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Marc Pillarz*, Axel von Freyberg, and Andreas Fischer

Scalable multi-distance measurement approach for the optical assessment of tooth-individual shape parameters of large gearings

Skalierbarer Multi-Distanz-Messansatz für die optische Erfassung von zahnindividuellen Formparametern von Großverzahnungen

Abstract: The demand for scalable measuring systems for the fast measurement of tooth-individual shape parameters of large gears is increasing. Due to a limited measuring speed and a limited measuring volume, standard tactile gear measuring systems are only conditionally suitable for comprehensive measurements of large gears. Therefore, the applicability of a scalable optical multi-distance measurement approach consisting of an optical sensor and a rotary table with a model-based evaluation is studied. As a fundamental gear shape parameter, the base circle radius of a large gearing is evaluated and the achievable measurement uncertainty is estimated. As a result, the tooth-individual base circle radius is determined on average with an expanded measurement uncertainty ($k = 2$) of one quarter of the required tolerance, and the realized measurement setup enables the measurement of all teeth within 1 minute. Hence, the scalable optical multi-distance measurement approach is appliacable for the fast measurement of tooth-individual shape parameters of large gearings.

Keywords: Optical large gear measurements, fast multi-distance measurements, model-based evaluation, confocal-chromatic sensor, measurement uncertainty, scalability.

Zusammenfassung: Die Nachfrage nach skalierbaren Messsystemen für die schnelle Messung zahnindividueller Formparameter von Großverzahnungen steigt. Herkömmliche taktile Verzahnungsmesssysteme sind aufgrund einer begrenzten Messgeschwindigkeit und eines begrenzten Messvolumens nur bedingt für umfassende Messungen an Großverzahnungen geeignet. Daher wird die Anwendbarkeit eines skalierbaren optischen Multi-Distanz-Messansatzes bestehend aus einem optischen Sensor und einem Drehtisch mit modellbasierter Auswertung untersucht. Als grundlegender Formparameter wird der Grundkreisradius einer Großverzahnung ausgewertet und die erreichbare Messunsicherheit abgeschätzt. Im Ergebnis wird der zahnindividuelle Grundkreisradius im Mittel mit einer erweiterten Messunsicherheit ($k = 2$) von einem Viertel der geforderten Toleranz bestimmt, und der realisierte Messaufbau ermöglicht die Messung aller Zähne innerhalb 1 Minute. Der skalierbare optische Multi-Distanz-Messansatz ist somit für die schnelle Messung zahnindividueller Formparameter von Großverzahnungen anwendbar.

Schlagwörter: optische Großverzahnungsmessungen, schnelle Multi-Distanz-Messungen, modellbasierte Auswertung, konfokal-chromatischer Sensor, Messunsicherheit, Skalierbarkeit.

1 Introduction

1.1 Motivation

The involute is the most commonly used shape for the tooth flank of spur gears in gear technology [10]. The geometry of the involute is determined by the shape parameter base circle radius. For the evaluation of the tooth flank geometry, the deviation parameter profile slope deviation is considered as a standard in gear metrology instead of the base circle radius, but note that both variables correlate with each other through a linear relationship. Already micrometer deviations of the base circle radius from the nominal geometry can lead to a non-uniform transmission behavior and a premature wear of the gears. The manufacture of gears with involute tooth flanks is therefore subject to tight manufacturing tolerances [6]. Derived from

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[6], base circle radius deviations in the low two-digit micrometer range are tolerated depending on the gear quality and dimensions. According to [14], to ensure uniform power transmission under load, a measurement uncertainty ($k = 2$) of 20 %–30 % of the tolerances is necessary for measuring the tooth flank geometry. For a small gear with 26 teeth, a normal module of 3.75 mm and a tip diameter of 105 mm, the base circle radius tolerance for gear quality grade 6 is 20.4 µm. For a large gearing with 38 teeth, a normal module of 12 mm (three times larger), and a tip diameter of 480 mm (four times larger), the base circle radius tolerance is only a factor of 1.8 larger and amounts to 37.5 µm. The required uncertainty of measurement for the large gearing is 11.2 µm ($k = 2$). Thus, large gears with a tip diameter ≥ 1 m or large gearings, independent of the workpiece size, with a normal module ≥ 10 mm are particularly challenging for dimensional metrology. As the diameter and normal module of the gear increase, the ratio of the gear diameter to the required measurement uncertainty increases, although the tolerances also increase.

Due to the larger chip volume and longer machining time compared to small gears, asymmetric heat input and tool wear occur during manufacturing. It must therefore be assumed that the individual teeth have individual shape deviations and surface qualities. To ensure uniform power transmission, the geometry of each individual tooth must be inspected accordingly. However, as the number of teeth increases, the measuring time also increases proportionally, which is why fast measuring systems are required to measure all teeth.

Furthermore, geometric measurements on large gears represent an additional logistical challenge due to the sometimes meter-sized dimensions and masses greater than 500 kg. Therefore, measuring systems with a scalable measuring volume are required for the quality inspection of large gears.

In summary, fast gear measuring systems with a scalable measuring volume are required that record the tooth flanks of all teeth and quantify tooth-individual shape parameters such as the base circle radius with single-digit or low two-digit micrometer uncertainty.

1.2 State of the art

The current state of the art for measuring the gear geometry is evaluated in this article on the basis of three criteria: measurement uncertainty, measurement time and scalability.

