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Quantum Physics Explained by Gravity and Relativity

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Quantum Physics

Explained by Gravity and Relativity

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Book Series: Universe: Unified from Microcosm to Macrocosm, Volume 7

Hans-Otto Carmesin

February 14, 2022

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Since Planck discovered quantization in 1900, the nature of quanta was a mystery. That problem is solved in this book.

We derive the postulates of quantum physics from the equivalence principles, gravity and relativity, by analyzing the vacuum.

We clarify various conundrums of quanta:

- We derive nonlocality.

- We find the origin of the Schrödinger equation.

- We find the origin of the probabilities of quanta.

- We find the basis of the Planck constant h.

- We find a generalized Schrödinger equation.

- We find the origin of quantum gravity.

- We discover how quanta, vacuum and curved space are related.

Using the concepts of space, time, gravity and vacuum, we discover how vacuum propagates at the velocity of light. We realize how that propagation causes quantization. We apply that propagation to the calculation of the density parameter Ω_Λ of the vacuum of the universe. Our result is in precise accordance with observation, whereby we do not apply any fit.

Invited to discover the nature of quanta are classes from grade 10 or higher, courses, research clubs, enthusiasts, observers, experimentalists, mathematicians, natural scientists, researchers, etc.

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Contents

Chapter 1

Introduction

1.1 Two great basic concepts of physics

Planck (1900) discovered the **quantization of light** and the corresponding Planck constant h (table 7.1). Moreover, he made clear that quantization is a new and general physical concept. Accordingly, he proposed the Planck units or natural units, based on three universal constants: the Newton (1686) constant G of gravitation, the velocity c of light, see Rømer (1676), and the Planck constant h. Planck's discovery of the particle property of light, combined with the wave property of light, see e. g. Young (1802), demonstrated the wave particle duality of quantum objects.

Quantization is a key discovery and made possible a series of further essential results: Einstein (1905) explained the emission and absorption of photons via quantization. Bohr (1913) explained the atomic spectra on the basis of quantization. Thomson (1927) and Davisson and Germer (1927) discovered matter waves, see figure (1.2), that de Broglie (1925) had proposed on the basis of quantization, Heisenberg (1925) developed fundamental kinematic equations based on operators representing observables of quantum objects, see also Dirac (1925). Schrödinger (1926a) proposed the differential equation describing the time evolution of quantum objects, the **Schrödinger equation**. It

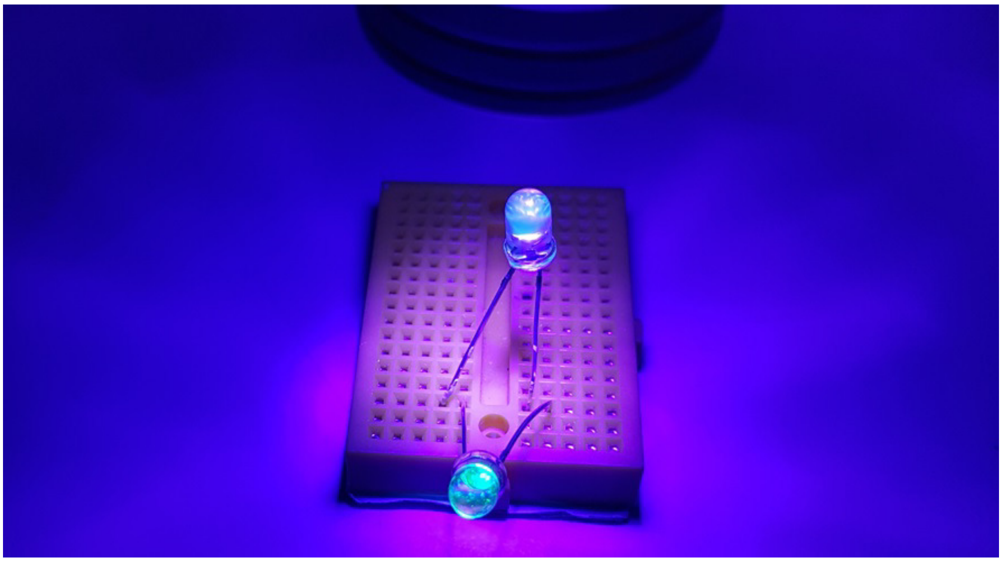


Figure 1.1: The upper LED absorbs quanta of light, photons, and transforms the energy of light into electric energy. The lower LED transforms that electric energy into emitted photons. The experiment is an example for the absorption and emission of photons, whereby the band structure of the LEDs determines the green color of the emitted light.

is physically equivalent to the dynamics proposed by Heisenberg (1925). Born (1926) proposed a **probabilistic interpretation** of quantum objects that is in precise accordance with observation.

Fermi (1926) derived the energy distribution in quantum gases, which is a basis for the understanding of the band structure used in electronics and computers, see figure (1.1). Heisenberg (1927) derived the uncertainty of quantum objects. Dirac (1927) elaborated wave functions in spacetime and thereby proposed antimatter, which was discovered by Anderson (1933). These results establish a predictive, successful as well as useful experimental and theoretical basis of quantum physics, QP, see e. g. Sakurai and Napolitano (1994), Ballentine (1998), Kumar (2018).

However, Einstein et al. (1935) pointed out that according to

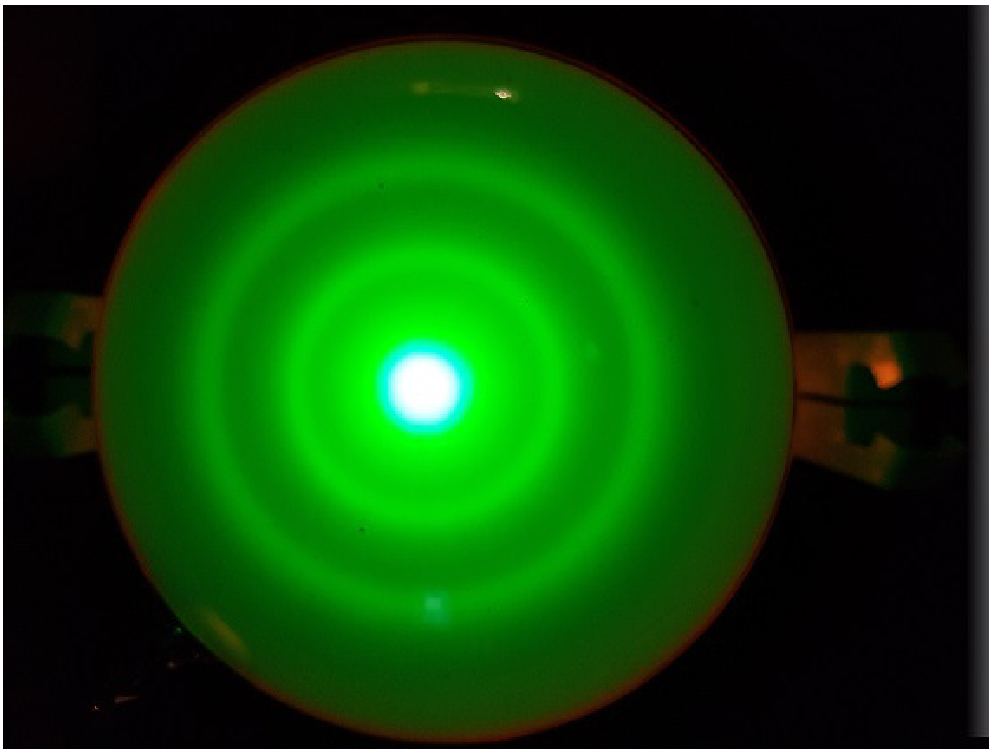


Figure 1.2: The two concentric rings indicate electron waves arriving at a screen: For it, electrons have been accelerated by a voltage of 5000 Volts. Then the electron beam has been diffracted at a slice consisting of many crystals of graphite. Behind that slice, the electrons propagate either at the original direction and form the central light at the screen. Or they propagate at one of two cones around the central beam, whereby these cones cause the two concentric rings when arriving at the screen.

the theory of QP outlined above, a measurement of a quantum object could instantly cause a correlation with a measurement of another distant quantum object. Moreover, Einstein et al. (1935) presumed that no correlation could correspond to a velocity larger than the velocity c of light. This presumption corresponds to the fact that the velocity c is the maximal velocity in special relativity, SR, and (in most frames) in general relativity, GR.

Indeed, de Sitter (1913) confirmed that light emitted from binary stars propagates to Earth at the same velocity of light, irrespective of the velocities of the two stars of the binary, see figure (1.3). You can confirm that observation on your own by using a small telescope, see Carmesin (2006). For further tests of relativity, see e. g. Bailey et al. (1977), Will (2014). Moreover, the invariance of c confirmed via observation by de Sitter (1913) or Carmesin (2006) can alternatively be obtained via a thought experiment, see section (7.8).

In fact, Einstein (1905) proposed SR and Einstein (1915) initiated GR, which includes gravity. Today, **quantum physics and relativity** are regarded as the two great and basic concepts of physics, see e. g. Weinberg (2017), Bricmont and Goldstein (2019).

According to the above outlined instant correlations of QP, Einstein et al. (1935) proposed that the above theory of QP should be incomplete, as a consequence. More generally, a physical effect corresponding to a velocity $v \leq c$ is called local, whereas other effects are called nonlocal.

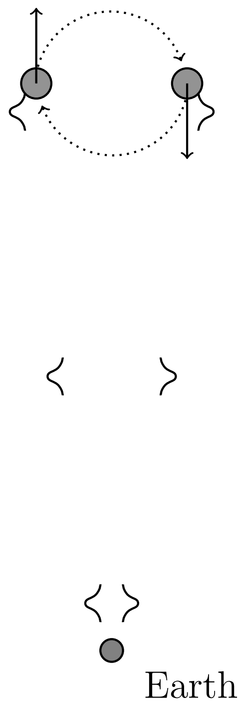


Figure 1.3: Binary star: two stars rotate around their center of mass. For instance, when the stars have the same distance to Earth, they emit one light signal each. These signals arrive at Earth simultaneously, though the emitting stars move in opposite directions. Such observations confirm that light propagates at a constant velocity relative to an observer, irrespective of the velocity of the light emitting source relative to the observer, see e. g. de Sitter (1913), Carmesin (2006).

Correspondingly, Einstein et al. (1935) pointed out that the theory of QP outlined above is nonlocal. Additionally, they presumed that nature should be local, and so they constructed a paradoxical situation, the so-called EPR paradox.

Bell (1964) proposed experimental tests of such correlations, and Aspect et al. (1982) confirmed these correlations experimentally. So it is clear that nature is nonlocal, and nonlocal effects become visible in QP. However, the relation between QP and the theories of relativity, SR and GR, remains a conundrum, even after the experimental results achieved by Aspect et al. (1982), see e. g. Weinberg (2017), Bricmont and Goldstein (2019).

In this book, we resolve the relation between QP and relativity: A first hint was provided by Carmesin (2021d) by showing that even relativity is nonlocal. That finding opened the possibility that QP could be included in GR. Accordingly, that finding made conceivable that GR might be used in order to explain QP. In fact, that is possible.

