to capture the empirical distribution of asset returns (for a comprehensive list of studies with empirical tests invalidating the CAPM, see Harvey et al., 2016). This discrepancy prompts the pragmatic exploration of multifactor models, as discussed in the next subsubsection.

2.4.3 Multifactor models

Myriad empirical studies demonstrate that, beyond exposure to systematic market risk, asset returns appear to be influenced by specific asset characteristics. For instance, Banz (1981) observes that a strategy involving the purchase of small-cap stocks and the sale of large-cap stocks yields abnormally high returns compared to CAPM predictions. Similarly, Rosenberg et al. (1985) finds that purchasing stocks with high book-to-market ratios and selling those with low ratios also leads to abnormal returns. Additionally, Jegadeesh & Titman (1993) report abnormal returns for strategies that buy assets with high returns over multiple months while selling assets with low returns over the same period. These empirical findings suggest that asset characteristics play a crucial role in understanding the distribution of asset returns.

The traditional literature refers to these cross-sectional patterns related to asset characteristics as *anomalies* as they should not exist if the CAPM fully explains asset price movements. Assuming that assets are priced rationally, with investors diversifying unsystematic risks and seeking premiums only for exposure to nondiversifiable systematic risks, the existence of anomalies indicates that the CAPM may overlook systematic risk factors, prompting the exploration of multifactor models (see, e.g., Fama & French, 1992).[13] The empirically motivated notion that further risk factors beyond the market return contribute to the distribution of asset returns can be reconciled with the asset pricing theory based on the Arbitrage Pricing Theory (APT) of Ross (1976).

One of the most prominent multifactor models in the classical asset pricing literature is Fama & French (1993)'s 3-factor model. In addition to the CAPM's market factor, the 3-factor model incorporates a size factor SML (Small Minus Big) and a value factor HML (High Minus Low). It will soon become apparent, why these factors are denoted as SML and HML. But first, note that the 3-factor model can be expressed imilarly to Equation (22) as

---

the risk factors and to the idiosyncratic return of all other assets.

[13]Alternatively, some authors propose that anomalies indicate mispricing due to behavioral biases and limits to arbitrage.

$$
\begin{aligned}
r_{i,t+1} - r_{f,t+1} = \alpha_i &+ \beta_i^{Mkt}\left(r_{Mkt,t+1} - r_{f,t+1}\right) \\
&+ \beta_i^{SMB} SMB_{t+1} + \beta_i^{HML} HML_{t+1} \\
&+ \varepsilon_{i,t+1}.
\end{aligned} \tag{23}
$$

Here, SMB_{t+1} and HML_{t+1} represent the returns of portfolios meant to mimic the returns related to size and value, respectively.[14] These mimicking portfolios are constructed using a double-sorting procedure. First, all assets are sorted into two groups based on the median market capitalization (size characteristic). Second, all assets are divided into three groups based on the 30% and 70% percentiles of the book-to-market ratio (value characteristic). Fama & French (1993) create then six value-weighted portfolios from the intersection of the two size portfolios and the three value portfolios. The portfolio meant to mimic the size-related returns is obtained by taking a long position in the three small-stock portfolios, each weighted equally, and a short position in the three large-stock portfolios, each also equally weighted. Similarly, the portfolio meant to mimic the value-related returns is obtained by taking equally weighted long positions on the two high-value stocks, and equally weighted short positions in the two low-value stocks.[15] Since the risk factors are constructed as returns of tradeable portfolios, these are also typically referred to as *factor returns*.

Following evidence that profitability and investment characteristics of assets are also related to the cross-section of asset returns and that this relationship is not reflected in Fama & French (1993)'s 3-factor model (see, e.g. Fama & French, 2006), Fama & French (2015) extend the 3-factor model to a 5-factor model

$$
\begin{aligned}
r_{i,t+1} - r_{f,t+1} = \alpha_i &+ \beta_i^{Mkt}\left(r_{Mkt,t+1} - r_{f,t+1}\right) \\
&+ \beta_i^{SMB} SMB_{t+1} + \beta_i^{HML} HML_{t+1} \\
&+ \beta_i^{RMW} RMW_{t+1} + \beta_i^{CMA} CMA_{t+1} \\
&+ \varepsilon_{i,t+1}.
\end{aligned} \tag{24}
$$

where RMW_{t+1} (Robust Minus Weak) and CMA_{t+1} (Conservative Minus Aggressive) represent the returns of portfolios meant to mimic the returns related to profitability and investment, respectively. These factor returns, constructed similarly to SMB_{t+1} and HML_{t+1}, add to the ability of the 5-factor model to capture systematic price movements.

[14] The market factor is the excess return of a value-weighted market portfolio.

[15] Note that the double-sorting procedure is done to separate the influence of the size effect from the influence of the value effect. Fama & French (1993) point out that their breakpoints are arbitrarily chosen based on empirical evidence without exploring alternatives. However, they perform plausibility tests to demonstrate the suitability of their choice.

However, since the prominent momentum effect is not captured by the 3- nor the 5-factor model (see, e.g., Carhart, 1997), Fama & French (2018) more recently extend the 5-factor model to a 6-factor model

$$
\begin{aligned}
r_{i,t+1} - r_{f,t+1} = \alpha_i &+ \beta_i^{Mkt} \left(r_{Mkt,t+1} - r_{f,t+1}\right) \\
&+ \beta_i^{SMB} SMB_{t+1} + \beta_i^{HML} HML_{t+1} \\
&+ \beta_i^{RMW} RMW_{t+1} + \beta_i^{CMA} CMA_{t+1} \\
&+ \beta_i^{UMD} UMD_{t+1} \\
&+ \varepsilon_{i,t+1}.
\end{aligned} \tag{25}
$$

where UMD_{t+1} (Up Minus Down) represents the return of a portfolio meant to mimic the returns related to momentum. In comparative analyses, Fama & French (2018) find that the 6-factor model is more suitable to describe systematic price movements than previous versions. This offers practical guidance for portfolio managers who use factor models to estimate asset return distributions for mean-variance analysis and portfolio optimization.

2.4.4 Factor investing

Over the years, empirical investigations into factor models have led to an extensive array of risk factors in asset pricing models, a phenomenon coined by Cochrane (2011) as the *factor zoo*. This proliferation of factors extends beyond enhancing our understanding of asset return distributions. Recently, it has sparked new portfolio management strategies where factors serve as building blocks for constructing portfolios instead of relying solely on individual assets, allowing for investment strategies to be analyzed in terms of risk-return properties at the factor level rather than the asset level (see, e.g., Calluzzo et al., 2019). While the primary focus of this thesis is on the development of enhanced asset-level portfolio optimization approaches, Paper (1) addresses this newer paradigm of factor-level investing, which presents a significant challenge for practical portfolio managers: how to effectively navigate the factor zoo?

Recently, scholars have began to uncover predictability in the cross-section of factor returns with the goal of discerning future winners from losers (factor selection).[16] Existing studies primarily focus on documenting individual cross-sectional predictors. One of the most prominent known cross-sectional predictors of factor returns is factor momentum, which is defined as the cumulative factor return over prolonged periods of time (Avramov et al., 2017; Zaremba & Shemer, 2018; Arnott et al., 2023). Empirical evidence shows that factors with larger momentum tend to outperform factors with low momentum. The

[16] Studies exploring systematic patterns in factors over time (factor timing) also exist. Subsubsection 3.3.2 provides some insights into the related literature.

intuition behind this observation is that factors which performed well in the recent past have a tendency to continue performing well in the immediate future, whereas factors that performed poorly tend to continue underperforming, suggesting that investors can discern winning factors from losers by their momentum. However, scholars continue to find further cross-sectional predictors for factor returns (Keloharju et al., 2016; Blitz, 2023; Mercik et al., 2023; Anginer et al., 2024), leading to the fundamental question of what predictors should guide portfolio managers in selecting factors.