The gold standard for gear quality inspection is represented by tactile coordinate measuring machines (CMM) and gear measuring instruments (GMI) [3, 9, 17]. Tactile measurement methods are precise, but limited in terms of speed and measurement volume. Due to both the sequential acquisition and the limited speed of tactile measuring methods, only four teeth distributed around the circumference of the gear are measured in standard gear inspections [3]. However, this random inspection scope does not allow a reliable quality inspection of all teeth, which becomes increasingly important for the measurement of large gears or large gearings. Moreover, no further significant developments are expected in tactile gear measurements [11].

For this reason, faster optical measurement approaches for gear measurement of all teeth have been developed and investigated in recent years [13]. Essentially, current optical gear measurement approaches are based on the measurement principle of interferometry or laser triangulation.

In 2011 and 2014, Fang et al. presented an interferometric approach using phase shift technique for complete two-dimensional detection of the geometry of a tooth flank of small gears [7, 8]. However, while profile and flank line measurements agreed with reference measurements and the two-dimensional data acquisition significantly reduced measurement time, the measurement setup is only suitable for measuring a single tooth flank. In 2015 and 2017, Balzer et al. presented the point-by-point scanning acquisition of the gear geometry using a frequency-modulated continuous wave laser radar sensor moved by a CMM [2, 3]. Experimental investigations showed that the measurement results only partially agreed with the tactile reference measurements. Moreover, no significant speed advantage was achieved compared to the gold standard. As a result, no optical gear measurement method based on interferometry is currently suitable for the fast measurement of all tooth flanks.

Laser triangulation based optical gear measurement can perform point, line and area oriented measurements. In 2005, Younes et al. presented a measurement approach using point triangulation sensors in combination with a rotary table to measure the gear geometry of all teeth within one minute [25, 26]. However, a sufficient measurement uncertainty for the evaluation of the tolerances could not be achieved. Despite the fast and extensive measurement of all teeth, the measurement approach is not suitable for a reliable gear inspection. Line-oriented measurement approaches for gear measurements based on triangulation are presented in [1, 13, 24]. Tian et al. used two opposing triangulation sensors in combination with a rotary table to acquire the geometry of all tooth flanks of a small gear in one measurement run [24]. After the compensation of systematic measurement deviations due to an unknown measurement setup, remaining measurement deviations of up to 59 µm were determined. With respect to the required tolerances for gears, the achieved measurement deviation is not sufficient, which is why the measurement approach of Tian et al. is not suitable for fast gear measurements. In 2019, Auerswald et al. presented a laser line triangulation approach for measuring the geometry of a tooth flank of a large helical gear [1]. Combined with a linear axis as a scanning unit, the entire surface of a tooth flank was measured with the laser line triangulation sensor. Auerswald et al. determined the tooth flank shape with a mean measurement deviation of ±8.2 µm. The two-dimensional measurement approach also theoretically accelerates gear inspection by a factor of 5700 compared to classic tactile gear inspection. However, the measurement setup is currently not designed for a fast gear inspection of all teeth. In 2020, Guo et al. presented a successful geometry measurement of a small gear using a laser line triangulation sensor and a rotary table. In one measurement run, pitch and profile deviations of all tooth flanks were measured and subsequently validated with tactile reference measurements. The determined pitch deviations differ by 3 µm and the profile deviations by only 1 µm from the tactile reference measurements [13]. A measurement duration for the measurement of all tooth flanks was not quantified in detail, but the components used for the measurement setup indicate possible measurement times < 10 minutes.

Fringe projection and moiré techniques are known area-oriented measurement approaches for gear measurements based on the triangulation principle. As early as 2000, Peter et al. presented a measurement approach based on a fringe projector combined with a rotary table for measuring the geometry of the tooth flanks of small helical gears [18]. The shape of the tooth flank could be measured with a measurement deviation of approx. ±10 µm. Peter et al. emphasized the fast data acquisition due to the two-dimensional measurement approach, but did not specify a concrete measurement duration. In 2006, Meeß et al. demonstrated another measurement approach based on fringe projection for the two-dimensional gear measurement by integrating the fringe projector into an opto-tactile CMM [16]. The measurement uncertainty achieved for characterizing the gear geometry is insufficient in relation to the required tolerance. Since Meeß et al. performed sequential pitch-related area-oriented measurements of the individual tooth flanks, a significant speed advantage over classic CMMs can be expected. However, the measuring time was not quantified in more detail. The suitability of the moiré technique for gear measurement was investigated by Sciammarella et al. and Chen et al. In 2005, Sciammarella et al. validated that the two-dimensional measurement of the gear geometry using the moiré technique combined with a rotary table achieves results comparable to a CMM. They reached a measurement deviation for the profile measurement of 3 µm [22]. A measurement duration was not specified. Chen et al. achieved measurement deviations < 5 µm with their moiré approach for measuring the geometry of gear teeth [4]. However, the approach is currently designed only for measuring a single tooth flank and not for measuring multiple tooth flanks in one measurement run. In summary, current optical triangulation techniques show great potential for fast measurements of the geometry of gears with measurement uncertainties down to the single-digit micrometer range.

In 2020 and 2021, Pillarz et al. proposed a scalable multi-distance measurement approach for the fast measurement of larger gears based on an optical sensor in combination with a rotary table [19–21]. Here, a confocal-chromatic distance sensor was established for the measurement of gears. Using a model-based evaluation of the sensor signal, Pillarz et al. determined the mean base circle radius over all teeth as well as the tooth-individual base circle radii of all teeth with single-digit micrometer uncertainty of a small gear.