Indeed, we derive the quantization and the postulates of QP as a consequence of GR. Hereby, we apply an especially clear formulation of relativity and gravity as well as relatively simple mathematical tools. Accordingly, we use the very transparent understanding of space and time in order to explain QP. Correspondingly, students and interested people can now understand QP in an especially transparent and elucidating manner on the basis of space and time.

1.2 Our aim

In this book we derive the quantum physics from gravity and relativity.

1.3 Our method

We achieve our aim as follows. Firstly, we introduce our general model, based on gravity and relativity, see section (2.1). Secondly, we derive quantum physics from our model.

1.3.1 On the derivation of quantum physics

After the definition of our model, including relativity and gravity, we derive the **postulates of quantum physics**, as they have been summarized in Kumar (2018).

For it, we derive the **duality of particles and waves** by developing a new **duality transformation**. Thereby, we discover the nature of the wave functions in a very precise manner.

1.3.2 Summary of postulates of quantum physics

In this part, we summarize the postulates of quantum physics, as they have been summarized in Kumar (2018). Other formulations are essentially equivalent and can be found in Sakurai and Napolitano (1994) or Ballentine (1998), for instance.

1.3.2.1 Postulates of quantum physics

There are four postulates describing the **quantum state**, the **dynamical variable or observable**, the **time evolution** and the **probability rule**:

(Postulate 1) A **quantum state** is defined by a wave function $\psi(t, \vec{r})$, see e. g. (Kumar, 2018, p. 14).

(Postulate 2) *'To each* **dynamical variable or observable** A *there corresponds a linear operator* $\hat{A}$*, and the possible values of the dynamical variable are the eigenvalues of the operator'*, see (Ballentine, 1998, p. 43), see also (Kumar, 2018, p. 18, 36).

(Postulate 3) The **time evolution** of the wave function is described by the Schrödinger equation, SEQ, see Schrödinger (1926a), Schrödinger (1926b), (Kumar, 2018, p. 32) or section (1.3.2.3). Thereby, the Schrödinger equation sets $i\hbar\partial_t\psi(t, x)$ equal to the operator of the energy term, consequently denoted by $\hat{E}_{term}$, multiplied by $\psi(t, x)$.

$$i\hbar\partial_t\psi(t, x) = \hat{E}_{term}\psi(t, x) \tag{1.1}$$

(Postulate 4) Born (1926) discovered the **probability rule**. It states that the probability to find a quantum object in a state $\psi(t, \vec{r})$ is proportional to the square of the absolute value of the wave function $|\psi(t, \vec{r})|^2$, see e. g. (Ballentine, 1998, p. 46) or (Kumar, 2018, p. 169).

1.3.2.2 Conventional operators of dynamical variables

In this section, we introduce and test some conventional operators typically used in quantum physics.

The momentum operator is as follows:

$$\hat{p}_x = -i\hbar\partial_x \tag{1.2}$$

In order to test it, we use the wave function of a freely propagating object, whereby we apply the usual sign convention in the exponent, see e. g. (Kumar, 2018, Eq. 3.2.11), and we denote the amplitude or normalization factor by f_n:

$$\psi(t,x) = f_n \cdot e^{-i\cdot\omega\cdot t + i\cdot k\cdot x} \tag{1.3}$$

Next we test whether the eigenvalue is the momentum $p = \hbar \cdot k$:

$$\hat{p}_x\psi(t,x) = -i\hbar\partial_x\psi(t,x) = -i\hbar \cdot i \cdot k\psi(t,x) \tag{1.4}$$

We simplify the above equation:

$$\hat{p}_x\psi(t,x) = \hbar \cdot k\psi(t,x) = p \cdot \psi(t,x) \tag{1.5}$$

Obviously, the momentum operator in Eq. (1.2) operates in the physically correct manner.

The following operator provides the energy:

$$\hat{E} = i\hbar\partial_t \tag{1.6}$$

Next we test whether the eigenvalue is the energy $E = \hbar \cdot \omega$:

$$\hat{E}\psi(t,x) = i\hbar\partial_t\psi(t,x) = i\hbar\partial_t f_n \cdot e^{-i\cdot\omega\cdot t + i\cdot k\cdot x} \tag{1.7}$$

We evaluate the above equation:

$$\hat{E}\psi(t,x) = i\hbar(-i)\omega \cdot \psi(t,x) = \hbar \cdot \omega \cdot \psi(t,x) = E \cdot \psi(t,x) \tag{1.8}$$

Obviously, the operator in Eq. (1.6) operates in the physically correct manner, as the eigenvalue is the energy.

In the above tests, we used the wave function of a freely propagating wave in Eq. (1.3). According to the concept of the Fourier analysis, we can form a complete class of wave functions by (discrete or continuous) linear combinations of wave functions of a freely propagating waves. Correspondingly, our tests of the operators are very general.

1.3.2.3 Schrödinger equation for two typical systems

In this section, we elaborate the Schrödinger equation for two typical systems.

For instance, the non-relativistic kinetic energy is as follows:

$$E_{kin,non-relativistic} = \frac{p^2}{2m} \tag{1.9}$$

We insert the momentum operator, see Eq. (1.2):

$$\hat{E}_{kin,non-relativistic}\psi(t,x) = -\frac{\hbar^2}{2m}\partial_x^2\psi(t,x) \tag{1.10}$$

So the corresponding Schrödinger equation is as follows, for the case of a one dimensional system, for instance:

$$i\hbar\partial_t\psi(t,x) = -\frac{\hbar^2}{2m}\partial_x^2\psi(t,x) \tag{1.11}$$

For the case of an object with zero rest mass m_0, for instance, the relativistic kinetic energy is as follows:

$$E_{kin,relativistic} = p \cdot c \tag{1.12}$$

We insert the momentum operator, see Eq. (1.2):

$$\hat{E}_{kin,relativistic}\psi(t,x) = -c \cdot i\hbar\partial_x\psi(t,x) \tag{1.13}$$

So the corresponding Schrödinger equation is as follows:

$$i\hbar\partial_t\psi(t,x) = -c \cdot i\hbar\partial_x\psi(t,x) \tag{1.14}$$

1.3.3 On the explanation of quantum physics

Using the derivation of the postulates of quantum physics from our general model including gravity and relativity, we derive the explanation of quantum physics. Thereby, it turns out that the wave function ψ in QP represents the rate of change of vacuum $\dot{\varepsilon}$, see the glossary in section (7.7).

1.3.4 On the formation of space

According to the explanation of the wave function ψ in QP by the rate of change of vacuum $\dot{\varepsilon}$, we analyze the integrated rate of change of vacuum $\dot{\varepsilon}$. It turns out that the integral of the rate of change of vacuum $\dot{\varepsilon}$ ranging from here towards the Hubble radius R_H, see the glossary in section (7.7), represents the present day vacuum. This result is in precise accordance with observation, whereby no fit is applied, see chapter (4).

We regard this finding as a great result: The same rate of change of vacuum $\dot{\varepsilon}$ explains quantum physics as well as the formation of the vacuum and space in the universe.

Chapter 2

Universal Model

In this chapter, we introduce the basics of a general model for a physical object. Additionally, we summarize the used physical concepts.

2.1 Our general model

In this section, we introduce the basics of a general model for a physical object.

Firstly, the object has an **energy** E or an equivalent dynamical mass or mass M.

Secondly, it is possible to analyze or measure the **object's gravitational and relativistic effects upon its vicinity**, even without considering the **object's internal structure** explicitly.

- Thereby, essential **object's effects** upon its vicinity include the gravitational field G^*, the curvature of space or of spacetime and properties of the vacuum, see e. g. Carmesin (2021d) or chapter (3).

- Moreover, the object's **internal structure** may be the electrons of a helium atom, or the atoms of a molecule, or the quarks of a neutron, and so forth. For instance, if large molecules are diffracted at a grating, then the diffraction pattern can be derived from the momentum of the molecules, without considering the atoms of the molecule, the electrons, neutrons and protons of the atoms or the quarks inherent to neutrons and protons, see figure (3.14) or Nairz et al. (2003).

- I emphasize that our model describes the behavior of very different objects such as photons, electrons, neutrons or even atoms and molecules, though we only apply gravity, relativity and the vacuum, see chapter (3).
- The fact that our model describes the behavior of very different objects means that our model is fundamental, see chapter (3).

Thirdly, the object may be **localized**.

- In that case, we might illustrate the object with help of a center of mass or of dynamical mass, see figure (2.1).
- In that case, we might illustrate the distribution of the mass or of dynamical mass by an unstructured grey region, representing the object's internal structure, see figure (2.1).
- In the case of a **localized object**, we analyze object's effects upon its vicinity with help of a small mass m as a probe.

Fourthly, several objects may form a **homogeneous density**, whereby we do not measure or analyze the internal structure of the objects, or a possible location of the centers of mass of the objects.

Fifthly, we generalize the above two cases of a **localized object** and of a **homogeneous density of objects** with help of tensors, see chapter (3).

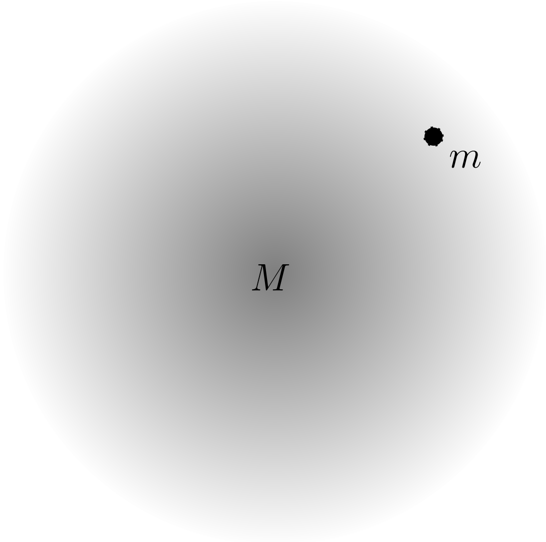


Figure 2.1: Our model: In the case of a localized object, mass or dynamic mass M is distributed around a center of mass. The object's effect upon its surroundings is analyzed with help of a small mass, similar as in a gedankenexperiment. Internal structures of the object are possible, but they are not shown here in an explicit manner.

Moreover, the theoretical basis of our model is constituted by the following quadruple, we call it the **spacetime-quadruple, SQ**:

1. Principles of free fall, PFF, see section (2.2.2)

2. Gaussian gravity, GG, see section (2.3)

3. Special relativity, SR, see e. g. Einstein (1905). Here we apply SR for the case of non-quantized objects, since we derive quantization therefrom, see chapter (3).

4. Formed vacuum with a corresponding volume, see e. g. Carmesin (2021d) or e. g. Carmesin (2021a), Carmesin (2021e) or section (2.6).

$$\text{spacetime} - \text{quadruple}, \text{SQ} = \{PFF, GG, SR, \text{formed vacuum}\} \quad (2.1)$$

2.2 Principles of free fall

In this section, we treat principles that hold for a freely falling object or system.