In an attempt to answer this question, Paper (1) in Table 1, entitled "Picking Winners in the Factor Zoo", endeavors to contribute with a novel, comprehensive approach to forecasting the cross-section of factors. Leveraging the extensive dataset provided by T. I. Jensen et al. (2023), encompassing five decades and 153 factors, Paper (1) simultaneously examines 242 predictors, aiming to extract the pertinent predictors and gain insights into the key drivers of cross-sectional predictability in factor returns. To achieve this goal in a data-driven fashion, machine learning techniques are employed. The empirical findings of Paper (1) robustly demonstrate significant cross-sectional predictability for factor returns and that investors can harness this predictability in practical applications. Specifically, it exhibits that investment strategies that are long on factors with the highest predicted returns and short on factors with the lowest predicted returns yield in average positive returns.

A notable contribution of Paper (1) is its analysis of cross-sectional factor predictability by simultaneously exploiting multiple predictors, as opposed to prior studies that focus on forecasts derived from individual predictors. This approach allows for a comprehensive examination of the primary drivers behind cross-sectional factor predictability. For instance, the study conducts analyses of variable importance, which reveal that factor momentum emerges as the most influential predictor among the 242 forecasting variables examined.

This finding prompts a critical question: do portfolio managers truly need to incorporate a multitude of cross-sectional predictors into their investment strategies, or is it adequate to rely solely on factor momentum? To address this query, regressions are conducted comparing the returns of factor selection strategies against factor momentum strategies. The results notably reveal that factor momentum explains a significant portion of the returns observed in the factor selection strategies exploiting multiple predictors. Consequently, Paper (1) argues that factor momentum alone is sufficient to assist practical portfolio managers in identifying winning factors.

In contrast to Paper (1), which explores factor-level investment strategies, Papers (2), (3), and (4) delve into portfolio optimization approaches focused on classical asset-level investment strategies. To provide the necessary context, the next section offers a detailed description of the classical paradigms in portfolio management.

3 Portfolio management

As noted by Roll (1992), practitioners often neglect the guidance provided by mean-variance analysis regarding optimal risk-returns trade-offs because, in reality, they are confronted with a different task: optimizing the relative performance of their portfolios compared to a benchmark such as a market index.[17] An index serves as a tangible and accessible reference for portfolio performance and is often facilitated by cost-efficient Exchange-Traded Funds (ETFs). If a portfolio manager achieves a higher risk-adjusted performance than the index, the portfolio manager is praised for a good job. Conversely, if the portfolio yields a lower risk-adjusted performance than the index, questions are raised regarding the skills of the portfolio manager and whether it would be more appropriate to substitute the portfolio manager's strategy with a ETF of the index which is more simple and less expensive to implement.

While it might be natural to seek to beat a certain benchmark, this evaluation approach overlooks the possibility that such a benchmark may have suboptimal risk-adjusted performance and, hence, represent a poor reference (Dybvig & Ross, 1985). Although relative portfolio optimization can lead to contradictions of models of rational investors maximizing expected utilities, restricting feasible solutions to suboptimal portfolios, relative performance measures are the standard used in practice to determine the success or failure of portfolio management strategies.

Recognizing the need for portfolio construction guidelines that conform to the reality of investors and the financial industry, scholars have developed portfolio optimization techniques tailored to two distinct paradigms of portfolio management: passive and active. The main objective of this thesis is contributing to these two strands of the portfolio management literature by introducing novel models that outperform standard approaches proposed in prior research. Specifically, Papers (2) and (3) make contributions to the active portfolio management literature, while Paper (4) contributes to the body of work on passive portfolio management.

Before delving into the two paradigms of portfolio management in Subsections 3.2 and 3.3, respectively, Subsection 3.1 provides a brief discussion on the foundational assumptions underpinning both paradigms.

[17]Recognizing that practical portfolio management focuses on relative portfolio performance compared to a benchmark, Roll (1992) formalizes a relative mean-variance optimization as the minimization of the variance of active returns with a constraint on the expected active return, where the active return is defined as the portfolio return minus the benchmark return. Roll (1992) shows that a portfolio with optimal relative performance is not necessarily optimal in the traditional sense of Markowitz (1952).

3.1 Efficient market hypothesis

Before delving into the specifics of passive and active portfolio management, it is important to note that these two paradigms are founded on contrasting assumptions regarding the efficiency of capital markets. The efficient market hypothesis (EMH) essentially states that market prices *fully reflect all available information.* The EMH plays a profound role in portfolio management as it implies, in its strongest form, that asset prices react only to new information and that prices changes, hence, cannot be consistently predicted a priori. Under this assumption, it is not possible to use current information to discern which assets will exhibit superior performance in the future or when will be the best point in time to execute specific trades. In particular, this means that no investment strategy derived from current information can consistently yield a better risk-adjusted performance than a given benchmark.

Although the efficiency of markets has been a subject of study dating back at least to Bachelier (1900), it was not until approximately 55 years ago that testable predictions were formerly introduced by the influential work of Fama (1970) where a more concrete definition of the phrase *"fully reflect all available information"* is presented. Specifically, Fama (1970) introduces three forms of tests for the EMH examining whether different information subsets are reflected in the market prices: (1) *weak form* tests in which the information set consists only of historical prices, (2) *semi-strong form* tests in which the information set contains, in addition to the historical prices, all publicly available data on the assets (e.g., earning reports, etc.), and (3) *strong form* tests, which are the most comprehensive and consider not only all historical and publicly available information, but also private information (e.g., insider information).[18]

Despite the considerable influence of the EMH on portfolio managers' decision-making, the efficiency of capital markets remains a topic of debate. Nevertheless, portfolio managers are required to take a stance on this matter before selecting a specific strategy. In the scenario where the EMH is believed to be invalid, portfolio managers are assumed to be potentially able of exploiting relevant information, not factored into asset prices, to gain an advantage over other market participants unaware of such information. This forms the foundation of active portfolio management, which aims to outperform a predetermined benchmark by identifying mispriced assets. In contrast, passive portfolio management operates on the assumption that the EMH is valid, asserting the absence of mispricing possibilities. As in this case it is assumed that portfolio managers cannot consistently outperform the benchmark, this strategy seeks to closely replicate the benchmark returns

[18]If markets are efficient in the weak form, utilizing historical trading data, as seen in technical analysis, does not provide individual investors with a reliable means to consistently outperform the market portfolio. In the semi-strong form of market efficiency, fundamental analysis also proves ineffective for outperforming the market. Finally, in the strong form, even insider information fails to confer an advantage over the market.

with minimal costs.

The assessment of the validity of the EMH falls outside the scope of this thesis. In the context of my cumulative dissertation, I investigate portfolio optimization methodologies for both the active and passive paradigms of portfolio management and develop novel approaches that achieve superior performance compared to methods proposed in previous literature.

3.2 Passive portfolio management

Under the assumption that markets are efficient, there is no possibility for portfolio managers to earn abnormally high returns relative to their benchmarks on a consistent basis because the asset prices already reflect every possible information that investors might want to exploit. In such an environment, actively searching for mispriced assets in an attempt to outperform the benchmark is a futile undertaking. A more logical approach is to adopt a passive portfolio management strategy aimed at tracking the benchmark with a focus on minimizing costs. For illustrative purposes, throughout the current subsection, arguments will be formulated in terms of tracking a target index (e.g., S&P 500), using *index tracking* as a special case of general tracking applications.