In conclusion, almost all measurement approaches for optical gear measurements have been validated on small gears so far. Only Auerswald et al. demonstrated a tooth flank measurement on a large helical gear, but the measurement approach was only designed for the measurement of a single tooth flank. Pillarz et al. presented a scalable multi-distance measurement approach for the measurement of large gears and large gearings, but the applicability to large gearings and the estimation of the achievable measurement uncertainty in the determination of tooth-individual shape parameters still need to be clarified.

1.3 Aim and structure of the article

In this article, the applicability of a scalable multi-distance measurement approach for the fast assessment of tooth-individual shape parameters with a sufficiently uncertainty of < 30 % of the required tolerances is validated for large gearings. As a fundamental shape parameter, the tooth-individual base circle radius is evaluated using a two-stage approximation based on a geometric model. The measurement concept is realized by using a confocal-chromatic sensor in combination with a rotary table to perform continuous distance measurements to all tooth flanks. To this end, the scalable multi-distance measurement approach is explained in Section 2. Section 3 describes the experimental measurement setup to validate the applicability of the scalable multi-distance measurement approach for measuring large gearings. The measurement results are discussed in Section 4 and validated with tactile reference measurements based on a standard gear evaluation. The article closes with a summary and an outlook in Section 5.

2 Measurement principle

2.1 Measurand base circle radius

The validation of the scalable multi-distance measurement approach is carried out in this article on an unmodified large gearing with involute profile. The involute describes a trajectory curve that is created when a thread is unwound on a base circle. The curvature of the resulting involute depends on the radius of the base circle and decreases steadily as the thread is unwound. The base circle radius is thus the decisive parameter for the geometry of the involute and will be considered as a fundamental shape parameter in this article. In standard gear measurements, the geometry of the tooth flank is typically evaluated using the deviation parameter profile slope deviation. However, the profile slope deviation linearly correlates with the actual base circle radius

$$r_\mathrm{b} = r_\mathrm{b,n} \cdot \frac{f_{\mathrm{H}\alpha}}{L_\alpha} + r_\mathrm{b,n}, \tag{1}$$

which can be calculated from the nominal base circle radius $r_\mathrm{b,n}$, the profile slope deviation $f_{\mathrm{H}\alpha}$ and the evaluation range L_α. The profile slope deviation describes the deviation of the actual slope of the profile of the tooth compared to the nominal slope [15].

To validate the multi-distance measurements and to estimate the achievable measurement uncertainty in the measurement of tooth-individual base circle radii, reference measurements are performed using a large CMM. The geometry of the individual teeth is measured tactilely and the profile slope deviations are then evaluated classically. By applying Eq. (1), reference values for the tooth-individual base circle radii can then be determined.

2.2 Model-based evaluation of the tooth-individual base circle radius

In [19, 20], Pillarz et al. have already presented the approximation of the mean base circle radius based on a geometry model of involute gears according to [11, 12]. This evaluation approach is extended in this article by another approximation level to determine the tooth-individual base circle radii of all teeth.

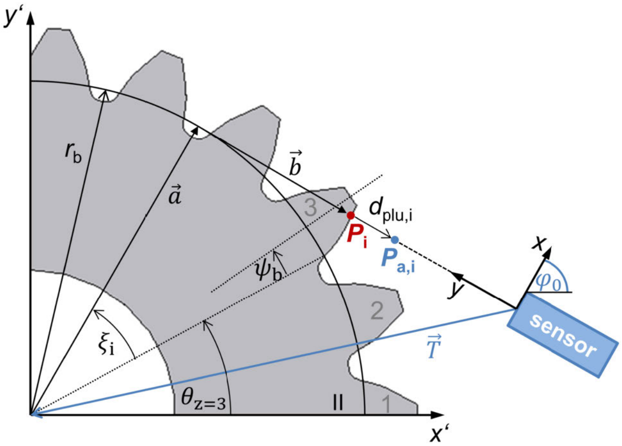

Figure 1: Geometry model for calculating the shape parameter base circle radius r_b of a non-modified gear with involute profile in a measurement coordinate system (x, y). A point on an involute can be calculated from a vector addition of a radial vector $\vec{a}$ and tangential vector $\vec{b}$, in which the root point P_i of the nominal geometry of the gear for the plumb line distance $d_\mathrm{plu,i}$ and the position parameters ξ_i, θ_z, ψ_b, φ_0, $\vec{T}$ as well as the required base circle radius r_b are included. To calculate the base circle radius, the inverse problem must be solved. The plumb line distance of the measuring point to the nominal geometry of the gear is shown enlarged for illustration.