2.2.1 Galileo's equivalence principle

Galileo (1638) considered experiments with objects with different masses falling from the tower at Pisa, see figure (2.2). While Aristotle (C350) thought that bodies with a large mass would fall faster than bodies with a smaller mass, Galileo realized that both bodies fall equally fast, if the friction of the air is negligible. Galileo (1638) obtained his result by a gedankenexperiment: If a body with a large mass m is divided into two parts with masses $m/2$ each, then theses parts must fall at the same velocity as m, as the parts and the body arrive at the bottom at the same time. Probably, Galileo did not perform these experiments in reality, see e. g. Schlichting (1999).

In principle, he realized the following: If a mass M generates a gravitational field G^* (Eq. 2.4), and if a freely falling probing mass m experiences the corresponding force $F = m \cdot G^*$, then m exhibits an acceleration a that is equal to the field G^*. So the equality of the inertial mass and the gravitational mass explains the observation. Accordingly, he regarded that equality as a principle:

$$\vec{G}^* = \vec{a} \rightarrow \quad \text{Galileo's equivalence principle, GEP} \tag{2.2}$$

Also many modern tests confirmed that, see e. g. Will (2006).

2.2.2 Einstein equivalence principle

Einstein used Galileo's equivalence principle and extended it by two statements. So the corresponding Einstein equivalence principle, EEP, includes three items, see e. g. Will (2014):

1. Galileo's equivalence principle

2. The outcome of any local non-gravitational experiment is independent of the velocity of the freely falling reference frame in which it is performed.

3. The outcome of any local non-gravitational experiment is independent of where and when in the universe it is performed.

Also the EEP has been confirmed by many experiments, see e. g. Will (2014).

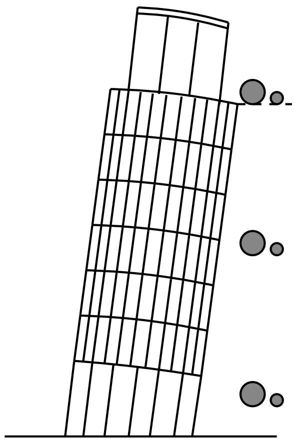


Figure 2.2: Galileo analyzed experiments with different falling objects at the tower in Pisa, see Galileo (1638), Schlichting (1999). If two bodies with different masses are started at the top at the same time, then they arrive at the same time in the middle and near the bottom. This fact holds in the ideal case of zero friction.

2.2.3 Principle of energy conservation at free fall

Energy conservation is a very general principle of nature. However, the energy depends on the chosen frame. For instance, if you ride on your bicycle on a road, then your kinetic energy in the frame of the bicycle is zero, whereas your kinetic energy is nonzero in the frame of the road. This example shows that the principle of conservation of energy makes sense only in a particular frame.

In what frame is the energy conserved, if a mass or dynamical m falls freely towards a mass or dynamical mass M?

An appropriate frame is a HUF, as external fields and external accelerations add up to zero in a HUF. Then the center of mass frame of M and m is an inertial frame. In that frame, the energy is conserved.

Moreover, the treatment of the free fall becomes especially simple, if the mass m is very small compared to M. Then the energy of the mass m is conserved in the frame of M, whereby M falls freely relative to any external gravitational fields or effects.

This energy conservation includes the case of an isotropically distributed mass M interacting with itself, see figure (3.4) and section (3.5).

2.2.4 Summarized principles of free fall

In the following, we combine Galileo's equivalence principle, Einstein's equivalence principle and the principle of energy conservation at free fall to the **principles of free fall, PFF**:

$$PFF = \{GEP, EEP, \text{energy conservation at free fall}\} \quad (2.3)$$

2.3 On Gaussian gravity

The first essential theory of gravity is Newton's gravity, NG, see e. g. Newton (1686). We identify four essential parts of NG: Firstly, according to Newton, (Newton, 1686, p. 78), space is absolute and at absolute rest Secondly, Newton (Newton (1686)) used Euclidean geometry, which presumes flat space, see e. g. Euklid (C325). Thirdly, Newton presumed absolute time that goes on at a constant rate and in the same manner everywhere in space, see (Newton, 1686, p. 79). Fourthly, a mass is the source of gravity, see (Newton, 1686, p. 397) and Gauss (1809).

The third part about time has been generalized in special relativity, SR. The first and second part about space have been generalized in general relativity, GR. The fourth part has been generalized only slightly by the fact that mass is equivalent to energy and both (mass and energy) are sources of gravity. However, the essential part of gravity did not change: there are sources of gravity, these are mass as well as energy.

Accordingly, we will use that fourth part of NG, whereby we include energy as an additional source of gravity. We denote that fourth part of NG by *Gaussian gravity, GG.*

The idea of Gaussian gravity is simple and robust: A mass M generates a gravitational field $\vec{G}^*$, spreading uniformly in the vicinity. For an illustration see figure (2.3). We apply GG locally in a freely falling system, so it is applicable without any loss of generality. Accordingly, the field G^* generated by a mass M at a distance r is as follows:

$$|\vec{G}^*| = \frac{G \cdot m}{r^2} \tag{2.4}$$

Hereby G denotes the gravitational constant (Sect. 7.1).

Gaussian gravity was discovered on the basis of the motions of the planets as follows: Tycho Brahe observed the motions of the planets, see Brahe and Kepler (1627). Analyzing these results, Kepler (1619) discovered the Kepler laws of planetary motions. Huygens (1673) discovered the law of radial force. Newton (1686) combined the radial force with Kepler's laws of planetary motions and discovered Newton's law of gravitation. Note that this combination can be derived at a single page, see e. g. (Carmesin et al., 2021, p. 108-109). Gauss (1809) elaborated the essence of the generation of gravity by sources such as masses.

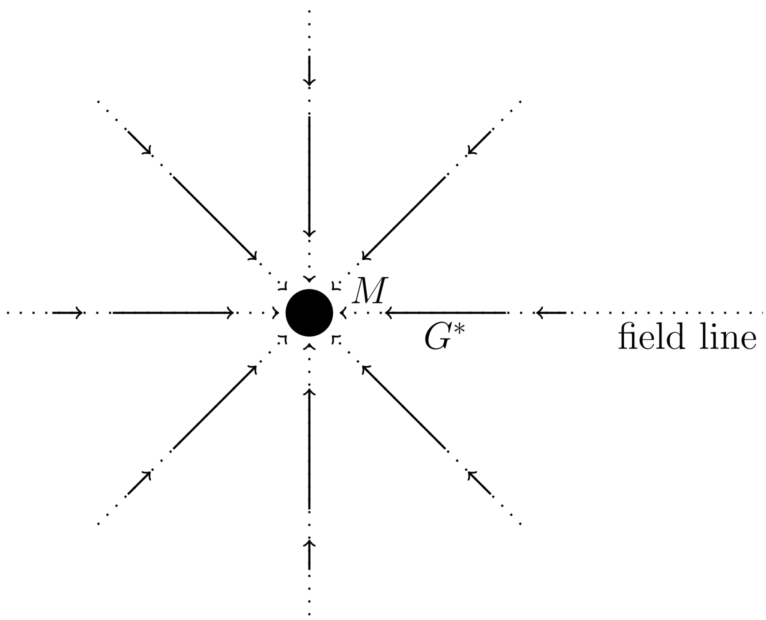


Figure 2.3: Mass M with field lines (dotted) and vectors (solid) of the gravitational field $\vec{G}^*$.

2.3.1 Field G^* as a function of the radial coordinate r

In this section, we derive the field[1] in the vicinity of a mass M. Thereby, the field is a function of the radial coordinate r, whereby M is at the coordinate $r = 0$. In general, the space can be elongated in the radial direction. Thereby, a coordinate difference dr may be elongated to a length dL, as a function of r. In the following we show that this has no effect on the function $G^*(r)$.

There is no gravity in the horizontal direction, by definition. Therefore there is no spatial elongation in this direction. Thus a circle with a radius r and with its center at a field-generating mass M at the **radial coordinate** $r = 0$ has the following

[1] Usually, we emphasize a field generating mass by a large letter M. Of course, all masses are in principle equal in physics. The distinction between a field generating mass and a probing mass is just a method of the analysis. It can easily be avoided by considering both masses as field generating masses and probing masses simultaneously. The above distinction may be appropriate, when one mass is relatively large compared to the other. Whenever a high accuracy is essential, then this distinction is not appropriate, of course.

circumference U:

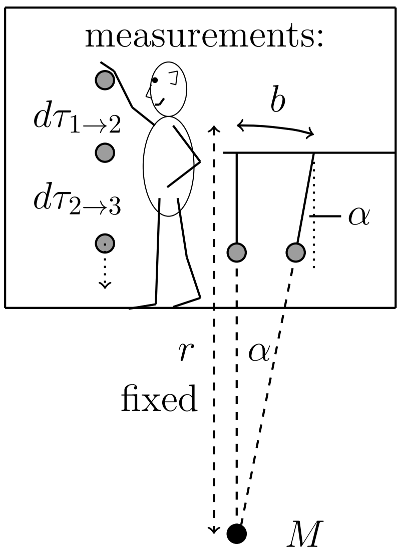
$$U = 2\pi \cdot r \tag{2.5}$$

Likewise, a sphere with the center at $r = 0$ and with the **radial coordinate** r has the following surface A:

$$A = 4\pi \cdot r^2 \tag{2.6}$$

With it we derive G^*:

$$\boxed{G^*(r) = -\frac{G \cdot M}{r^2}} \tag{2.7}$$



evaluation:

$r = \frac{b}{\alpha}$

for $j = 1$ and $j = 2$:

$dr_{j\to j+1} = r_{j+1} - r_j$

$v_{j\to j+1} = \frac{dr_{j\to j+1}}{d\tau_{j\to j+1}}$

$dv = v_{2\to 3} - v_{1\to 2}$

$d\tau = \frac{d\tau_{1\to 2}}{2} + \frac{d\tau_{2\to 3}}{2}$

$a = \frac{dv}{d\tau} = G^*$

$M = -\frac{G^* \cdot r^2}{G}$

Figure 2.4: A local observer localized at an object at r measures: Two hand leads provide the angle α and the arc length b. A falling ball yields time intervals in the observer's frame $d\tau_{j\to j+1}$. Therefrom r, v, a, G^* and M are evaluated.

2.3.2 Local measurements in curved spacetime

In this section, we derive physical quantities that can be measured locally in the vicinity of a mass M. In particular, the field can be measured. An object at a coordinate r can be investigated in the object's own frame: In particular, a local observer localized at the object can measure the **radius** r, the 'object's own time' $d\tau$, the velocity $v = \frac{dr}{d\tau}$ relative to the mass M, the acceleration $a = \frac{dv}{d\tau}$ and the mass M as elaborated in Fig. (2.4). We summarize our results:

$$v = \frac{dr}{d\tau} \text{ and } a = \frac{dv}{d\tau} \text{ can be measured locally in GR} \quad (2.8)$$

2.4 On special relativity

Einstein (1905) introduced special relativity, SR, in order to describe **non-quantized objects** that move at relatively high velocity v and $v \leq c$. (see also Hobson et al. (2006), Carmesin et al. (2022), Straumann (2013), Moore (2013), or Carmesin (2020b)).