The direct approach to replicating a target index involves employing a full replication strategy, where the same assets are held with identical portfolio weights as those in the index. While this ensures precise tracking of the target index, practical portfolio management often finds this strategy impractical due to potentially substantial transaction costs. Moreover, factors such as customer preferences, regulatory requirements, and statutory constraints may render a full replication strategy untenable. Consequently, passive portfolio managers opt for a more selective approach, choosing a smaller subset of assets compared to the constituents of the target index (see, e.g., Beasley et al., 2003). This selective strategy results in what is known as a *tracking portfolio*, which, by its nature, deviates from the exact composition of the index. As a consequence, differences arise in their returns, leading to what is termed as a *tracking error*. Passive portfolio managers endeavor to minimize this tracking error while simultaneously adhering to constraints on the size of the tracking portfolio. This balancing act is crucial as the size of the tracking portfolio significantly impacts the costs associated with implementing and monitoring the portfolio strategy. The optimization of these two concurrent objectives—minimizing tracking error while constraining the number of assets in the portfolio—determines the methodologies and techniques applicable in passive portfolio management.

First, note that the quantification of tracking error in the literature is not unique. For example, Rudd (1980) and Roll (1992), in the spirit of Markowitz (1952), posit that an optimal tracking portfolio is one that minimizes the variance of active returns subject to the constraint that the expected active return must be zero. Instead of minimizing the variance of active returns, Beasley et al. (2003) minimize the mean squared error of returns of the tracking portfolio with respect to the target returns.[19] The minimization of the mean absolute error can be similarly applied (see, e.g., Rudolf et al., 1999). To specifically reduce the one-sided instead of the two-sided risk of the tracking portfolio relative to the benchmark, Gaivoronski et al. (2005) investigates the minimization of downside risk measures applied to the active returns, such as value-at-risk and conditional value-at-risk. In summary, the literature mostly defines tracking error using risk measures on the active returns, but the choice of the specific risk measure rests on the hands of the portfolio manager.

Second, notice that passive portfolio managers must address tracking error minimization while constraining the number of assets in their tracking portfolios, known as the *cardinality constraint*.[20] A significant strand of the literature investigates heuristic algorithms for asset selection in tracking portfolios subject to cardinality constraints (see, e.g., Rudd, 1980; Haugen & Baker, 1990; Beasley et al., 2003; Derigs & Nickel, 2003; Corielli & Marcellino, 2006; Krink et al., 2009; Guastaroba & Speranza, 2012). While these heuristics permit the use of nonlinear objective functions and constraints, they often yield suboptimal solutions. In contrast, a more recent and expanding body of research has turned to mixed-integer optimization as a more precise framework for constructing fixed-size portfolios that minimize tracking errors optimally (see, e.g., Canakgoz & Beasley, 2009; C. Chen & Kwon, 2012; Kwon & Wu, 2017; Strub & Baumann, 2018). Mixed-integer optimization offers considerable appeal for constructing tracking portfolios since it ensures satisfaction of optimality conditions, unlike heuristic approaches producing suboptimal results. Furthermore, commercially available mixed-integer optimizers offer a more accessible solution for practical portfolio management, compared to specialized heuristics that can be more complex to implement.

The characteristic-based portfolio index tracking framework developed and demonstrated in Paper (4) contributes significantly to the ongoing research of optimal tracking portfolios incorporating cardinality constraints through mixed-integer optimization. The methodology introduced in Paper (4) and its contributions are detailed in Subsection 4.5.

3.3 Active portfolio management

Contrary to the philosophy of passive portfolio management, active portfolio management operates under the assumption that markets are not efficient, potentially displaying asset mispricing reflected in available financial data. Accordingly, an active portfolio manager aims to construct a portfolio, which differs from the benchmark portfolio, seeking superior

[19] While Beasley et al. (2003) define tracking error in general with an exponent of $\alpha > 0$, they restrict the empirical demonstration of their methodology to the quadratic case with $\alpha = 2$.

[20] In his mean-variance analysis of tracking error, Roll (1992) does not incorporate a cardinality constraint, much like the original mean-variance framework by Markowitz (1952).

risk-adjusted performance compared to the benchmark, while ensuring that the costs of actively managing such a portfolio strategy do not surpass the potential gains.

As elaborated by Grinold & Kahn (2000), investors have two potential avenues for achieving this goal. The first approach involves selecting assets expected to exhibit strong future performance (e.g., buying future winners and selling future losers), which relates to the cross-section of asset returns and is termed *selection*. The second approach, known as *timing*, revolves around anticipating optimal times to trade specific assets (e.g., buying low and selling high) based on the time-series of asset returns.

Grinold & Kahn (2000)'s fundamental law of active management quantifies that selection strategies are typically profitable because their performance can be enhanced by adjusting the number of assets, providing ample opportunities for optimization given the large cross-section of assets. In contrast, successful timing strategies are more challenging as their performance relies on manipulating trading frequency, which is inherently more difficult to predict and control. Addressing this challenge, Papers (2) and (3) add to the active portfolio management literature by leveraging time-series predictability to enhance the performance of state-of-the-art selection strategies. To set the stage for discussing the contributions of Papers (2) and (3), the next two subsubsections delve into the fundamental concepts of selection and timing, along with a review of related literature.

3.3.1 Selection

Effectively selecting assets by exploiting mispricing in inefficient markets requires predicting the cross-section of asset returns. Modern portfolio theory emphasizes that is not sufficient to simply determine which assets will yield higher returns. Rather, active portfolio managers must search for assets offering higher risk-adjusted performance. Existing literature prominently advocates using factor models to assess asset mispricing.

Assuming that the CAPM truly captures the mechanism of how asset prices are produced, M. C. Jensen (1968) states that the difference between the realized portfolio returns and the returns expected due to exposure to the systematic market risk (see Equations (21) and (22)) reflect the selection skills of portfolio managers in inefficient markets. Making use of ordinary least square estimation, M. C. Jensen (1968) proposes to estimate the linear regression model

$$r_{P,t+1} - r_{f,t+1} = \alpha_P + \beta_P(r_{Mkt,t+1} - r_{f,t+1}) + \varepsilon_{P,t+1} \tag{26}$$

of the excess portfolio returns $r_{P,t+1} - r_{f,t+1}$ on the excess market returns $r_{Mkt,t+1} - r_{f,t+1}$ to statistically evaluate selection performance. The coefficient β_P, which represents the exposure of the portfolio to the systematic market risk and is assumed constant over time,[21] determines the expected return of the portfolio based on the market return. Since

[21]As argued by M. C. Jensen (1968), a time-invariant portfolio beta is unrealistic as portfolio man-

the random error-term $\varepsilon_{P,t+1}$ is assumed to have a mean of zero, if portfolio returns differ from the values suggested by the CAPM, it is captured by the intercept α_P. If the portfolio return is consistently higher (lower) than predicted by the CAPM in the sample, a statistically significant positive (negative) alpha must be observed. Otherwise, the alpha coefficient will not be statistically distinguishable from zero.

However, in reality, active portfolio managers are evaluated relative to benchmark portfolios rather than to a theoretical market factor. Since the concept of Jensen's alpha is specifically developed to compare a portfolio against the CAPM's market factor, this measure of selection performance must be adapted to account for the actual benchmarks in practical scenarios. Substituting the market return $r_{Mkt,t+1}$ in Equation (26) with the return $r_{B,t+1}$ of the benchmark, one similarly obtains

$$r_{P,t+1} - r_{f,t+1} = \alpha_P + \beta_P(r_{B,t+1} - r_{f,t+1}) + \varepsilon_{P,t+1}. \tag{27}$$

The coefficient α_P in Equation (27) reflects whether the portfolio is equivalent to simply holding a scaled version of the benchmark (i.e., $\alpha_P = 0$) or if the portfolio truly exhibits a return component that is uncorrelated to the benchmark (i.e., $\alpha_P \neq 0$), and hence, a result achieved by the selection strategy of the portfolio manager.