Figure 1 shows the tooth flank geometry of an involute gear in the transverse section. In a measurement coordinate system (x, y), an ideal point

$$\begin{aligned} P_\mathrm{i} &= \begin{bmatrix} x_\mathrm{i} \\ y_\mathrm{i} \end{bmatrix} = \vec{a} + \vec{b} + \vec{T} \\ &= r_\mathrm{b} \cdot \begin{bmatrix} \cos(\xi_\mathrm{i} + \theta_\mathrm{z} - \psi_\mathrm{b} + \phi_0) \\ \sin(\xi_\mathrm{i} + \theta_\mathrm{z} - \psi_\mathrm{b} + \phi_0) \end{bmatrix} \\ &\quad + r_\mathrm{b} \cdot \xi_\mathrm{i} \cdot \begin{bmatrix} \sin(\xi_\mathrm{i} + \theta_\mathrm{z} - \psi_\mathrm{b} + \phi_0) \\ -\cos(\xi_\mathrm{i} + \theta_\mathrm{z} - \psi_\mathrm{b} + \phi_0) \end{bmatrix} + \begin{bmatrix} x_\mathrm{t} \\ y_\mathrm{t} \end{bmatrix} \end{aligned} \tag{2}$$

on a tooth flank can be described by a vector addition of a radial vector $\vec{a}$ and a tangential vector $\vec{b}$ consisting of the position parameters (ξ_i: rolling angle, θ_z: position of the centerline of tooth z, ψ_b: tooth thickness half angle, ϕ_0: rotation to workpiece coordinate system and $\vec{T} = (x_\mathrm{t}, y_\mathrm{t})$: translation to the workpiece coordinate system and center of the gear) and the shape parameter base circle radius r_b. To calculate the base circle radius, the inverse problem of Eq. (2) must be solved, accordingly. It must be taken into account that the parameters $(\phi_0, x_\mathrm{t}, y_\mathrm{t}, r_\mathrm{b})$ and additionally for each point i the parameter ξ_i are unknown, which result in $(4 + i)$ unknown parameters. To solve the inverse problem, at least four measurement points ($i = 1 \ldots 4$) and, thus, 8 known coordinates are required to calculate the 4+4 unknown parameters.

The actual measured tooth flanks typically deviate from an ideal involute geometry. Therefore, Pillarz et al. have chosen an iterative approach to calculate the actual mean base circle radius [19, 20]. Here, a mean ideal involute is approximated by taking into account the deviations (plumb line distances $d_{\mathrm{plu,i}}$) between the actual geometry $P_{\mathrm{a,i}}$ and the ideal nominal geometry P_{i} of the tooth flank in the measuring points of all measured tooth flanks. The calculation of the plumb line distances is based on [23]. By minimizing the sum of the squared plumb line distances to the measured points

$$\min_{\xi_{\mathrm{i}},\phi_0,x_{\mathrm{t}},y_{\mathrm{t}},r_{\mathrm{b}}} \left(\sum_{i=1}^{k} d^2_{\mathrm{plu,i}} \right), \tag{3}$$

an ideal mean involute is calculated by the least squares method as a function of the parameters (ξ_{i}, ϕ_0, x_{t}, y_{t}, r_{b}), i. e. including the sought mean base circle radius r_{b}. In comparison to the classic tactile measurement of involute gears, where the gear center is first determined on the basis of reference datums and then the gear geometry is checked in a further measurement, here, the center of the gear is determined directly on the basis of the gear geometry measurement.

For a reliable quality inspection of large gears or large gearings, it is useful to evaluate the geometry of all teeth individually. For this purpose, the tooth-individual base circle radii must be determined. However, the approximation of the measuring points by an involute is a complex nonlinear optimization problem with a multidimensional solution space. Especially in the approximation of tooth-individual base circle radii, the boundary conditions of an incomplete coverage of the theoretical mathematical involute and the local clustering of the measurement points additionally increase the probability of not finding the global minimum (see Fig. 2).

As a result, the determined tooth-specific base circle radii and the other free position parameters systematically deviate. For this reason, in order to reduce the systematic deviations in the tooth-individual base circle radius calculation, a two-stage approximation is used and the multidimensional solution space is solved step by step. First, the free parameters (ξ_{i}, ϕ_0, x_{t}, y_{t}, r_{b}) are calculated over all measured teeth according to Eq. (3). Subsequently, in the second approximation stage, the calculation of the tooth-individual base circle radii is performed on the basis of Eq. (3), with the previously determined parameters ϕ_0, x_{t}, y_{t} locked. The second approximation stage thus has fewer free parameters, which reduces systematic deviations in the approximation of the tooth-individual base circle radius based on the measurement data from only one involute.

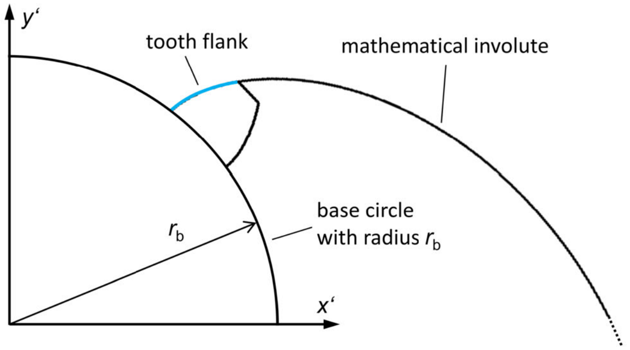

Figure 2: Principle sketch for generating a tooth flank from a mathematical involute on the basis of a base circle with radius r_{b}. The actual tooth flank (blue) represents only a small part of the mathematical involute. If a measured tooth flank is to be approximated by an ideal mathematical involute, the incomplete coverage of the mathematical involute and the local clustering of the measuring points on the mathematical involute in relation to the gear center (= center of the workpiece coordinate system) increase the probability of not finding the global minimum.

2.3 Scalable multi-distance measurement approach

In order to calculate the tooth-individual base circle radii, the actual geometry of the respective tooth flanks must be measured. For this purpose, a multi-distance measurement approach consisting of n optical distance sensors in combination with a rotary table for a scanning measurement is presented (see Fig. 3).