Einstein (1905) introduced the **special relativity theory, SR**, in order to describe objects with high velocity in various **inertial frames**, these are frames that are not accelerated. Thereby, Einstein assumed that the velocity of light c is an invariant. This has been confirmed, for instance by de Sitter (1913) or by Will (2014), see Fig. (1.3). As a consequence, space and time are no longer invariant, instead they form a four dimensional **spacetime**, see e. g. Einstein (1905) or Carmesin (2020c), Carmesin (2020b).

For instance, if two events occur within an object resting in its **own inertial frame**, then the time interval Δt beginning at the first event and ending at the second event depends on the inertial frame measuring Δt. The shortest Δt is measured in the own frame of the object, while the corresponding intervals are longer in external frames moving at a velocity v relative to the object:

$$\Delta t_{own} \leq \Delta t_{external} = \Delta t_{own} \cdot \gamma \text{ with } \gamma = \frac{1}{1 - v^2/c^2} \quad (2.9)$$

Thereby γ is called **Lorentz factor**, and v is the corresponding velocity.

2.5 On general relativity

Einstein (1915) introduced general relativity, in order to describe acceleration and gravity, in addition to special relativity (see also Hobson et al. (2006), Carmesin (1996), Carmesin et al. (2022), Straumann (2013), Moore (2013)).

2.5.1 General relativity is mesoscopic

The usual theory of GR is based on curvature. In general, curvature can be measured in terms of radii of curvature, see figure (2.5). For it, at least three smallest regions are necessary. In this sense, the usual theory of GR is mesoscopic.

As GR is mesoscopic, while we derive a theory of elementary objects, we do not use results of GR here. However, we use the essential concept of GR that spacetime is modified by mass and energy. If we need results in GR, we derive these results on our own.

In fact, we derive the mesoscopic curvature of spacetime on the basis of our microscopic description of the vacuum, see e. g. Carmesin (2021d) or section (2.6). So we confirm that spacetime is curved at a mesoscopic level.

2.6 Formed vacuum

We realized that the curvature of GR is a mesoscopic concept, see figure (2.5). Accordingly, we need a really microscopic concept. For it, we realize that vacuum is permanently formed, according to the expansion of space since the Big Bang. Accordingly, we use the volume δV of the formed vacuum at one microscopic location per time δt and per existing volume dV. Carmesin (2021d) proposed and analyzed that concept.

Thereby, formed vacuum with its corresponding volume δV can be added and integrated. This fact is very deeply founded:

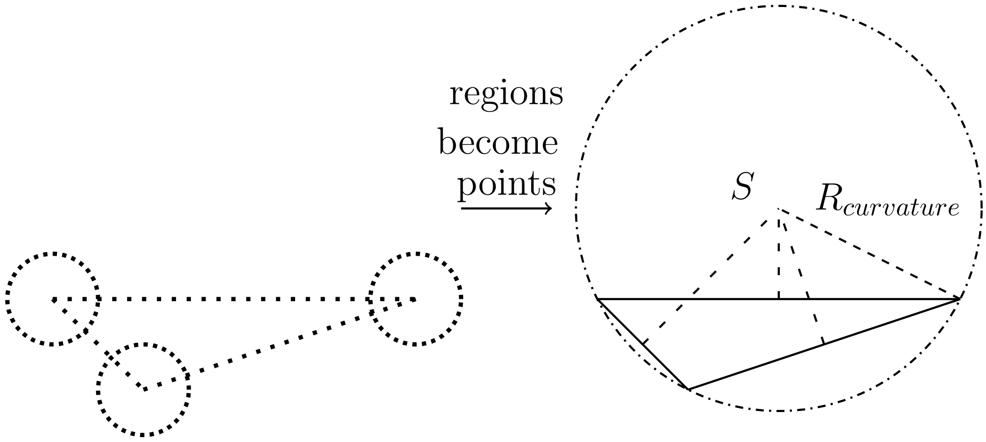


Figure 2.5: Three smallest regions are marked by three balls (dotted) and form a triangular construct (loosely dotted). The circumcircle (dashdotted) with its circumcentre S and the circumradius $R_{curvature}$ can be constructed. That curvature can be used as a radius of curvature. In that manner, a radius of curvature can be measured by using three smallest regions.

Volume can be added. An independent foundation of the addition of vacua is the addition of energies, in particular of the dark energy, which is the energy of the vacuum. Correspondingly, the principle of linear superposition holds for formed vacuum and for formed volume.

Moreover, the formed vacuum propagates at the velocity of light c, for the following reason: If the formed vacuum would propagate at a smaller velocity $v_{vac} < c$, then it would be possible to measure a velocity $v < c$ of an object relative to the vacuum. However, such a velocity $v < c$ relative to the vacuum cannot be measured, according to SR. According to SR, non-quantized objects do not exhibit velocities $v > c$. Note that interesting consequences of SR are derived in section (5.2.5.9).

2.7 Homogeneous universe frame HUF

In this section, we develop a tool that can be used for an analysis of gravity in the universe: In the universe, there are

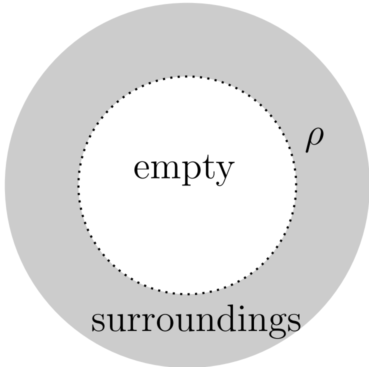


Figure 2.6: Empty ball embedded in a homogeneous surrounding: It establishes the **homogeneous universe frame, HUF**.

many sources of gravitational fields, such as masses or dynamical masses. While in electromagnetism, the fields do usually cancel or screen each other as there are two signs of the charges, this is not so in gravity. However, fields emerging at different parts of the universe can cancel each other, as vectors can have opposite directions. This mechanism is treated in the present section.

While in figure (2.3), we investigated the field of a mass without analyzing the surroundings, we consider the surroundings in this section. In particular, we analyze the field in an empty ball, embedded in surroundings with a homogeneous density, a **homogeneous fluid** (Fig. 2.6, Carmesin (2020b), Carmesin (2021d)). We emphasize that there is not even vacuum in this ball, so it is a purely mathematical model, as physical space is constituted by vacuum. So the empty ball is a tool[2] used for the analysis of the vacuum.

Newton (1686) showed that there is no field in such a sphere. For the case of the GR, Birkhoff (1921) derived that there is no field in that sphere. We introduce a corresponding frame:

[2]In GR, results are often derived by using an appropriate frame (see for instance Straumann (2013), Stephani (1980), Moore (2013)).

Definition 1 Homogeneous universe frame, HUF

(1) If an empty ball is embedded in homogeneous surroundings ranging from the ball to the light horizon[3]*, and if that ball is not accelerated, then the frame with the origin at the center of the ball is called* **homogeneous universe frame** *(Fig. 2.6).*

(2) A **vacuum HUF,** HUF_v *is a HUF for which the surroundings have the following property: The density parameters of radiation* Ω_r *and of the matter* Ω_m *(table 7.3) tend to zero. So the surroundings of a* HUF_v *consist of vacuum, up to an infinitesimal amount of radiation and matter, while* $\Omega_K \approx 0$ *(table 7.3).*

The field $\vec{G}^*$ is zero in the HUF. More realistically, the density of the surroundings exhibit fluctuations. These are analyzed in quantitative detail in Carmesin (2021d). As a result, the average of the field $\langle \vec{G}^* \rangle$ is zero in the HUF, and the variance $(\Delta \vec{G}^*)^2$ is nonzero. However, the variance $(\Delta \vec{G}^*)^2$ is a function of the radius R of the HUF, and that function decreases according to a power law: $(\Delta \vec{G}^*)^2 \propto R^{5-2D}$ for each dimension $D \geq 3$. The field variance $(\Delta \vec{G}^*)^2$ is particularly small for the case of a vacuum HUF. We summarize our findings:

Proposition 1 The HUF has the following properties

The gravitational field is zero in the empty ball of the HUF.

A single object that might be added in the HUF does not experience any force or acceleration.

If there are fluctuations of the density in the surroundings, then the average of the field $\langle \vec{G}^* \rangle$ *is zero in the HUF, and the variance of the field* $(\Delta \vec{G}^*)^2$ *tends to zero as the radius* R *of the HUF tends to infinity.*

In the vacuum HUF, the variance of the field $(\Delta \vec{G}^*)^2$ *is particularly small.*

---

[3]According to the principle of translation invariance, neighboring HUFs include space beyond the HUF.

Chapter 3

Derivation of Quantum Physics

In this section, we derive quantum physics from the spacetime-quadruple, SQ.

3.1 Our derivation of the Schwarzschild metric: energy factor

In this section, we derive the energy factor $\epsilon(R)$. That factor characterizes the Schwarzschild metric, SM, based on the spacetime-quadruple.

Definition 2 Field generating mass frame, FMF
If a mass M (Fig. 2.4) is in a HUF, then there is a frame with M at its origin and with a radial coordinate r. We call it the **field generating mass frame, FMF**.

According to the PFF, we obtain the following results:

Proposition 2 Local observer in a freely falling frame
If a mass M and a local observer at a fixed distance r relative to M (Fig. 2.4) are in a HUF, then the following holds:

(1) The situation can be analyzed in the FMF.

(2) The mass M generates a radial gravitational **field** *with the value $|\vec{G}^*| = \frac{GM}{r^2}$.*

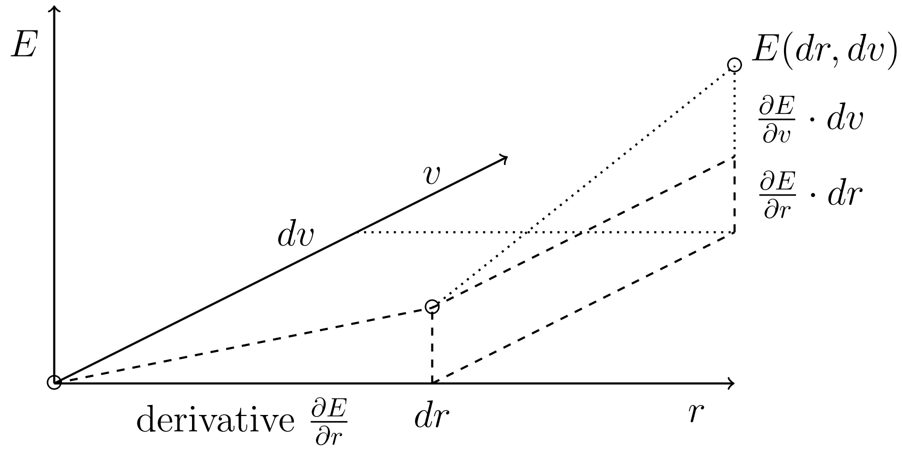


Figure 3.1: Change dE of $E(r,v)$ ($\circ$): The two slope triangles result in the changes $\frac{\partial E}{\partial v} \cdot dv$ and $\frac{\partial E}{\partial r} \cdot dr$. The total change $dE = E(r+dr, v+dv) - E(r,v)$ is the sum $dE = \frac{\partial E}{\partial v} \cdot dv + \frac{\partial E}{\partial r} \cdot dr$.