The approach of Roll (1992), discussed in Section 3.2 in the context of index tracking, can be also employed in active portfolio management to generate positive alphas with respect to benchmark portfolios by minimizing the variance of active returns subject to a positive rather than a zero active return. Nevertheless, this approach possesses two major shortcomings. First, it is a mean-variance technique with the same pitfalls as Markowitz (1952)'s optimization, in particular, regarding the challenge of estimating the distribution of asset returns. Second, it neglects the advanced knowledge from the literature regarding cross-sectional effects related to the asset characteristics which can guide investors in selecting the best assets. A formal portfolio optimization approach that does not require estimations of the return distribution and exploits information provided by asset characteristics to outperform a predefined benchmark is given by the parametric portfolio policies introduced by Brandt et al. (2009). A detailed description of Brandt et al. (2009)'s methodology is provided in Subsection 4.2. Papers (2) and (3) build directly upon this optimal asset selection framework and extend it by incorporating timing, as discussed in 4.3 and 4.4, respectively.

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agers constantly change the composition of their portfolios. While the approach of M. C. Jensen (1968) straightforwardly applies to stationary betas, it can also be extended to incorporate nonstationary betas, which the author proposes as a market timing test.

3.3.2 Timing

While selection is concerned with the cross-section of asset returns, timing is related to their time-series. The related literature focuses on predictability in the time-series of aggregated returns within asset classes (e.g., bonds or stocks), rather than attempting to forecast the returns of individual assets. Predicting aggregated returns of a certain asset class exploits the tendency for prices of all assets in that class to move together,[22] and is commonly referred to as *market timing*.

Treynor & Mazuy (1966) argue that, if portfolio managers are able to correctly predict at least the direction of the benchmark returns (i.e., whether these will be positive or negative) and invest accordingly, their portfolios are characterized by elevated returns relative to the benchmark in times of positive benchmark returns, and lower return magnitudes when negative benchmark returns are realized. The principle of Treynor & Mazuy (1966) can be captured in an statistical test by adding a quadratic term to Equation (27) in the form of

$$r_{P,t+1} - r_{f,t+1} = \alpha_P + \beta_P(r_{B,t+1} - r_{f,t+1}) + \gamma_P(r_{B,t+1} - r_{f,t+1})^2 + \varepsilon_{P,t+1}, \qquad (28)$$

where a statistically significant positive γ_P reflects a successful timing, and alpha is attributed to the portfolio manager's selection strategy as in Equation (27).[23]

A considerable amount of research studies advocate for market timing predictability through the exploitation of time-series predictors including inflation rates (see, e.g., Nelson, 1976; Fama & Schwert, 1977; Pesaran & Timmermann, 1994; Campbell & Vuolteenaho, 2004), default spreads (see, e.g., Fama & French, 1989; N.-F. Chen, 1991), term spreads (see, e.g., Fama & French, 1989; N.-F. Chen, 1991), dividend yields (see, e.g., Ball, 1978; Rozeff, 1984; Campbell & Shiller, 1988; Fama & French, 1989; Pesaran & Timmermann, 1994; Kothari & Shanken, 1997), book-to-market ratios (see, e.g., Kothari & Shanken, 1997; Pontiff & Schall, 1998; Baker & Wurgler, 2002) and volatility (see, e.g., French et al., 1987; Guo, 2006), among many others.

In recent decades, researchers have focused on clarifying whether the traditional evidence for the predictability of the aggregated market return, primarily obtained from in-sample regression models, can be translated into actionable investment strategies for practical portfolio management. Challenging the inferences from in-sample analyses, Welch & Goyal (2008) comprehensively revise the prediction models in out-of-sample tests. Concretely, Welch & Goyal (2008) compare out-of-sample market return forecasts from ordinary least square estimations of regressing annual excess market returns on different predictors against a naive forecast simply consisting of the historical market return.

[22]This is a widely accepted premise in capital markets, particularly reflected in the CAPM.

[23]Alternative statistical market timing tests are proposed, e.g., by M. C. Jensen (1968) and Merton (1981).

As they observe that the historical average displays superior forecast performance in their study, the authors argue that time-series predictors lack the necessary predictive power for real-world applications. However, Campbell & Thompson (2008) reveal that constraining the sign of the coefficients and the sign of the expected return in the regression models of Welch & Goyal (2008) to signs suggested by investment theory enables the predictive regressions to beat the historical average. Rapach et al. (2010) additionally find that combining the individual forecast models in the methodology of Welch & Goyal (2008), which reduces the uncertainty associated with the individual models (Bates & Granger, 1969) much like diversification across assets reduces the variance of portfolio returns, also leads to statistically and economically significant gains relative to historical average forecasts. Moreover, addressing the issue that such regression-based approaches are ill-suited for time-series predictions when the number of predictors nears or even exceeds the number of observations, which becomes necessary as the number of predictors suggested by the literature keeps growing,[24] Gu et al. (2020) apply machine learning techniques and obtain superior market timing forecasts than with linear regression methods.

The studies referenced above primarily focus on prediction models and employ simplistic investment strategies to evaluate the economic significance of their forecasts. These approaches do not directly address the needs of active portfolio managers seeking to optimize their portfolios by exploiting market timing predictability. Traditional mean-variance optimization theory formulates portfolio optimization as a single-period problem, modeling each asset with the help of a single random variable representing its return. In contrast, market timing involves a multi-period framework and requires modeling asset returns as stochastic processes.[25] To tackle this challenge formally, the research literature draws upon the theory of dynamic programming (see, e.g., Bellman, 1954), which is applied to the optimization of multi-stage decision processes, referred to as *policies*.[26]

Early studies in dynamic portfolio optimization, such as those by Merton (1969) and Samuelson (1975), assume that returns are independent over time, characterizing optimal portfolio policies in the absence of market timing, a tradition still followed in later research (see, e.g., Li & Ng, 2000; Leippold et al., 2004). Recognizing the empirical evidence supporting market timing predictability, Campbell & Viceira (1999) develop a dynamic portfolio optimization approach that incorporates the stochastic return process as a function of an autoregressive state variable of order one.[27] Nevertheless, this approach

[24]For example, Kelly & Pruitt (2013) finds that the cross-section of book-to-market ratios predicts aggregated market returns.

[25]A stochastic process is a series of various random variables.

[26]In contrast to the more classical approach of finding the optimal policy over the space of all possible decision sequences, which very often proves impractical, dynamic programming determines the optimal policy by deriving the necessary optimal conditions for any stage of the process in terms of the current state of the system in a recursive fashion.

[27]In a similar spirit but without applying dynamic programming, Ferson & Siegel (2001) analyze

accounts only for a special case of possible market timing predictors. Other approaches to dynamic portfolio optimization that account for market timing potential exist (see, e.g., Çelikyurt & Özekici, 2007; Basak & Chabakauri, 2010), but they involve complex numerical techniques that are often out of reach for practical portfolio managers.

To address the optimal dynamic portfolio problem while ensuring computational feasibility, Brandt & Santa-Clara (2006) propose an approximation solution with a complexity comparable to Markowitz (1952)'s mean-variance analysis. Specifically, they expand the asset universe to include naively managed portfolios following market state variables and determine the classical static mean-variance optimum within the augmented asset universe. Each naive portfolio invests in a risky asset for one period and in the riskless asset for all other periods. Any dynamic portfolio policy can be constructed as a combination of these mechanically managed portfolios, albeit representing an approximation that overlooks compounding effects that result, for example, from holding one asset for multiple periods. The authors argue that compounded returns are significantly smaller than the returns of the managed portfolios, making this approximation relatively accurate, although they do not examine more in detail the loss associated with this approximation. Brandt & Santa-Clara (2006) suggest finding first the coefficients that optimize the parametric policy and recovering then the weights invested in the individual assets from this parametric function.[28]

Despite its advancement towards more tractable techniques for optimal market timing portfolios, the approach of Brandt & Santa-Clara (2006) exhibits various shortcomings. First, as discussed in detail by the authors, their approach relies on data-intensive samples for long horizons. Second, it incorporates a different coefficient for each individual asset, or in other words, each individual asset has a distinct parametric function, aiming at timing individual assets rather than timing the market, as opposed to the predominant literature which focuses on predicting aggregated market returns. Third, Brandt & Santa-Clara (2006) neglect the effect of compounded returns on market timing decisions based on the argument that considering only one-period returns approximates the optimal portfolio policy. However, it is not clear how good this approximation is. Fourth, the authors develop their approach so that it exploits either cross-sectional or time-series predictors but not both at the same time. Thus, practical portfolio managers seeking to employ both selection and timing cannot employ this technique. The market timing approach developed in Paper (2) overcomes all these shortcomings. A detailed description of Paper (2) is provided in Subsection 4.3.