The individual sensors are distributed around the circumference of the gear and are arranged tangentially to the nominal base circle in the transverse section, i. e. perpendicular to the tooth flank surface. Depending on the sensor arrangement, the measuring approach permits detecting of both the left and right tooth flanks. At the center of rotation, the sensors and the rotary table together form a common measurement coordinate system (x, y). While the gear rotates, the tooth flanks are continuously measured in the form of distances d_{i} as a function of the angle of rotation α_{i}. To evaluate the base circle radius, the measured distances must be converted into coordinates of the general measurement coordinate system, which requires the knowledge of the exact sensor position and orientation. However, the exact positioning and orientation of the sensors are not known, which leads to significant systematic

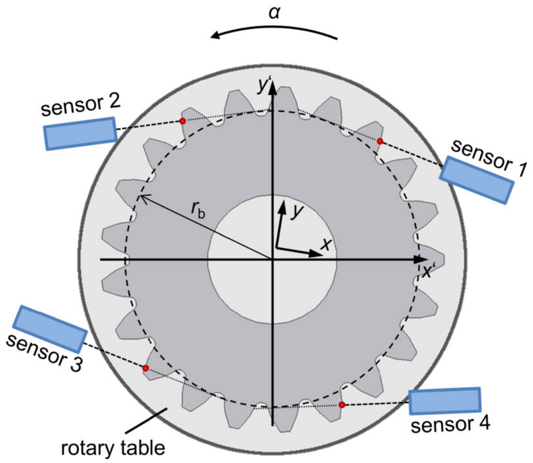

Figure 3: Model-based scalable multi-distance measurement approach consisting of $n = 4$ optical distance sensors combined with a rotary table in a common measurement coordinate system (x, y), which continuously measures the geometry of both the left and right tooth flank of a gear with a workpiece coordinate system (x', y') as a function of the rotation angle α.

measurement deviations in the base circle radius approximation. Accordingly, a calibration of the multi-distance measurement approach is required to reduce these systematic deviations. The sensor system is therefore calibrated according to the approach of [20] by means of an offset correction. For this purpose, Pillarz et al. use a gear with a known geometry, which fulfills similarity conditions to the gear to be measured according to [5]. Using the multi-distance measurement approach, a mean base circle radius is determined for the known gear over all teeth and compared with the corresponding reference geometry. The offset between the measured mean base circle radius and the reference value then represents a correction value for subsequent measurements. It should be noted that the calibration is valid for a specific sensor arrangement. If the sensor arrangement is changed, the sensor system must be recalibrated. Also to be considered during calibration is a reproducible gear clamping. If the positions of the calibration gear and the gears to be measured subsequently differ significantly, a larger proportion of systematic deviations will remain despite compensation. In contrast to the offset correction with respect to the mean base circle radius [20], in this article the sensor system is calibrated individually for each tooth.

The scanning data acquisition then achieves an overdetermined system of equations, which enables the iterative evaluation of the tooth-individual base circle radii according to Section 2.2.

By using a rotary table, the profile geometry of all teeth can be acquired within one rotation, which significantly reduces the measurement time. If several optical sensors are also used, the measuring time can be further reduced by a parallel data acquisition and depending on the sensor arrangement. The perpendicular alignment of the sensors to the tooth flank surface also enables the use of optical sensors with a low acceptance angle. With perpendicular sensor alignment, however, it must be taken into account that the entire flank cannot be optically detected due to a limited accessibility through neighboring teeth. Furthermore, the measurement volume can be adapted to the size of the measured object by a flexible positioning of the sensors, which is why the multi-distance measurement approach is suitable for measuring various large gears.

3 Experimental setup

The applicability of the multi-distance measurement approach for measuring tooth-individual shape parameters of large gears and large gearings is investigated in this article on the basis of a multi-distance measurement approach consisting of one sensor ($n = 1$) in combination with a rotary table. The experimental setup is described in the following sections.

3.1 Measurement object

A non-modified spur gear with involute profile is used for the gear measurements. The gear has 38 teeth, a nominal base circle radius of 214.25 mm and a normal module of 12 mm. Due to its large normal module, the measurement object can be considered as a large gearing. Based on [6] and Eq. (1), the manufacturing tolerance for the base circle radius for gear quality 6 for this gear geometry is 37.5 µm. According to [14], a measurement uncertainty of at least 11.2 µm ($k = 2$) is therefore required. To validate the applicability of the multi-distance measurement approach for large gear and large gearing measurements and to estimate the achievable measurement uncertainty, a reference measurement with a Leitz PMM-F 30.20.7 CMM is used. In a standard gear measurement, the profile slope deviations of all teeth are measured with a previously estimated measurement uncertainty of 1 µm ($k = 1$), and based on Eq. (1), reference values for the tooth-individual base circle radii are determined.

The calibration of the scalable model-based multi-distance approach requires a known calibration gear which, according to [5], fulfills similarity conditions to the gear to be measured. However, within the scope of this research, no second comparable gear was available. For this reason, the approach taken in this article is to perform the calibration on all odd teeth and to evaluate the actual validation on all even teeth.

3.2 Measurement arrangement

The realized measurement setup of the scalable multi-distance measurement approach consisting of a rotary table and an optical sensor is illustrated in Fig. 4. The hydro static rotary table of a Leitz PMM-F 30.20.7 CMM is used to rotate the gear. The rotary table enables dynamic scanning measurements at 1.1 min^{-1}. The entire profile geometry of all teeth can thus be captured within one minute. A special clamping unit is used to position the gear to be measured as concentrically as possible at the center of rotation of the rotary table.