(3) A the local observer at r falling freely in the radial direction (Fig. 2.4) can **locally observe** *the body's radial velocity* $v(r) = \frac{\partial r}{\partial \tau}$ *and its radial coordinate* r *of the FMF.*

3.1.1 Freely falling mass m

In this section we derive the **energy function** $E(r,v)$ of a mass m that is falling in the field of a mass M, and that starts at $r \to \infty$ and $v = 0$. Thereby, the velocity v and the radius r are measured relative to the mass M, and the own mass or rest mass is denoted by m_0. Solutions with more general initial conditions are elaborated in (Carmesin (2020b)).

For it we apply the **principle of energy conservation** (see Mayer (1842) or PFF). In particular, we apply the **relativistic energy** derived in SRT (Einstein (1905) or Carmesin (2020b)):

$$E(v) = m_0 \cdot c^2 \cdot \gamma(v) \quad \text{in SR and with} \quad \gamma(v) = \frac{1}{\sqrt{1 - v^2/c^2}} \tag{3.1}$$

As m is falling, the velocity v increases and r decreases. Hence the energy would increase by the factor $\gamma(v)$ according to

Eq. (3.1). Correspondingly, the energy decreases by a **position factor** $\epsilon(r) = 1/\gamma(v)$, so that the energy is conserved. So we get:

$$E = m_0 \cdot c^2 \cdot \gamma(v) \cdot \epsilon(r) \quad \text{with} \quad \gamma(v) = 1/\epsilon(r) \tag{3.2}$$

The functional term of $\epsilon(r)$ must be determined. We consider the change dE of the energy, which obviously depends on r and v (Fig. 3.1). Accordingly we get:

$$dE = \frac{\partial E}{\partial r} dr + \frac{\partial E}{\partial v} dv \tag{3.3}$$

From this equation we obtain a differential equation, DEQ, for $\epsilon(r)$. According to the principle of energy conservation, dE is zero. The derivative regarding v is $\frac{\partial E}{\partial v} = E \cdot \gamma^2 \cdot v/c^2$, while the derivative regarding r is $\frac{\partial E}{\partial r} = E \cdot \epsilon'/\epsilon$ with $\epsilon' = \frac{d\epsilon}{dr}$. So we get:

$$0 = E \cdot \frac{\epsilon'}{\epsilon} \cdot dr + E \cdot \gamma^2 \cdot \frac{v}{c^2} \cdot dv \tag{3.4}$$

We divide by E and $d\tau$ and use $v = \frac{dr}{d\tau}$ and $a = \frac{dv}{d\tau}$ (Eq. 2.8 and Fig. (2.4). We also resolve for ϵ'. Therefore we obtain:

$$\epsilon' = -\frac{\epsilon \cdot \gamma^2}{c^2} \cdot a \tag{3.5}$$

We use $\gamma(v) = 1/\epsilon(r)$ (Eq. 3.2). We utilize the equivalence principle of the GR $a = -G^* = -\frac{G \cdot M}{r^2}$ (Eq. 2.7, here a is directed downwards, see Fig. 2.4), too. So we derive:

$$\epsilon' = \frac{1}{\epsilon \cdot c^2} \cdot \frac{G \cdot M}{r^2} \tag{3.6}$$

We use the well known term $R_S = \frac{2G \cdot M}{c^2}$ for the **Schwarzschild radius**. So we get the following DEQ for $\epsilon(r)$:

$$\boxed{\epsilon' = \frac{1}{\epsilon} \cdot \frac{R_S}{2r^2}} \tag{3.7}$$

Solution of the DEQ for ϵ: For the case of a constant mass M, we solve the DEQ for ϵ with the following Ansatz:

$$\boxed{\epsilon(r) = \sqrt{1 - \frac{R_S}{r}}} \tag{3.8}$$

The derivative corresponds to the DEQ (3.7). So Eq. (3.8) is a solution. We use the two factors $\epsilon(r)$ and $\gamma(v)$ in Eqs. (3.2, 3.8, 3.1)). So we get a term for the **invariant** energy depending on r and v:

$$\boxed{E(r,v) = m_0 \cdot c^2 \cdot \frac{\sqrt{1 - \frac{R_S}{r}}}{\sqrt{1 - v^2/c^2}}} \tag{3.9}$$

This term generally represents the functional dependence of the energy on r and v. Landau and Lifschitz (1971) obtain the same result (page 299), this confirms our derivation. We summarize:

Proposition 3 Energy in the FMF

If a **field generating mass** *M is in a HUF, then an* **own mass** *m_0 has the following properties:*

(1) The mass m_0 can be analyzed in the FMF.

(2) In the FMF, M generates a radial gravitational **field** *with the value $G^* = |\vec{G}^*| = \frac{GM}{r^2}$.*

(3) A local observer at r can **locally observe** *the body's radial velocity $v(r) = \frac{\partial r}{\partial \tau}$ and its radial coordinate r of the FMF (see proposition 2 and Fig. 2.4).*

(4) If the probing mass falls freely in the field of M, and if $v = 0$ at $r \to \infty$, then the **energy function** *$E(r,v)$ of m_0 is described by Eq. (3.9):*

$$E(r,v) = m_0 \cdot c^2 \cdot \frac{\sqrt{1 - \frac{R_S}{r}}}{\sqrt{1 - v^2/c^2}} \tag{3.10}$$

(5) In particular, that energy function $E(r,v)$ of m_0 represents an **invariant** *of the motion in the FMF.*

3.2 Our derivation of the FLE

In this section, we present our derivation of the **Friedmann Lemaître equation, FLE** (Friedmann (1922) and Lemaitre (1927)).

3.2.1 Expansion of space

Einstein (1917) analyzed a possible expansion of the space. Slipher (1915) discovered the redshift of distant galaxies, Wirtz (1922) analyzed empirical evidence for the expansion of space, and Hubble (1929) obtained a convincing empirical basis for that expansion of space.

That expansion of space since the Big Bang is usually described by a **uniform scaling**. In this section we derive the DEQ for the case of a homogeneous ball embedded in a HUF (Fig. 3.2).

3.2.1.1 DEQ of uniform scaling: derivation

The surroundings do not generate a field $\vec{G}^*$ in the embedded sphere (sect. 2.7). A homogeneous sphere with a mass M generates a field in its vicinity that is equal to the field generated by the mass M in the center of the ball (Gauss (1840)). So the Schwarzschild solution applies (Eq. 3.9), and thus the energy of a probing mass with the condition $(r|v) = (r|\dot{r}) = (\infty|0)$ at some time is as follows (other conditions are analyzed in Carmesin (2020b)):

$$E(r,v) = m_0 \cdot c^2 \cdot \gamma(v) \cdot \epsilon(r) = E_0 \text{ or } E_{ref} \qquad (3.11)$$

Thereby the factors are:

$$\gamma(v) = \frac{1}{\sqrt{1 - v^2/c^2}} \; ; \; \epsilon(r) = \sqrt{1 - \frac{R_S}{r}} \text{ and } m_0 \cdot c^2 = E_0 \qquad (3.12)$$

The Eq. (3.11) represents a DEQ, as it contains v, which in turn represents a derivative. This DEQ describes the dynamics of the probing mass. Next we transform this DEQ, in order to obtain a transformed DEQ, still describing the dynamics of m and $r(t)$.

3.2.1.2 Structured energy function

In this section we derive a **structured energy function**. This may be interpreted as a result of a mathematical transformation of the DEQ, or it may be interpreted physically in addition:

The structured energy function might be interpreted as a normalized **excess energy** (Carmesin (2020b)) as follows:

In SR, the difference of the square E^2 of the energy and of the square of the own energy $m_0^2 \cdot c^4 = E_0^2$ represents the square of the kinetic energy $p^2 \cdot c^2$. By construction, it represents the square of the excess energy that the mass m has compared to its own mass m_0.

In GR, that excess energy contains the kinetic energy and, additionally, a gravitational energy in the field.

Correspondingly, we derive the excess energy in GR as follows: We take the square of Eq. (3.11), and we subtract the squared own energy $m_0^2 c^4$ (so we obtain the square of the generalized excess energy):

$$E(r,v)^2 - m_0^2 c^4 = m_0^2 \cdot c^4 \cdot (\epsilon(r)^2 \cdot \gamma(v)^2 - 1) \qquad (3.13)$$

As the rest mass is positive, $m_0 > 0$, the velocity v is smaller than c, so the Lorentz factor $\gamma(v)$ is nonzero. Thus we can divide by $\gamma^2(v)$:

$$\frac{E(r,v)^2 - m_0^2 c^4}{\gamma^2} = m_0^2 \cdot c^4 \cdot (\epsilon(r)^2 - \gamma(v)^{-2}) \qquad (3.14)$$

In order to simplify, we insert the factors $\epsilon(r)$ and $\gamma(v)$:

$$\boxed{\frac{E(r,v)^2 - m_0^2 c^4}{\gamma^2} = m_0^2 c^4 \cdot \left(\frac{v^2}{c^2} - \frac{R_S}{r}\right)} \qquad (3.15)$$

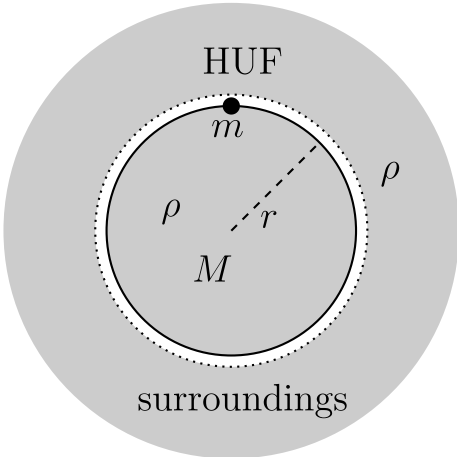


Figure 3.2: Ball with mass M and radius r embedded in homogeneous surroundings and exhibited to a probing mass m.