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the static mean-variance optimization problem with conditioning information by modeling conditional expected returns, conditional variances and conditional covariances as functions of some variables.

[28] Unlike Campbell & Viceira (1999), who parameterize the moments of the distribution of asset returns, Brandt & Santa-Clara (2006) parameterize portfolio weights based on state variables, following Aït-Sahalia & Brandt (2001)'s recommendation.

Before concluding this section, it is important to highlight that in addition to market timing (time-series predictability in aggregated market returns), there is also a dedicated literature strand focused on factor timing (time-series predictability in factor returns). A notable example is the phenomenon of momentum crashes (Daniel & Moskowitz, 2016). While the momentum factor exhibits a strong positive average return over time, it is susceptible to large negative returns during market declines. However, Barroso & Santa-Clara (2015b) find that the variance of daily returns of the momentum factor can be predicted, enabling investors to manage the time-varying risks associated with the momentum factor.

Since factors often represent specific investment strategies that exploit asset characteristics (see Subsubsection 2.4.3), factor timing can be viewed as a specialized form of time-series predictability within characteristic-based investment strategies. Throughout this thesis, the broader category of time-series predictability related to the relationship between asset returns and characteristics is referred to as *characteristic timing*. Pioneering works by Asness et al. (2000) and Cohen et al. (2003) laid the foundation of characteristic timing by demonstrating the predictable time variation in the returns of value strategies. Specifically, these studies show that the value spread, defined as the difference of the value characteristic between a diversified portfolio with value stocks and a diversified portfolio with growth stocks, acts as a predictor of the returns of a strategy buying value stocks and selling growth stocks. This strategy is known for its positive average return and yields disproportionately higher returns when the value spread is notably large, suggesting that gains from a value-based strategy intensify when undervalued (overvalued) assets are cheaper (more expensive) than usual. Building on this, Baba Yara et al. (2021) unveil that value spreads not only forecast value returns in stocks but also across industries, commodities, currencies, global government bonds, and global stock indexes. Furthermore, an expanding body of research continues to uncover predictable time variation in investment strategies associated with other characteristics as well (see, e.g., Lewellen, 2002; Greenwood & Hanson, 2012; Kelly & Pruitt, 2013; Barroso & Santa-Clara, 2015b; Daniel & Moskowitz, 2016; Moreira & Muir, 2017; Zaremba & Shemer, 2018; Gu et al., 2020; Haddad et al., 2020; Ehsani & Linnainmaa, 2022; Huang, 2022; Anginer et al., 2024). In Paper (3), I endeavor to address the question of how active portfolio managers, using the parametric portfolio policies proposed by Brandt et al. (2009), can integrate characteristic timing into their portfolio strategies. A detailed description of Paper (3) is provided in Subsection 4.4.

4 Characteristic-based portfolio optimization

Even sophisticated extensions of traditional portfolio optimization approaches face challenges due to their sensitivity to inaccuracies in estimating asset return distributions. For instance, using factor models to estimate inputs for mean-variance analysis, as discussed in Subsubsection 2.4.1, remains susceptible to estimation errors because the parameters of factor models must be estimated from asset characteristics.

To address these issues and develop more robust portfolio optimization strategies, some research studies advocate for one-step portfolio construction approaches. The related literature is discussed in Subsection 4.1. The main idea of such one-step approaches is to directly model optimal portfolio choice as a function of asset characteristics, unlike traditional two-step methods that use characteristics to estimate asset return distributions and then determine optimal portfolios based on imperfect estimations. A prominent example is Brandt et al. (2009)'s framework of parametric portfolio policies. This methodology is described in detail in Subsection 4.2.

Recognizing the significance of one-step portfolio construction strategies, the present thesis aims to advance the work on characteristic-based portfolio optimization approaches.[29] The primary objective is to devise enhanced portfolio strategies compared to existing methods. Subsections 4.3 and 4.4 describe how Papers (2) and (3) contribute to extending Brandt et al. (2009)'s framework by incorporating market timing and characteristic timing, respectively. Subsection 4.5 discusses the contributions of the mixed-integer, characteristic-based tracking portfolio optimization approach developed in Paper (4) to the field of passive portfolio management.

4.1 Relationship between optimal portfolios and characteristics

The traditional approach for portfolio optimization in the literature uses asset characteristics as predictors of the distribution of asset returns. This approach aims to enhance the accuracy of input estimations for mean-variance optimization or more general expected utility maximization compared to using sample estimates directly computed from realized returns (see Subsubsection 2.4.1). While this enhances the first stage of the two-step portfolio construction procedure outlined by Markowitz (1952), challenges arise due to potential misspecification in the underlying models linking the distribution of asset returns to the asset characteristics and other types of forecasting variables.

To investigate the impact of such misspecification, Brandt (1999) adopts a nonparametric approach, analyzing the relationship between optimal portfolios and macroeconomic predictors associated with time variation in aggregated market returns. In the context of constant relative risk aversion (CRRA) utility and focusing on four timing predictors, the author finds a significant relationship between optimal multiperiod portfolio choice and these forecasting variables. This implies that it is possible to directly model optimal portfolio weights as a function of predictors linked to the asset returns, bypassing

[29] The only exception is Paper (1) which studies the problem of factor selection as discussed in Subsubsection 2.4.1.

the inherent challenges of estimating the distribution of asset returns.

Since the study of Brandt (1999) applies nonparametric techniques, a question naturally emerges: how can portfolio managers directly exploit forecasting variables for portfolio formation? It's worth noting that different moments of the return distribution are associated with distinct forecasting variables. For instance, a predictor could anticipate both increasing expected returns and volatility. In such a case, deriving actionable insights from the forecasting variable might not always be straightforward if it is not clear whether the objectives of the investor can be met using this information.

A method that endogenously selects only the predictors that are relevant for the optimization of the investor's objective function is proposed by Aït-Sahalia & Brandt (2001). Their study, however, does not delve into how portfolio managers can concretely compute the portfolio weights.

First parametric models offering concrete guidance on how to derive optimal portfolio weights from forecasting variables are given by the two companion papers Brandt & Santa-Clara (2006) and Brandt et al. (2009). As described already in Subsubsection 3.3.2, Brandt & Santa-Clara (2006) models the weight of each distinct asset as a different parametric function of common timing predictors. In contrast, Brandt et al. (2009) models the weights of all assets as one single parametric function of asset characteristics.[30] Since Brandt et al. (2009)'s approach offers several significant advantages, it is discussed in more depth in the next subsection.