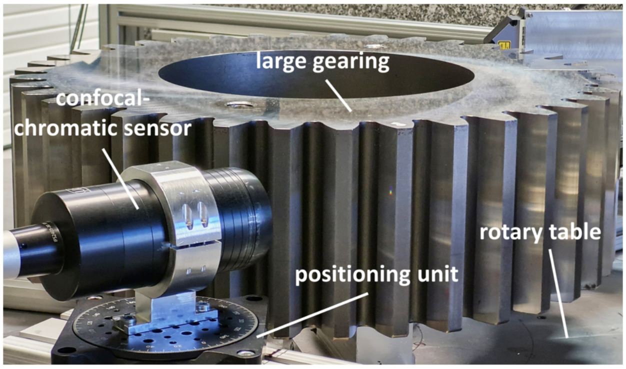

Figure 4: Measurement setup for model-based multi-distance measurement of the tooth-individual base circle radii of a non-modified large gearing with involute profile. The investigated large gearing has 38 teeth, a normal module of 12 mm and a tip diameter of 480 mm. The multi-distance measurement approach is realized in the form of a single confocal-chromatic distance sensor and a rotary table. To align the sensor as perpendicularly as possible to the tooth flanks, a manual positioning unit is used.

A confocal-chromatic distance sensor IFS2405-30 from MicroEpsilon is used as the sensor. The sensor is specified for a measuring range of 30 mm at a working distance of 100 mm, so that large gearings with normal modules $\geq$ 10 mm in particular can also be measured. The acceptance angle of the sensor is ±9°. The sensor is also specified with a linearity error $< \pm 7.5\,\mu\mathrm{m}$. The measuring spot has a mean diameter of 50 µm on the tooth surfaces. Combined with a control unit confocalDT IFC2422 also from MicroEpsilon, measuring rates of 6.5 kHz can be achieved. The sensor is positioned in such a way that the entire measuring range is used as far as possible when detecting the tooth flanks. To align the sensor tangentially to the nominal base circle of the gear or perpendicular to the tooth flank, a manual rotating unit is used. However, due to wobble resulting from eccentric and possibly slightly inclined clamping of the large gearing on the rotary table, the real sensor alignment deviates from the ideal perpendicular alignment. The sensor system is calibrated by using all odd teeth of the gear to be measured as calibration objects. Accordingly, the gear is clamped only once at the beginning of the measurements and its position is not changed. Systematic contributions to the measurement uncertainty due to the different positioning of the calibration gear and the gear to be measured are thus not considered in this article. The measurements are performed in an air-conditioned measuring room, which is specified with a temperature change over time of less than 0.4 K per hour.

Currently, the synchronization of the rotation table with the measurement signal of the confocal-chromatic sensor in a dynamic scanning measurement is not possible. The rotation angle is not detected synchronously with the measured distances. In this article, therefore, the approach of a stepwise scanning measurement is pursued. The rotary table is rotated in defined angular steps and the distance to the tooth flank is measured for each setting. For the model-based evaluation of the tooth-individual base circle radii according to Section 2.2, the angle-dependent measured distances are then converted into coordinates.

4 Results

Due to the unknown sensor arrangement and the associated systematic deviation in the base circle radius approximation, the multi-distance measuring system must be calibrated in a first step. The sensor system is calibrated on the basis of the geometries of all odd numbered teeth of the available large gearing, which were previously determined with tactile reference measurements and on the basis of Eq. (1). The uncertainty estimation for the tooth-individual reference base circle radii is based on a propagation calculation with the estimated uncertainty for the profile slope deviation of 1 µm. The measurement uncertainty for the tactile measured reference base circle radii is thus 4.91 µm for $k = 1$.

To calibrate the multi-distance measurement system, a mean tooth-individual correction value for the tooth-individual base circle radii is calculated from 4 repeated measurements with identical boundary conditions with regard to the sensor and measurement object arrangement. The corresponding calibration uncertainty is then composed of the uncertainty of the tactile reference measurement and the uncertainty of the optical multi-distance measurement. The calibration uncertainty is 4.93 µm for $k = 1$ on average and is currently dominated by the tactile reference measurement.

The achievable measurement uncertainty in the tooth-individual base circle radius determination is then quantified on the even numbered teeth of the same large gearing. The gear is not repositioned on the rotary table. Likewise, the geometry of the even numbered teeth is previously referenced tactile, so that a reference base circle radius is available for each even numbered tooth. For the measurement uncertainty estimation of the achievable measurement uncertainty in the measurement of tooth-individual base circle radii of large gearings, 4 repeated measurements are performed with the multi-distance measurement approach. The experimental results of the tooth-individual base circle approximation are illustrated in Fig. 5. The red crosses symbolize the averaged tooth-individual base circle radii for the 4 repeated measurements determined with the model-based multi-distance measurement approach. The corresponding tactile measured tooth-individual reference base circle radii are shown as black circles. In addition, the expanded measurement uncertainty ($k = 2$) is given for both the optically measured base circle radii and the tactilely measured reference base circle radii.