Conventional form: In this paragraph, we derive a conventional energy function with a conventional kinetic and potential energy term. For it we divide by $2m_0c^2$. So we get:

$$\frac{E(r,v)^2 - m_0^2c^4}{2\gamma^2 m_0 c^2} = m_0 \cdot c^2 \cdot \left(\frac{v^2}{c^2} - \frac{R_S}{r}\right) \cdot \frac{1}{2} \tag{3.16}$$

We denote that energy function with a bar, $\bar{E}(r,v)$. We apply the Schwarzschild radius $R_S = \frac{2GM}{c^2}$: So the result is a conventional structured energy function:

$$\boxed{\frac{E(r,v)^2 - E_0^2}{2\gamma^2 E_0} =: \bar{E}(r,v) = \frac{m_0 \cdot v^2}{2} - \frac{G \cdot M \cdot m_0}{r}} \tag{3.17}$$

Form with the Hubble parameter: In this part we transform the DEQ (3.15) further so that we obtain a term for the **Hubble parameter**:

$$H = \frac{\dot{r}}{r} \tag{3.18}$$

For it, we multiply with $\frac{1}{m_0^2 \cdot c^4} \cdot \frac{c^2}{r^2}$, and we use the density $\rho = \frac{M}{r^3 \cdot 4\pi/3}$. So we get:

$$\frac{E(r,\dot{r})^2 - m_0^2 c^4}{m_0^2 \cdot c^4 \gamma^2} \cdot \frac{c^2}{r^2} = \frac{\dot{r}^2}{r^2} - \frac{8\pi G \cdot \rho}{3} \tag{3.19}$$

We identify the scaled squared energy $-\frac{E(r,\dot{r})^2 - m_0^2 c^4}{m_0^2 \cdot c^4 \gamma^2}$ or the scaled energy term $-\frac{2\bar{E}(r,\dot{r})}{m_0 \cdot c^2}$ with the so-called **curvature parameter** k (Friedmann (1922), Lemaitre (1927), Stephani (1980)), we identify $\frac{\dot{r}^2}{r^2}$ by the squared Hubble parameter H^2, and we solve for H^2. So we get the **Friedmann Lemaître equation, FLE** (Friedmann (1922) and Lemaitre (1927)), the DEQ for the homogeneous system:

$$\boxed{H^2 = \frac{8\pi G \cdot \rho}{3} - k \cdot \frac{c^2}{r^2}} \tag{3.20}$$

Observations and theory, see e. g. Planck-Collaboration (2020), Bennett et al. (2013), Carmesin (2020b), show that the curvature parameter k is zero, which means the space is globally flat. We summarize our derivation:

Theorem 1 Direct derivation of the FLE from the SM

The expansion of the universe has the following properties:

(1) In classical GR, it is described by a **uniform scaling** *with a scale factor $r(t)$ Fig. (3.2).*

(2) In classical GR, the time evolution of the scale factor $r(t)$ is described by the FLE:

$$H^2 = \left(\frac{\dot{r}}{r}\right)^2 = \frac{8\pi G \cdot \rho}{3} - k \cdot \frac{c^2}{r^2} \tag{3.21}$$

(3) The FLE of that uniform scaling can be derived from the time evolution of a **microscopic probing mass** *m as follows:*

(3a) In the HUF with density ρ, there is a homogeneous ball of the universe with the same density and generating a field $\vec{G}^*$, and m is at the surface of that ball (Fig. 3.2).

(3b) The time evolution of m is derived from the SM, see the DEQ (3.11) as well as the transformed DEQ (3.17).

(4) Thereby, these above two DEQs use a **structured energy function** *$\bar{E}(r, \dot{r})$ with $\bar{E}(r, \dot{r}) = 0 = k =$ invariant:*

$$\boxed{-k := \frac{2\bar{E}(r,\dot{r})}{m_0 \cdot c^2} \quad \text{with} \quad \bar{E}(r,\dot{r}) = \frac{m_0 \dot{r}^2}{2} - \frac{GMm_0}{r}} \tag{3.22}$$

(5) That **structured energy function** *is defined as follows and proportional to E_0 and a normalized energy $E_{norm} = \frac{\bar{E}}{E_0}$:*

$$\boxed{\frac{E(r,\dot{r})^2 - E_0^2}{2\gamma^2 E_0} =: \bar{E}(r,\dot{r}) = E_0 \cdot \left(\frac{\dot{r}^2}{2c^2} - \frac{G \cdot M}{r \cdot c^2} \right)} \tag{3.23}$$

After we analyzed the expansion of space by using the concepts of the uniform scaling, the HUF and the law of energy conservation, we analyze the additional vacuum in the following section.

3.2.2 Homogeneous metric: new vacuum

The expansion of space is usually described by a mathematical transformation: the uniform scaling. In this section we analyze, how that transformation is generated by the permanent and ubiquitous formation of new vacuum. This novel analysis is based on the fundamental concept of linear superposition of volume, see Carmesin (2021d).

3.2.2.1 Rate of formed vacuum

The increase of the radius corresponds to an increase of the volume. Hence additional vacuum is formed. In this section we summarize the rate at which the vacuum forms. This rate is derived from the FLE[1]. The flat, isotropic and homogeneous space expands according to the Hubble parameter:

$$H = \frac{\partial a}{\partial t \cdot a} = \sqrt{8\pi \cdot G/3} \cdot \sqrt{\rho} \tag{3.24}$$

The volume of a ball of the universe with radius a is $V = \frac{4\pi}{3}a^3$. With it we derive the rate of increase of the volume V by applying the chain rule:

$$\frac{\partial V}{\partial t \cdot V} = \frac{1}{V} \cdot \frac{\partial V}{\partial t} = \frac{1}{V} \cdot \frac{\partial V}{\partial a} \cdot \frac{\partial a}{\partial t} = \frac{3}{a} \cdot \frac{\partial a}{\partial t} = 3H \tag{3.25}$$

So the flat, isotropic and homogeneous space expanding according to the Hubble parameter exhibits the following DEQ for the rate of increase of the volume:

$$\boxed{\left(\frac{\partial V}{\partial t \cdot V}\right)^2 = 24\pi \cdot G \cdot \rho} \tag{3.26}$$

We denote the formed volume per volume and time by $\underline{\delta} V$, see Eq. (3.29). Correspondingly, we denote the time difference by δt. Moreover, we may consider infinitesimal amounts of volume dV rather than V So we derive the following DEQ:

$$\left(\frac{\underline{\delta} V}{\delta t \cdot dV}\right)^2 = 24\pi \cdot G \cdot \rho \tag{3.27}$$

Furthermore, we denote the **relative volume** by ε:

$$\frac{\delta V}{dV} = \varepsilon \tag{3.28}$$

If we consider an additional volume or vacuum that forms per volume dV and per time δt, we denote it with an underline: Furthermore, we denote a **relative volume** by ε:

$$\frac{\underline{\delta} V}{dV \cdot \delta t} = \dot{\varepsilon} \tag{3.29}$$

[1] Carmesin (2018b), Carmesin (2018a), Carmesin (2019b)

With it we derive the **rate of the formation of relative volume**:

$$\dot{\varepsilon} = \sqrt{24\pi \cdot G \cdot \rho} \tag{3.30}$$

We summarize our novel concept and our derivation:

Theorem 2 Formed vacuum causes expansion of space

The uniform scaling that describes the expansion of space is caused by a rate of additionally formed vacuum with the following properties:

(1) The density ρ in a ball causes the permanent formation of additional vacuum.

(2) For a ball with radius R, the volume of the additional vacuum $\underline{\delta}V$ per volume dV and per time δt is described by the following rate:

$$\frac{\underline{\delta}V}{\delta t \cdot dV} = \dot{\varepsilon} = \sqrt{24\pi \cdot G \cdot \rho} \tag{3.31}$$

3.3 Possible unidirectional elongation

In this section we introduce the concept of a possible unidirectional elongation of space. Thereby we describe the elongation by tensors. As an example, we analyze the case of the Schwarzschild metric, SM, see e. g. Schwarzschild (1916), Carmesin (2021d) or Sect (3.9). We emphasize that we do not presume the SM in our derivation of the SM.

In GR, the spacetime in the vicinity of a mass M experiences a curvature. It can be described by using polar coordinates $dx_1 = r$, $dx_2 = \theta$ and $dx_3 = \phi$ and with the time coordinate $dx_0 = t$. The curvature can be described with help of an underlying *metric tensor* g_{ij}, so that the square of an infinitesimal line element ds is as follows:

$$ds^2 = \Sigma_{i=0}^{3}\Sigma_{j=0}^{3}\, g_{ij} \cdot dx_i \cdot dx_j \tag{3.32}$$

In the vicinity of a mass M, the metric tensor is as follows, see e. g. Schwarzschild (1916), Landau and Lifschitz (1971), Straumann (2013), Hobson et al. (2006), whereby we use the sign convention outlined in equation (3.108):

$$g_{ij} = \begin{pmatrix} -(1 - \frac{R_S}{r}) \cdot c^2 & 0 & 0 & 0 \\ 0 & \frac{1}{1-\frac{R_S}{r}} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \cdot \sin^2(\theta) \end{pmatrix} \tag{3.33}$$

Hereby, the metric tensor describes the Schwarzschild metric, SM, and R_S is the Schwarzschild radius:

$$R_S = \frac{2GM}{c^2} \tag{3.34}$$

Note that there are two different sign conventions in the literature. Hereby, we use the sign convention described by the Cartesian metric tensor of flat space as follows:

$$\eta_{ij,Cartesian} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{3.35}$$

Note that the opposite signs are used in Landau and Lifschitz (1971) or in Stephani (1980), for instance. For an overview of various signs used in the literature, see Hobson et al. (2006).

3.3.1 Change tensor

In this section we analyze possible unidirectional changes that are caused by the mass M. For it we introduce a change tensor $\hat{\varepsilon}_{ij}$, more generally. As above and as an example, we use the metric tensor of the SM.

The mass M changes the metric tensor g_{ij}, whereby there are only diagonal nonzero elements g_{ii}. In particular, we consider the radial direction in space only, so $d\theta$ and $d\phi$ are both zero. So Eq. (3.32) takes the following form:

$$ds^2 = g_{00} \cdot dt^2 + g_{rr} \cdot dr^2 \tag{3.36}$$

As the Schwarzschild metric is stationary, we may consider $dt = 0$. So we derive:

$$ds^2 = g_{rr} \cdot dr^2 \tag{3.37}$$

We insert $g_{rr} = \frac{1}{1-\frac{R_S}{r}}$, see Eq. (3.33). So the length dr is elongated to the length ds or dr' as a result of the mass M as follows:

$$ds = \frac{1}{\sqrt{1 - \frac{R_S}{r}}} \cdot dr = dr' \tag{3.38}$$

So the difference or *displacement* δr_{SM} is as shown below:

$$\delta r_{SM} = dr' - dr = \left(\frac{1}{\sqrt{1 - \frac{R_S}{r}}} - 1 \right) \cdot dr \tag{3.39}$$

That displacement δr_{SM} is illustrated in figure (3.3).

The derivative of such a displacement δr_{SM} with respect to the original length dr can be interpreted as an element of a *change tensor* $\hat{\varepsilon}_{rr}$, similarly to the strain tensor in elasticity theory, see (Landau and Lifschitz, 1975, equations 1.5, 1.8) or (Sommerfeld, 1978, equation 11):

$$\frac{\delta r_{SM}}{dr} = \hat{\varepsilon}_{rr} \tag{3.40}$$

Hereby, δr_{SM} and dr are regarded as differentials in the sense of the Leibniz calculus, see e. g. Bos (1974), Leibniz (1684) or Fig. (3.3).