4.2 Parametric portfolio policies

In their seminal work, Brandt et al. (2009) delve into the problem of intertemporal expected utility maximization

$$\max_{w_{1,t},\dots,w_{N_t,t}} \mathbb{E}_t \left[u\left(\sum_{i=1}^{N_t} w_{i,t} r_{i,t+1} \right) \right], \tag{29}$$

where $\mathbb{E}_t$ represents the conditional expected value given information available at time t, N_t the number of investable assets at time t, $r_{i,t+1}$ the return of asset i from t to $t+1$, and $w_{i,t}$ the weight of asset i at time t. Equation (29) essentially portrays the investor's aim at maximizing the conditional expected utility of the portfolio's return across different periods t. The solution to this intertemporal expected utility maximization problem is termed the *optimal portfolio policy.*

To determine the optimal portfolio policy based on information derived from characteristics associated with the distribution of asset returns, Brandt et al. (2009) propose

[30] While these techniques offer explicit formulas for computing portfolio weights based on forecasting variables, they require that portfolio managers exogenously choose suitable predictors. DeMiguel et al. (2020) introduces a *screen-and-clean* method to select only significant characteristics in the framework of Brandt et al. (2009).

parameterizing the portfolio weights $w_{i,t}$ as a function f_θ of asset characteristics $x_{i,t}$, featuring time-invariant parameters θ. This parametric formulation can be generally expressed as[31]

$$w_{i,t} = f_\theta(x_{i,t}). \tag{30}$$

Given investors' inclination to exploit multiple characteristics, $x_{i,t}$ may constitute a K-dimensional vector with $K \geq 1$. Employing this parametric approach transforms the optimization program into[32]

$$\max_\theta \ \mathbb{E}_t \left[u \left(\sum_{i=1}^{N_t} f_\theta(x_{i,t}) r_{i,t+1} \right) \right], \tag{31}$$

where the objective is to maximize the conditional expected utility conditioned over the parameters θ. This solution yields the *optimal parametric portfolio policy* in the subset of all portfolio policies subject to the assumed parametric form.

Note that by assuming the parameters θ to be time-invariant, Brandt et al. (2009) conjecture that the relationship between optimal portfolio weights and characteristics remains constant over time. The rationale behind this modeling assumption is rather methodological than economical, since it implies that the parameters θ that maximize the conditional expected utility correspond to the parameters that maximize the unconditional expected utility, thereby simplifying Equation (31) to

$$\max_\theta \ \mathbb{E} \left[u \left(\sum_{i=1}^{N_t} f_\theta(x_{i,t}) r_{i,t+1} \right) \right] \tag{32}$$

which allows the optimal parameters to be estimated by solving

$$\max_\theta \ \frac{1}{T} \sum_{t=1}^{T} u \left(\sum_{i=1}^{N_t} f_\theta(x_{i,t}) r_{i,t+1} \right) \tag{33}$$

over a sample of size T. Brandt et al. (2009) demonstrate their approach primarily using the special case of a linear parametric function

$$f_\theta(x_{i,t}) = w_{i,t}^B + \frac{1}{N_t} \theta^T \hat{x}_{i,t}, \tag{34}$$

which has since become the established parameterization adopted in contemporary literature.[33] Here, $w_{i,t}^B$ denotes the weight of asset i in a benchmark portfolio adhering

[31] In contrast to Brandt & Santa-Clara (2006), who parameterize each individual asset i using a different function f_i, Brandt et al. (2009) parameterizes all N_t assets using the same function f.

[32] While Equation (29) is an optimization program over portfolio weights $w_{i,t}$, Equation (31) is an optimization program over the parameters θ.

[33] This parametric function yields long-short portfolios. Long-only portfolios can be obtained via truncation.

to the budget constraint $\sum_{i=1}^{N_t} w_{i,t}^B = 1$, $\hat{x}_{i,t}$ represents the cross-sectionally standardized version of $x_{i,t}$,[34] and θ is a K-dimensional vector comprising scaling factors. This linear parametric function models portfolio weights which deviate from the benchmark weights proportionally to the asset characteristics by choosing only the scaling factors θ, i.e., by choosing only the common deviation intensity among assets, whereas the direction and relative deviation of each particular asset is given by its characteristics.[35] Substituting Equation (34) in Equation (33) gives

$$\max_{\theta} \frac{1}{T}\sum_{t=1}^{T} u\left(\sum_{i=1}^{N_t} w_{i,t}^B r_{i,t+1} + \frac{1}{N_t}\theta^T \sum_{i=1}^{N_t} r_{i,t+1}\hat{x}_{i,t}\right). \tag{35}$$

Equation (35) delineates a relative portfolio optimization program for active portfolio management, aiming to identify scaling factors θ that yield a portfolio with higher estimated expected utility relative to the benchmark, under a linear relationship between portfolio weights and characteristics. In cases where no linear relationship exists for the chosen characteristics, or if the benchmark portfolio cannot be outperformed by some parametric portfolio, the scaling factors θ are left at zero, suggesting that the optimal portfolio policy consists of maintaining the benchmark allocation. Thus, one notable advantage of this relative portfolio optimization framework is its assurance that the portfolio manager cannot perform worse than the benchmark, assuming that the sample average utility effectively estimates the expected utility. Empirical studies consistently corroborate the effectiveness of this framework in facilitating benchmark outperformance across various markets and asset classes (Brandt et al., 2009; Plazzi et al., 2011; Hand & Green, 2011; Hjalmarsson & Manchev, 2012; Barroso & Santa-Clara, 2015a; Ammann et al., 2016; Fieberg et al., 2016; Fletcher, 2017; Dichtl et al., 2019; Z. Chen & Fei, 2021; Caldeira et al., 2023).

This approach presents several compelling benefits from both theoretical and practical standpoints. First, it is formulated as a one-step portfolio optimization framework that directly estimates optimal portfolio weights from asset characteristics, in contrast to the traditional two-step procedure in modern portfolio theory, which involves using characteristics to estimate the distribution of asset returns and optimizing the portfolio based on these error-prone estimations. As a result, parametric portfolio policies escape the error-maximization property that affects mean-variance optimization. Second, the parametric framework significantly reduces the number of variables requiring estimation. While mean-variance optimization necessitates estimating N_t means, N_t variances, and $N_t(N_t - 1)/2$ covariances at each time t, parametric portfolio policies entail estimating only one set of optimal parameters θ, substantially reducing the vulnerability against

[34]Thus, it holds $\sum_{i=1}^{N_t} \hat{x}_{i,t} = 0$ and $1/N_t \sum_{i=1}^{N_t} (\hat{x}_{i,t})^2 = 1$

[35]Dividing by the number N_t of assets is important to ensure that the overall portfolio investment depends only on the asset characteristics but not on the number of assets.

estimation errors.[36] Third, using the linear parametric function reduces the dimension from N_t weights in Equation (29) to $K \ll N_t$ in Equation (35), allowing for the efficient consideration of a virtually arbitrary number of assets in the optimization program. Fourth, with less optimization variables compared to the mean-variance approach there is also less potential for in-sample overfitting. Fifth, parametric portfolio policies yield less extreme weightings than the mean-variance approach as the optimal portfolio weights are constrained to adhere to a predetermined parametric form rather than being chosen arbitrarily. Sixth, this approach can easily incorporates the effect of transaction costs on the optimal portfolio policy as suggested by Brandt et al. (2009).

Despite these fundamental advantages, the original approach proposed by Brandt et al. (2009) presents two notable inconsistencies with the extensive empirical evidence in the literature regarding the predictability of asset returns. First, the framework of parametric portfolio policies, as initially conceived, focuses exclusively on exploiting predictability in the cross-section of asset returns, overlooking predictability in the time-series of aggregated market returns. In essence, parametric portfolio policies encompass selection but not timing, despite robust evidence of market timing predictability in the literature (see Subsubsection 3.3.2). Second, the linear portfolio policies established by Brandt et al. (2009) and subsequently adopted in later studies primarily assume time-invariant parameters θ.[37] Although this simplifies the estimation of the optimal portfolio policy, it overlooks documented time variations in the relationship between asset returns and characteristics, potentially resulting in performance loss.

In the context of my cumulative dissertation, I contribute to this strand of the literature by integrating market timing in Paper (2) and characteristic timing in Paper (3) into the methodological framework of parametric portfolio policies (see Subsubsection 3.3.2 for a discussion on the differences between market timing and characteristic timing). These contributions are elaborated upon in the next two subsections, respectively.