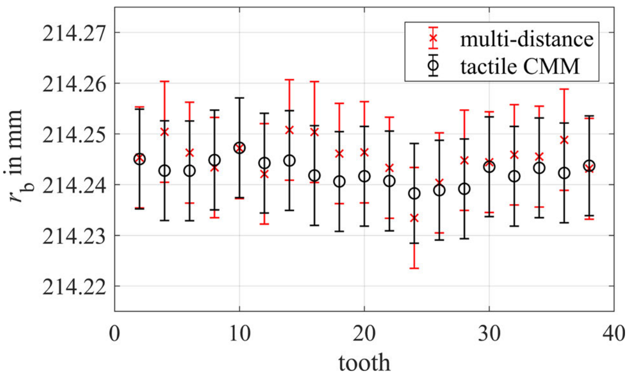

Figure 5: Results of the measurement of the tooth-individual base circle radii r_b of all even numbered teeth on a large gearing. The odd numbered teeth are used to calibrate the measuring arrangement and are not subsequently evaluated further. Hence, the mean base circle radii for all even numbered tooth for 4 repeated measurements based on the multi-distance measurement approach with an optical confocal-chromatic sensor (red crosses) and the tooth-individual reference base circle radii determined on the basis of a standard tactile tooth measurement (black circles) are shown. In addition, the expanded measurement uncertainty for $k = 2$ is given for the determined base circle radii.

The measurement results in Fig. 5 show that the model-based multi-distance measurements agree with the tactile reference measurements within the extended measurement uncertainties. Significant systematic measurement deviations are not visible and are thus successfully compensated for each individual tooth. It should be noted that the systematic influence of repositioning is not considered in these results. On average, the expanded measurement uncertainty ($k = 2$) of the tooth-individual base circle radius of the multi-distance measurements is 10.1 µm, which means that the goal of a measurement uncertainty < 30 % has been achieved. The achieved measurement uncertainty for $k = 2$ is 27 % of the required tolerance. The measurement uncertainty results from the uncertainty of the calibration of the measurement system and from the random measurement error from the optical multi-distance measurements. The uncertainty contribution from the calibration (4.93 µm for $k = 1$) significantly dominates the achieved measurement uncertainty, being larger by a factor of 5 than the random error of the optical multi-distance measurement. The contribution of the random measurement error resulting from the optical measurement is on average 1 µm and shows the potential of the scalable mutli-distance measurement approach for the applicability to large gearings with a normal module ≥ 10 mm. In order to decrease the achieved measurement uncertainty of the tooth-individual base circle radii in the future, a reduction of the calibration uncertainty by more precise reference technology or by an adaptation of the calibration strategy is to be aimed at.

In summary, the scalable multi-distance measurement approach is validated for tooth-individual large gearing measurements considering the uncertainty contributions currently regarded.

5 Summary and outlook

This paper examines the applicability of a scalable model-based multi-distance measurement approach for measuring tooth-individual shape parameters of large gears and large gearings with a measurement uncertainty < 30 % of the required tolerances. By combining an optical confocal-chromatic distance sensor with a rotary table, profile lines of all teeth of a large gearing with a normal module of 12 mm can be recorded within a one-minute rotation and subsequently evaluated based on a geometric model. To reduce uncertainties in the measurement setup, an offset correction is performed using a gear with known geometry so that systematic measurement deviations in the base circle radius approximation are compensated. The experimental investigations show that the systematic measurement deviations due to the unknown measurement setup are successfully corrected and the multi-distance measurements thereupon agree with the reference measurements within the expanded measurement uncertainty with a coverage factor of 2. Currently, a measurement uncertainty of 27 % of the required tolerance is achieved, so that the measurement approach is validated for the measurement of tooth-individual shape parameters of large gearings.

Future research will focus on extending the currently realized measurement setup with respect to the synchronization of the rotary table with the measurement signal. The use of several sensors must also be investigated so that the left and right tooth flanks can be recorded within a single measurement and the measurement time can be further reduced. In addition, for a complete validation of the multi-distance measurement approach, the influence of the systematic measurement deviation due to a repositioning of the gear teeth after calibration must be estimated. Furthermore, the investigation of the measurability of further gear shape parameters with the model-based multi-distance measurement method must be carried out in order to achieve a comprehensive evaluation of the gear geometry in the future.