For the case of other components, the change tensor takes the following form:

$$\frac{\delta r_i}{dr_j} = \hat{\varepsilon}_{ij} \tag{3.41}$$

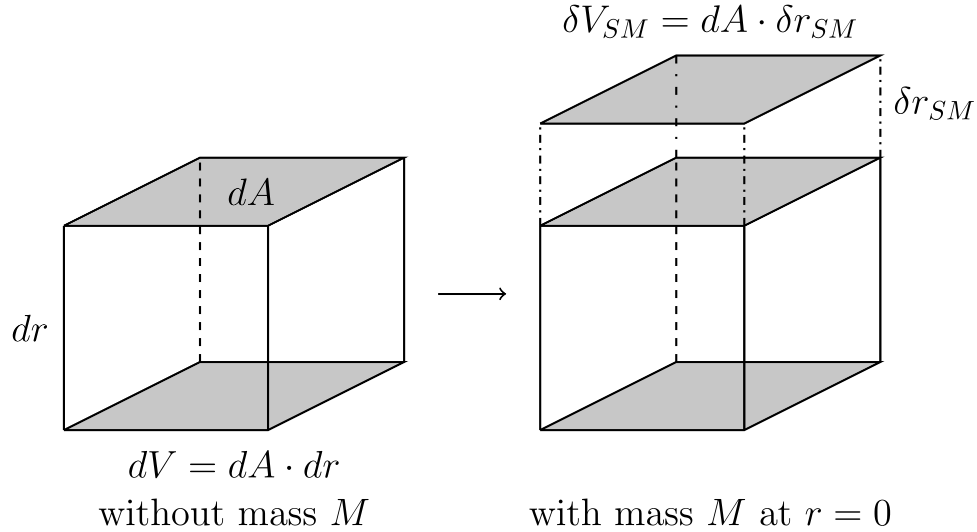


Figure 3.3: Unidirectional elongation in the radial direction: A cube with lower and upper surface dA is elongated by shifting the upper surface by an increment δr_{SM}.

Also the full change tensor is analogous to the strain tensor in elasticity theory, see (Landau and Lifschitz, 1975, equations 1.5, 1.8) or (Sommerfeld, 1978, equation 11).

3.3.2 Change of volume

Since the discovery of the dark energy, see e. g. Perlmutter et al. (1998), Riess et al. (2000), Smoot (2007), Spergel et al. (2007), Planck-Collaboration (2020), it is clear that the vacuum has a density ρ_Λ. Accordingly, the volumes dV and δV_{SM} in figure (3.3) correspond to respective energies. So it is interesting to analyze the relative change of the volume:

The change can directly be applied to the volume in figure (3.3), $dV = dA \cdot dr$. The change of the volume δV_{SM} is the product of the area dA with the change δr_{SM}:

$$\frac{\delta V_{SM}}{dV} = \frac{dA \cdot \delta r_{SM}}{dA \cdot dr} = \frac{\delta r_{SM}}{dr} = \hat{\varepsilon}_{rr} = \frac{1}{\sqrt{1 - \frac{R_S}{r}}} - 1 \qquad (3.42)$$

In general, the relative change of the volume is the sum of the changes for each Cartesian coordinate in a D dimensional space. So it is the sum of the diagonal elements of the change tensor:

$$\frac{\delta V}{dV} = \Sigma_{j=1}^{D} = \hat{\varepsilon}_{jj} \tag{3.43}$$

This result corresponds to respective terms in elasticity theory, see (Landau and Lifschitz, 1975, equations 1.5, 1.6) or (Sommerfeld, 1978, equations 18 - 20). Here we call the relative change of the volume ε:

$$\frac{dV' - dV}{dV} = \frac{\delta V}{dV} = \varepsilon \tag{3.44}$$

We summarize our derivation as follows:

Proposition 4 Elongation in the SM

A mass or dynamical mass M causes an elongation $\delta r_{SM,elo}$ of a radial coordinate distance dr. Thereby, $\delta r_{SM,elo}$ is a function of the distance r as follows:

$$\delta r_{SM,elo} = dr' - dr = \left(\frac{1}{\sqrt{1 - \frac{R_S}{r}}} - 1 \right) \cdot dr \tag{3.45}$$

That elongation can be expressed by the radial element of the change tensor:

$$\hat{\varepsilon}_{rr} = \frac{\delta r_{SM,elo}}{dr} = \frac{1}{\sqrt{1 - \frac{R_S}{r}}} - 1 \tag{3.46}$$

As a consequence, the volume $dV = 4\pi r^2 \cdot dr$ of the shell with radius r and thickness dr is increased by the volume $\delta V_{SM,elo} = 4\pi r^2 \cdot \delta r_{SM,elo}$ as follows:

$$\hat{\varepsilon}_{rr} = \frac{\delta V_{SM,elo}}{dV} = \frac{1}{\sqrt{1 - \frac{R_S}{r}}} - 1 \tag{3.47}$$

3.4 Isotropic expansion of space

In this section, we analyze an isotropic expansion. Physically, this describes the expansion of space since the Big Bang, see e. g. Wirtz (1922), Hubble (1929), Friedmann (1922), Lemaitre (1927), Einstein and de Sitter (1932), Carmesin (2017).

The increase of a scale factor or radius a can be described by the *Friedmann - Lemaître equation, FLE*, at the macroscopic level:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi \cdot G}{3} \cdot \rho \tag{3.48}$$

Hereby, the density ρ includes the density of the matter ρ_m, the density of the vacuum ρ_Λ, the density of the curvature ρ_K and the density of radiation ρ_r. Based on observation, see e. g. Spergel et al. (2007), Planck-Collaboration (2020), and based on theory, see e. g. Carmesin (2021d), ρ_K is negligible.

On a possible additional pressure p: The pressure p allows for an additional component, see e. g. Hobson et al. (2006). Some authors additionally represent an additional pressure in terms of an equation of state with an additional exponent w. We do not introduce any such additional parameters or fit parameters, as we derive everything directly from gravity, the EEP and SR. Accordingly, we do not execute any fit parameters, in particular, we do not use such an exponent w or such a pressure p as a fit parameter.

As an important example, we derived the FLE by application of the spacetime-quadruple, see above or (Carmesin, 2021d, theorem 3):

$$\text{SQ implies } FLE \tag{3.49}$$

In the FLE, the fraction $\frac{\dot{a}}{a}$ is called Hubble parameter H:

$$\frac{\dot{a}}{a} = H \tag{3.50}$$

3.4.1 Transformation of the Hubble parameter

In this section, we analyze the Hubble parameter $\frac{\dot{a}}{a}$ with help of the volume in figure (3.3). For it we name the scale factor a by dr:

$$\frac{\dot{dr}}{dr} = H \tag{3.51}$$

As a first step, we analyze an unidirectional expansion in the direction dr in figure (3.3). For it, we investigate the time evolution of the volume in figure (3.3), starting at $t = 0$. Thereby we consider the case of a short time δt in the sense of the Leibniz calculus. As an initial condition, the vertical length in figure (3.3) is dr at $t = 0$, and the change tensor $\hat{\varepsilon}_{rr}$ is zero at $t = 0$. At the later time δt, the length is named dr'. Accordingly, the differential fraction represents the time derivative $\dot{dr}$ in the sense of the Leibniz calculus:

$$\frac{dr' - dr}{\delta t} = \dot{dr} \tag{3.52}$$

We identify the difference $dr' - dr$ with δr in figure (3.3):

$$\frac{\delta r}{\delta t} = \dot{dr} \tag{3.53}$$

Here we identify the difference δr with the product $\hat{\varepsilon}_{rr} \cdot dr$, see Eq. (3.42):

$$\frac{\hat{\varepsilon}_{rr} \cdot dr}{\delta t} = \dot{dr} \tag{3.54}$$

According to the initial condition, we replace $\hat{\varepsilon}_{rr}$ by $\delta\hat{\varepsilon}_{rr}$:

$$\frac{\delta\hat{\varepsilon}_{rr} \cdot dr}{\delta t} = \dot{dr} \tag{3.55}$$

In the used framework of the Leibniz calculus, we identify $\frac{\delta\hat{\varepsilon}_{rr}}{\delta t}$ by the time derivative of the tensor:

$$\dot{\hat{\varepsilon}}_{rr} \cdot dr = \dot{dr} \tag{3.56}$$

We insert this equation into Eq. (3.51):

$$\dot{\hat{\varepsilon}}_{rr} = H_{unidirectional} \tag{3.57}$$

As a second step, we consider an expansion in all three Cartesian directions x, y and z. So the relative change of the volume is expressed by the sum of the three diagonal elements of the change tensor, see Eq. (3.43). Thus Eq. (3.57) is transferred to the case of isotropic expansion, if we replace the unidirectional change tensor $\hat{\varepsilon}_{rr}$ by the sum of the three Cartesian elements of the change tensor:

$$\Sigma_{j=1}^{3}\dot{\hat{\varepsilon}}_{jj} = H_{isotropic} = H = \frac{\partial}{\partial t}\frac{\delta V}{dV} = \dot{\varepsilon} \tag{3.58}$$

3.4.2 Rate of formation of vacuum

In this section, we analyze the rate at which the vacuum forms during the expansion of the universe since the Big Bang. For it, we express the volume V by the scale factor a:

$$V = \frac{4\pi}{3}a^3 \tag{3.59}$$

Using the chain rule, we obtain the derivative:

$$\dot{V} = 3\dot{a}\frac{V}{a} \tag{3.60}$$

So we derive:

$$\frac{\dot{V}}{V} = 3\frac{\dot{a}}{a} = \dot{\varepsilon} \tag{3.61}$$

We apply the square:

$$\dot{\varepsilon}^2 = 9\left(\frac{\dot{a}}{a}\right)^2 \tag{3.62}$$

We insert the FLE (equation 3.48):

$$\dot{\varepsilon}^2 = 24\pi G \cdot \rho \tag{3.63}$$

In order to simplify further, we apply Eq. (3.58) to Eq. (3.63):

$$\left(\Sigma_{j=1}^{3}\dot{\hat{\varepsilon}}_{jj}\right)^2 = 24\pi G \cdot \rho \tag{3.64}$$

We evaluate the square. Hereby the tensor product of unequal unidirectional tensors vanishes, for instance $\hat{\varepsilon}_{xx} \cdot \hat{\varepsilon}_{yy} = 0$:

$$\Sigma_{j=1}^{3}\dot{\hat{\varepsilon}}_{jj}^2 = 24\pi G \cdot \rho \tag{3.65}$$

As the expansion is isotropic, we derive:

$$3 \cdot \dot{\hat{\varepsilon}}_{xx}^2 = 24\pi G \cdot \rho \tag{3.66}$$

We divide by three:

$$\dot{\hat{\varepsilon}}_{xx}^2 = 8\pi G \cdot \rho \tag{3.67}$$

Moreover, we consider an arbitrary direction j instead of x:

$$\dot{\hat{\varepsilon}}_{jj}^2 = 8\pi G \cdot \rho \tag{3.68}$$

Note that no sum convention is applied here.