4.3 Market timing in parametric portfolio policies

This subsection outlines the contributions of Paper (2) in Table 1, titled "Market Timing in Parametric Portfolio Policies". The paper makes two fundamental contributions. Firstly, it introduces a novel parametric portfolio optimization approach for market timing, departing from Brandt & Santa-Clara (2006), who advocate timing individual assets rather than the aggregated market. Secondly, it extends the mechanism proposed by Brandt et al. (2009) for selection by incorporating simultaneous market timing.

[36]Even if factor models are used for estimating the distribution of asset returns, parametric portfolio policies are still more parsimonious.

[37]Brandt et al. (2009) additionally demonstrate a simple but unsuitable way for modeling time-varying parameters.

At its core, the extension proposed in Paper (2) unfolds in two stages. Initially, it introduces a risk-free asset with a weight expressed as a linear function of an M-dimensional vector z_t comprising market timing predictors, written as

$$w_{f,t} = \varphi \cdot \begin{pmatrix} 1 \\ z_t \end{pmatrix} = \varphi_0 + \sum_{m=1}^{M} \varphi_m z_{t,m} \tag{36}$$

where $\varphi = (\varphi_0, \varphi_1, \ldots, \varphi_M)^T$ is an $(M+1)$-dimensional parameter vector. Positive products $\varphi_m z_{t,m}$ lead to an increase of $w_{f,t}$ while negative products decrease it. Scaling factors φ_m ensure comparability among the magnitudes of the different market timing predictors, while the threshold parameter φ_0 facilitates the risk-free position to oscillate between negative (short) and positive (long) positions depending on the forecasts of the market state.

Subsequently, the initial variable $w_{f,t}$ is integrated into a portfolio already containing N_t weights, as defined by Equation (34). As the risk-free position in Equation (36) is generally nonzero, all weights (both risky assets and the risk-free position) must be renormalized to adhere to the budget constraint. This results in

$$w_{f,t}^{\text{ext}} = \frac{w_{f,t}}{1 + w_{f,t}} \tag{37}$$

and

$$w_{i,t}^{\text{ext}} = \frac{w_{i,t}}{1 + w_{f,t}}, \tag{38}$$

which represent a straightforward yet potent extension of Brandt et al. (2009)'s selection approach to incorporate market timing in a nonlinear manner. When the risk-free asset's weight is designated as zero (i.e., $w_{f,t} = 0$), the weights of the risky assets remain unchanged relative to Brandt et al. (2009)'s original parametric portfolio policy (i.e., $w_{i,t}^{\text{ext}} = w_{i,t}$). Opting for $w_{f,t} > 0$ uniformly diminishes the magnitude of the weights $w_{i,t}$ for all risky assets, beneficial during a bear market phase to mitigate market risk. As $w_{f,t} \to \infty$, the risk-free weight $w_{f,t}^{\text{ext}}$ approaches 1 while all risky weights $w_{i,t}^{\text{ext}}$ converge to 0, indicating the portfolio's exclusive investment in the risk-free asset. When $-1 < w_{f,t} < 0$, the denominator ranges between 0 and 1 and the risk-free weight $w_{f,t}^{\text{ext}}$ becomes negative. Here, the portfolio strategy uses a short position on the risk-free asset to uniformly leverage all risky assets, indicating anticipation of a bullish market and a desire to boost exposure to risky assets. Note that $w_{f,t}$ cannot equal -1 due to division by zero. Values $w_{f,t} < -1$ are feasible but should be excluded because they lead to the rather unrealistic scenario where the risk-free position is leveraged by a short position on the parametric portfolio policy.[38]

[38] In such instances, the denominator is negative and the weights $w_{i,t}^{\text{ext}}$ of the extended approach oppose

It is important to note that the approach proposed in Paper (2) encompasses the special case of pure market timing, where the portfolio manager decides only between holding the benchmark or the risk-free asset, akin to when the portfolio manager sets the parameters θ to zero (i.e., $w_{i,t} = w_{i,t}^B$). Notably, the market timing approach of Paper (2) diverges from that of Brandt & Santa-Clara (2006), as discussed in Subsubsection 3.3.2. The market timing approach of Paper (2) boasts multiple advantages over Brandt & Santa-Clara (2006)'s methodology. First, it simplifies complexity by formulating portfolio choice as an investment on the benchmark portfolio, requiring a single parametric function for all assets, unlike Brandt & Santa-Clara (2006), who employ a different parametric function for each individual asset, capturing timing of individual assets rather than aggregated market timing. Second, while Brandt & Santa-Clara (2006) approximate the exact solution to the optimal mean-variance portfolio policy by neglecting components of compounded returns, the approach of Paper (2) incorporates the effect of compounded returns, offering an exact solution. Third, being built upon the framework of Brandt et al. (2009), the approach of Paper (2) leverages its extensions as well, including the optimization of other objective functions and the incorporation of transaction costs.

Utilizing monthly data on stocks in the S&P 500 index between 1990 and 2019 and assuming long-only portfolios, Paper (2) illustrates, through simulation studies with ideally constructed market timing predictors, how the proposed approach captures market timing potential, both with and without characteristic-based asset selection. Empirical demonstrations using well-known market timing predictors from Welch & Goyal (2008) indicate that market timing with traditional predictors yields statistically significant gains after transaction costs only for portfolio managers with higher risk aversion levels. Specifically, the proposed approach effectively reduces portfolio risk by investing in the risk-free asset, also reducing the mean portfolio return. This outcome underscores that alternating between the benchmark and the risk-free asset particularly benefits portfolio managers who prioritize reducing risks. Similar results are observed in Paper (2) when combining asset selection and market timing. However, comparisons between strategies performing only selection and those combining selection and timing reveal relatively smaller gains for the latter, showcasing the difficulties of outperforming a selection strategy exploiting a well-diversified set of characteristics (see, e.g., Asness et al., 2017).

4.4 Characteristic timing in parametric portfolio policies

This subsection elucidates the contributions of Paper (3) listed in Table 1, entitled "Characteristic Timing in Parametric Portfolio Policies". The paper presents a pivotal contri-

the weights $w_{i,t}$ of the original parametric portfolio policy. By doing so, the initial parametric portfolio policy is sold short. This leads to the weight $w_{f,t}^{\text{ext}}$ of the risk-free asset, which remains positive, to be leveraged, converging to 1 for $w_{f,t} \to -\infty$.

bution: it introduces a novel parametric portfolio optimization approach that extends the framework of Brandt et al. (2009) to accommodate time-varying relationships between optimal portfolios and asset characteristics.

The approach delineated in Paper (3) revolves around crafting interaction terms between the characteristics $x_{i,t}$ and characteristic timing predictors z_t. Specifically, denoting Hadamard products (element-wise multiplication) as $\odot$, Paper (3) introduces a K-dimensional vector $y_{i,t}$, defined as

$$y_{i,t} = (z_t - \varphi) \odot x_{i,t}, \tag{39}$$

where all variables represent K-dimensional vectors. This operation entails multiplying the k-th raw characteristic in $x_{i,t}$, which is not cross-sectionally standardized, with the k-th characteristic timing predictor in z_t, shifted by the k-th threshold parameter in φ. This shifting ensures that these interaction terms can change sign, a crucial aspect as demonstrated in Paper (3), since the cross-sectionally standardized version $\hat{y}_{i,t}$ of $y_{i,t}$ exhibits the following property:

$$\hat{y}_{i,t} = \text{sign}(z_t - \varphi) \odot \hat{x}_{i,t}. \tag{40}$$

Subsequently, Paper (3) suggests substituting the vector $\hat{x}_{i,t}$ in the original parametric function (34) with the new vector $\hat{y}_{i,t}$, leading to the parametric function

$$f_{\theta,\varphi}(x_{i,t}, z_{i,t}) = w_{i,t}^B + \frac{1}{N_t}\theta^T \left(\text{sign}(z_t - \varphi) \odot \hat{x}_{i,t}\right). \tag{41}$$

A salient aspect of the approach posited in Paper (3) is that, assuming both θ and φ to be time-invariant, optimization employing the parametric function in Equation (41) is solved analogously to the original linear parametric function in Equation (34). This implies that the maximization of conditional expected utility is reduced to a maximization of unconditional expected utility, solved over both the scaling factors θ and the threshold parameters φ. Despite Paper (3) entailing a nonlinear parametric function that heightens the complexity of the optimization program, it offers the substantial advantage of determining suitable thresholds in a data-driven manner aligned with investor objectives, in contrast to the alternative of the investor independently selecting threshold parameters in a suboptimal manner.