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References

1. M. M. Auerswald, A. von Freyberg, and A. Fischer. Laser line triangulation for fast 3d measurements on large gears. *The International Journal of Advanced Manufacturing Technology*, 100(9):2423–2433, Feb 2019. doi:10.1007/s00170-018-2636-z.
2. F. Balzer, M. Schäfer, I. Lindner, A. Günther, D. Stöbener, and J. Westerkamp. Recent advances in optical gear measurements – a new approach for fast measurements of large gears. In VDI Wissensforum GmbH, editor, *VDI-Berichte*, volume 2255, pages 655–666, 2015.
3. F. Balzer, N. Steffens, M. Stein, and K. Kniel. Traceable measurement of large gears with micron accuracy: a mandatory basis for reliable wind energy systems. In *Engineering for a Changing World: Proceedings; 59th IWK, Ilmenau Scientific Colloquium*, volume 59, Technische Universität Ilmenau, September 11–15, 2017.
4. Y.-C. Chen and J.-Y. Chen. Optical inspection system for gear tooth surfaces using a projection moiré method. *Sensors*, 19(6): 1450, 2019. doi:10.3390/s19061450.
5. DIN EN ISO 15530-3. Geometrical product specifications (GPS) – Coordinate measuring machines (CMM): Technique for determining the uncertainty of measurement – Part 3: Use of calibrated workpieces or measurement standards (ISO 15530-3:2011), Sept. 2018.
6. DIN ISO 1328-1. Cylindrical gears – ISO system of flank tolerance classification – Part 1: Definitions and allowable values of deviations relevant to flanks of gear teeth (ISO 1328-1:2013), Mar. 2018.
7. S. Fang, X. Zhu, P. Yang, Q. Cai, M. M. Komori, and A. Kubo. Analysis and compensation method for installation error in measuring gear tooth flank with laser interferometry. *Optical Engineering*, 53(8): 084111, 2014.
8. S.-P. Fang, L.-J. Wang, M. Komori, and A. Kubo. Design of laser interferometric system for measurement of gear tooth flank. *Optik-International Journal for Light and Electron Optics*, 122(14): 1301–1304, 2011.
9. F. Franceschini, M. Galetto, D. Maisano, and L. Mastrogiacomo. Large-scale dimensional metrology (lsdm): from tapes and theodolites to multi-sensor systems. *International Journal of Precision Engineering and Manufacturing*, 15(8): 1739–1758, 2014. doi:10.1007/s12541-014-0527-2.
10. C. B. Fritz Klocke. *Zahnrad- und Getriebetechnik*. Hanser Fachbuchverlag, 2016. ISBN 3446430687. URL https://www.ebook.de/de/product/18112460/fritz_klocke_christian_brecher_zahnrad_und_getriebetechnik.html.
11. G. Goch. Gear metrology. *CIRP Annals*, 52(2): 659–695, 2003. doi:10.1016/S0007-8506(07)60209-1.
12. A. Günther, J. Peters, and G. Goch. Flächenhafte numerische Beschreibung, Ausrichtung und Auswertung von Zylinderrädern (3D-Surface-like numerical description, alignment, and evaluation of involute cylindrical gears). *tm – Technisches Messen Plattform für Methoden, Systeme und Anwendungen der Messtechnik*, 68(4/2001): 160, 2001.
13. X. Guo, Z. Shi, B. Yu, B. Zhao, K. Li, and Y. Sun. 3d measurement of gears based on a line structured light sensor. *Precision Engineering*, 61: 160–169, 2020. doi:10.1016/j.precisioneng.2019.10.013.
14. ISO 18653. Gears – Evaluation of instruments for the measurement of individual gears, Dec. 2003.
15. H. Linke, J. Börner, and R. Heß. 8 – ensuring the accuracy of cylindrical gears. In H. Linke, J. Börner, and R. Heß, editors, *Cylindrical Gears*, pages 554–589. Hanser, 2016. ISBN 978-1-56990-489-3. doi:10.3139/9781569904909.008.
16. K. Meeß, M. Kästner, and J. Seewig. Verringerung und abschätzung der messunsicherheit bei der optischen verzahnungsmessung mit streifenprojektion. *tm – Technisches Messen*, 73(11): 603–610, 2006.
17. G. N. Peggs, P. G. Maropoulos, E. B. Hughes, A. B. Forbes, S. Robson, M. Ziebart, and B. Muralikrishnan. Recent developments in large-scale dimensional metrology. *Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture*, 223(6): 571–595, 2009.

doi:10.1243/09544054JEM1284.

18. J. Peters, G. Goch, and A. Günther. Helical gear measurement using structured light. In *Proceedings of the XVI IMEKO World Congress, Wien*, pages 227–230, 2000.
19. M. Pillarz, A. von Freyberg, and A. Fischer. Determination of the mean base circle radius of gears by optical multi-distance measurements. *Journal of Sensors and Sensor Systems*, 9(2): 273–282, 2020. doi:10.5194/jsss-9-273-2020.
20. M. Pillarz, A. von Freyberg, and A. Fischer. Gear shape parameter measurement using a model-based scanning multi-distance measurement approach. *Sensors*, 20(14): 3910 (16 pp.), 2020. doi:10.3390/s20143910.
21. M. Pillarz, A. von Freyberg, D. Stöbener, and A. Fischer. Optische Messung zahnindividueller Formparameter am Beispiel von Geradverzahnungen. *tm – Technisches Messen*, 88(s1): s53–s58, 2021. doi:10.1515/teme-2021-0068.
22. C. A. Sciammarella, L. Lamberti, and F. M. Sciammarella. High-accuracy contouring using projection moiré. *Optical Engineering*, 44(9): 093605, 2005.
23. D. Stöbener, A. von Freyberg, M. Fuhrmann, and G. Goch. Areal parameters for the characterisation of gear distortions. *Materialwissenschaft und Werkstofftechnik*, 43(1–2): 120–124, 2012. doi:10.1002/mawe.201100898.
24. H. Tian, F. Wu, and Y. Gong. Gear tooth profile reconstruction via geometrically compensated laser triangulation measurements. *Sensors*, 19(7), 2019. doi:10.3390/s19071589. URL https://www.mdpi.com/1424-8220/19/7/1589.
25. M. Younes, A. M. Khalil, and M. Damir. Automatic measurement of spur-gear dimensions using laser light. Part 1: measurement of tooth thickness and pitch. *Optical Engineering*, 44(8): 087201, 2005.
26. M. Younes, A. M. Khalil, and M. Damir. Automatic measurement of spur gear dimensions using laser light. Part 2: measurement of flank profile. *Optical Engineering*, 44(10): 103603, 2005.

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