Utilizing the same dataset as Paper (2), Paper (3) conducts simulation studies with ideally constructed characteristic timing predictors to showcase the mechanics of the proposed characteristic timing approach. In empirical studies, the application of characteristic timing predictors known from the literature, including value spreads, momentum spreads and factor momentum, exhibit observations that are complementary to the market timing applications of Paper (2). While market timing in parametric portfolio policies adds value to investors by reducing risks, characteristic timing increases the portfolio return, specially benefiting portfolio managers with low risk aversion.

4.5 Characteristic-based tracking portfolio optimization

It is important to note that the foundational principle behind the framework of parametric portfolio policies, which involves utilizing asset characteristics directly in the portfolio optimization stage, thereby circumventing the challenge of errors associated with the estimation of the distribution of asset returns, extends its applicability to further scenarios, including index tracking. Beyond the characteristic-based portfolio optimization methods discussed in Subsections 4.3 and 4.4 for active portfolio management applications, my cumulative dissertation also introduces a novel characteristic-based approach tailored for passive portfolio management, presented next.

Paper (4) in Table 1, titled "Enhancing Index-Tracking Performance: Leveraging Characteristic-Based Factor Models for Reduced Estimation Errors", contributes to the literature discussed in Subsection 3.2 on passive portfolio management by introducing a mixed-integer, characteristic-based tracking portfolio optimization approach. This approach aims to enhance the out-of-sample performance of tracking portfolios through the reduction of estimation errors while adhering to the cardinality constraint.

Traditionally, the index tracking literature, akin to the principles laid out by Markowitz (1952), approaches tracking portfolio optimization as a two-step process (see, e.g., Roll, 1992). Firstly, it involves estimating the tracking error associated with the distribution of asset returns. Secondly, it minimizes this tracking error based on estimates that may be prone to errors. Similar to mean-variance analysis, various methodologies have been proposed to address estimation errors and enhance out-of-sample performance. Notably, the standard approach of using factor models to estimate the distribution of asset returns, as discussed in Subsection 2.4, has been employed to improve the performance of index tracking portfolios through effective financial modeling (see, e.g., Spronk & Hallerbach, 1997; Derigs & Nickel, 2003; Corielli & Marcellino, 2006). However, factor models rely on variables that require estimation, exposing such tracking approaches to potential estimation errors.

Paper (4) advocates for a direct utilization of asset characteristics in computing the optimal tracking portfolio. Essentially, the paper suggests tracking observable characteristics of the target portfolio instead of relying on the target returns estimated from these characteristics. By circumventing the estimation errors inherent in the latter approach, Paper (4) aims to enhance the out-of-sample performance. Utilizing the same dataset employed in Papers (2) and (3) for tracking the S&P 500, augmented with a short-term reversal characteristic, empirical demonstrations within Paper (4) strongly support the superiority of characteristic-based approaches over counterparts based on factor models estimated from the same characteristics. These findings exhibit robustness across various portfolio and model sizes, as well as different specifications of the objective function.

Paper (4) goes beyond assessing the raw performance of characteristic-based index tracking portfolios by delving into turnover and transaction costs, essential considerations for real-world implementation. Notably, the findings reveal that portfolios directly utilizing characteristics experience lower turnover compared to those relying on factor models, maintaining their out-of-sample superiority even after accounting for transaction costs, which bolsters their practical appeal.

Moreover, the approach outlined in Paper (4) offers a significant reduction in the complexity of the optimization process. Unlike traditional methods that necessitate the estimation of asset return distributions, the characteristic-based approach circumvents this requirement entirely. In summary, the characteristic-based tracking approach presents an attractive tool for practical portfolio managers, offering enhanced performance coupled with reduced costs and complexity.

5 Concluding remarks

The foundation of modern portfolio theory, based on the mean-variance framework introduced by Markowitz (1952), typically follows a two-step portfolio construction approach. Initially, this involves estimating the unknown distribution of asset returns, followed by determining the optimal portfolio allocation based on these estimates. However, estimation errors in the first stage significantly compromise the performance of strategies derived in the second stage, limiting their practical utility for portfolio managers in both active and passive management contexts. Despite extensive research efforts proposing techniques, including factor models, to mitigate estimation errors, these methods only partially alleviate the susceptibility of optimal portfolio choices to inaccuracies in estimation. To address this challenge, the studies featured in my thesis explore one-step formulations for optimal portfolio choice, bypassing the error-prone stage of estimating the distribution of asset returns.

In the context of active portfolio management, this thesis builds upon the seminal work of Brandt et al. (2009), who introduced parametric portfolio policies as a characteristic-based optimization framework exploiting cross-sectional return predictability associated with asset characteristics. However, the original parametric portfolio policies are limited to using cross-sectional predictors alone, overlooking potential benefits from incorporating time-series predictability. This thesis overcomes this limitation by integrating time-series predictability of aggregated market returns and the relationship between asset returns and characteristics. Through extensive simulation studies and empirical demonstrations, these enhancements are shown to deliver tangible value to real-world investors.

For passive portfolio management, this thesis introduces a new mixed-integer, characteristic-based optimization approach tailored for passive management subject to cardinality constraints. Empirical studies affirm the effectiveness of this approach, highlighting lower tracking errors and reduced portfolio turnover compared to conventional two-step tracking methods, offering substantial benefits for passive managers. By formulating tracking portfolio optimization as a one-step methodology, the complexity of portfolio construction is drastically reduced, easing adoption in practical applications.

Moreover, recognizing the trend of portfolio managers investing in factors rather than individual assets, this thesis also addresses factor-level investing strategies. Employing machine learning methodologies with a comprehensive dataset spanning decades and incorporating over a hundred factors, this research explores predictability in the cross-section of factor returns. The resulting data-driven empirical evidence reveals strong cross-sectional predictability, predominantly driven by factor momentum. This contributes to advancing our understanding of how portfolio managers can select factors for their portfolio strategies effectively and simply.

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Publication status of research papers

Here, Table 1 from Overview is repeated for clarity. Below the table, the publication status of the research papers included in this thesis is detailed.

#	Authors	Title	Referring to
(1)	Cakici, N. Fieberg, C. Osorio, C. Poddig, T. Zaremba, A.	Picking Winners in the Factor Zoo	2.4.4
(2)	Osorio, C. Poddig, T. Fieberg, C. Olschewsky, M. Falge, M.	Market Timing in Parametric Portfolio Policies	4.3
(3)	Osorio, C.	Characteristic Timing in Parametric Portfolio Policies	4.4
(4)	Fieberg, C. Osorio, C. Poddig, T. Varmaz, A.	Enhancing Index-Tracking Performance: Leveraging Characteristic-Based Factor Models for Reduced Estimation Errors	4.5

Table 1 (Repeated) This table presents an overview of each paper included in this thesis, detailing its title, authors, and the corresponding subsection that provides a deeper exploration of its contributions.

Maintaining the same numbering as in the table, the publication status of the research papers is as follows:

(1) This study has been accepted for publication in **The Journal of Portfolio Management** on November 30th, 2024. DOI: 10.3905/jpm.2025.1.688

(2) This study has been **published** in the **International Journal of Theoretical and Applied Finance** on July 16th, 2022. DOI: 10.1142/S0219024922500182

(3) This study has been **submitted for publication** in the **International Journal of Theoretical and Applied Finance**.

(4) This study has been **submitted for publication** in the **European Journal of Operational Research**.