Proposition 5 Expansion of space

If space expands with a Hubble parameter H, then the increase of the volume is equal to the following terms:

$$H = \partial_t \frac{\delta V}{dV} =: \dot{\varepsilon} \tag{3.69}$$

Thereby, the relative change of volume

$$\frac{\delta V}{dV} =: \varepsilon \tag{3.70}$$

is equal to the sum of the three Cartesian diagonal tensors

$$\frac{\delta V}{dV} = \hat{\varepsilon}_{xx} + \hat{\varepsilon}_{yy} + \hat{\varepsilon}_{zz} \qquad \text{with} \tag{3.71}$$

$$\frac{\delta x}{dx} = \hat{\varepsilon}_{xx};\ \frac{\delta y}{dy} = \hat{\varepsilon}_{yy}\ \frac{\delta z}{dz} = \hat{\varepsilon}_{zz} \qquad \text{and} \tag{3.72}$$

$$\frac{\delta V}{dV} = \Sigma_{j=1}^{3}\hat{\varepsilon}_{jj} = \text{Trace } \hat{\varepsilon}_{ij} \tag{3.73}$$

So the rates are as follows:

$$\dot{\varepsilon}^2 = \left(\frac{\delta V}{dV\delta t}\right)^2 = 24\pi G \cdot \rho = \Sigma_{j=1}^3 \dot{\hat{\varepsilon}}_{jj}^2 \qquad \text{or} \tag{3.74}$$

$$\dot{\hat{\varepsilon}}_{jj}^2 = 8\pi G \cdot \rho \tag{3.75}$$

3.5 Density ρ_f of the gravitational field G^*

Gravity can be described by several physical quantities. Note that these descriptions do not exclude each other. Firstly, Newton described a gravitational force of interaction of two masses with some distance r between these masses, hereby no concept is provided about the physics taking place in the space between the two masses. Secondly, Faraday introduced the concept of a field that describes the physics taking place in the space between two interacting bodies, see Faraday (1852), and Gauss adopted that concept for the case of gravity, see Gauss (1809). Thirdly, Einstein described gravity in terms of curvature of spacetime, see Einstein (1915), Hilbert (1915).

Which of the three theories is most adequate for a critical and microscopic analysis of gravity? NG is excluded, as it makes restrictive presumptions about space and time. Einstein's gravity is mesoscopic, but not microscopic. Gaussian gravity describes the formation of gravity by a mass most microscopically and in a manner that focuses on the essence most clearly. Accordingly, we use Gaussian gravity. Correspondingly, we apply the densities of the gravitational field.

3.5.1 Absolute value of ρ_f

In this section, we derive the absolute value $|\rho_f|$ of the energy density ρ_f of the field G^* located in a HUF. For it we analyze the energy ΔE_M that is necessary in order to lift a mass M in a shell with a radius R to a shell with a radius $R + \Delta R$, see Fig. (3.4). Thereby, the mass is lifted as follows: Differential parts

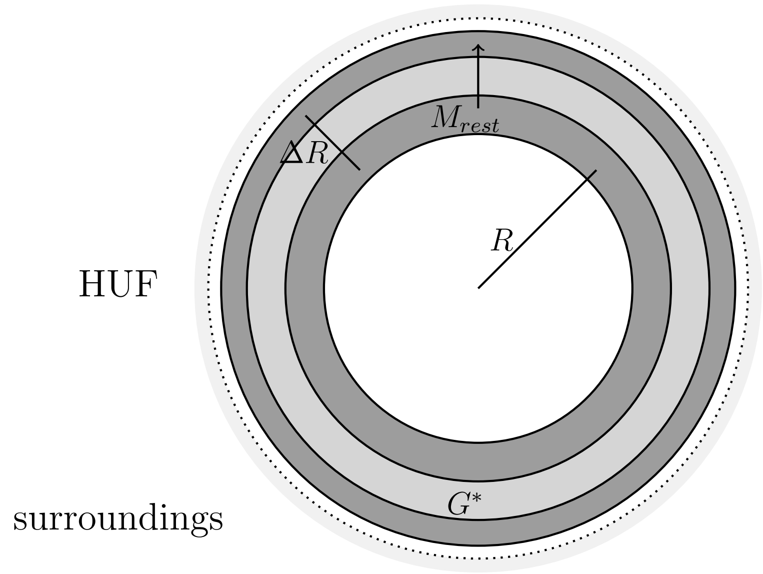


Figure 3.4: In a HUF, a mass M (dark grey) in a shell at a radius R is lifted to a radius $R + \Delta R$ as follows: Differential parts dM are lifted, while the rest M_{rest} is still at R. Thereby the field G^* (medium grey) in the shell with radius R and thickness ΔR becomes zero, when the whole mass is at $R + \Delta R$ (see Fig. 3.5).

dM are lifted, while the part M_{rest} is still at R. Moreover, the velocity of M remains zero, in an ideal manner. So a part dM is lifted at the gravitational field of the part M_{rest}:

$$|\vec{G}^*(M_{rest})| = \frac{G \cdot M_{rest}}{R^2} \tag{3.76}$$

So the field G^* is proportional to the part M_{rest} (Fig. 3.5). If a mass dM is lifted, and if the mass M_{rest} is still at R, then dM experiences the force[2] $F = G^*(M_{rest}) \cdot dM$, thus the energy $dE = F \cdot \Delta R$ is required:

$$dE = |G^*(M_{rest})| \cdot dM \cdot \Delta R = \frac{G \cdot M_{rest}}{R^2} \cdot dM \cdot \Delta R \tag{3.77}$$

We derive the full change in gravitational energy ΔE_M by integrating the above Eq.:

$$\Delta E_M = \int_0^E dE = \int_M^0 \frac{G \cdot M_{rest}}{R^2} dM \cdot \Delta R \tag{3.78}$$

[2]The force can be used instead of the position factor, as ΔR is infinitesimal.

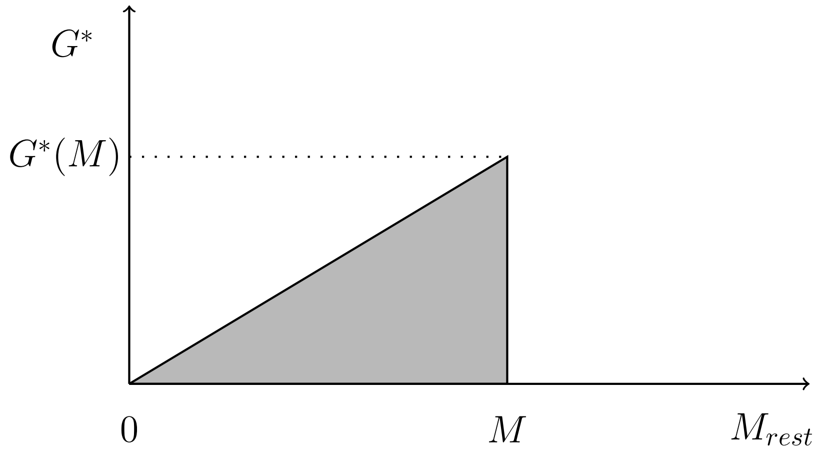


Figure 3.5: The field G^* is shown as a function of the mass M_{rest} that is still at the shell with the radius R.

When a mass dM is lifted, then the mass M_{rest} is decreased by dM. So $dM_{rest} = -dM$. Thus we get:

$$\Delta E_M = -\int_M^0 \frac{G \cdot M_{rest}}{R^2} dM_{rest} \cdot \Delta R \qquad (3.79)$$

We evaluate the integral:

$$\Delta E_M = \frac{G \cdot M^2 \cdot \Delta R}{2R^2} \qquad (3.80)$$

Location of the energy ΔE_M: While the mass M was lifted in the above process, the energy ΔE_M was added to the system. Where is this energy ΔE_M located in the system?

As the mass M is identical to the probing mass m and to the field-generating mass $M_f = M$, the mass M is not in an external field. So the position factor is 1 at the beginning and at the end of the process. Hence the energy ΔE_M is not located in the mass.

There is a **modification in the shell** between the radii R and $R + \Delta R$. It can be characterized by the additional curvature. That additional curvature can be characterized by formed volume flowing outwards at the velocity c. As we derived the

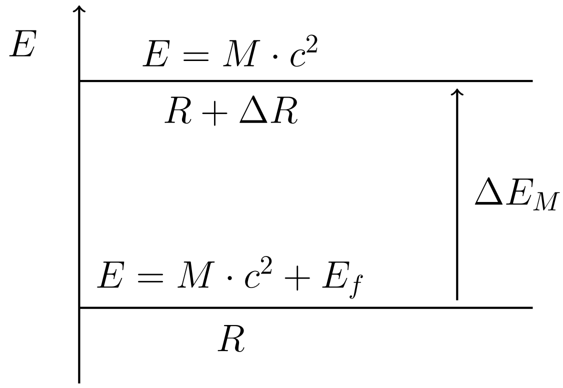


Figure 3.6: The energy of the mass is shown at the initial radius R and at the final radius $R + \Delta R$.

curvature from the EEP to which the field is inherent, that outflow can also be described by the field G^*.

Hence, **in the HUF the energy ΔE_M is located in the modifications in the shell between R and $R + \Delta R$, and it can be characterized by the field**.

Absolute value $|u_f|$ of the energy density u_f of the field: The field G^* is in the shell with radius R and thickness ΔR (see Fig. (3.4). The corresponding volume is $\Delta V = 4\pi \cdot R^2 \cdot \Delta R$. So we derive the energy density by the dividing the energy ΔE_M by the volume ΔV. So we get:

$$|u_f| = \frac{\Delta E_f}{\Delta V} = \frac{G \cdot M^2 \cdot \Delta R}{2R^2 \cdot 4\pi \cdot R^2 \cdot \Delta R} \tag{3.81}$$

We simplify the above term, we expand by G, and we apply the field $G^* = \frac{G \cdot M}{R^2}$. So we get:

$$\boxed{|u_f| = \frac{\vec{G}^{*2}}{8\pi \cdot G} = |\rho_f| \cdot c^2} \tag{3.82}$$

3.5.2 Sign of u_f

In this section, we derive the sign of the energy density u_f of the field.

For it, we analyze the gravitational energy of the field. The sign of an energy is determined from the law of energy conservation. Before the above process, the system has the energy of the mass $M \cdot c^2$, plus the energy E_f of the field in the HUF (Fig. 3.6):

$$E_{before} = M \cdot c^2 + E_f(r \geq R + \Delta R) + \Delta E_f \tag{3.83}$$

During the process, the energy ΔE_M is added to the system. So the energy E_{after} after the process is as follows:

$$E_{after} = E_{before} + \Delta E_M \quad \text{or} \tag{3.84}$$

$$E_{after} = M \cdot c^2 + E_f(r \geq R + \Delta R) + \Delta E_f + \Delta E_M \tag{3.85}$$

Moreover, we identify the energy E_{after} after the process directly: it consist of $M \cdot c^2$ and ΔE_f:

$$E_{after} = M \cdot c^2 + E_f(r \geq R + \Delta R) \tag{3.86}$$

We subtract Eq. (3.86) from Eq. (3.85). So we derive the following relation:

$$0 = \Delta E_f + \Delta E_M \tag{3.87}$$

As the sign of ΔE_M is positive, **the sign of the energy of the field ΔE_f is negative**. More generally, the energy of the gravitational field of a mass M is negative. Correspondingly, the energy density u_f of the field is negative.

3.5.3 Inertia inherent to ρ_f

In this section, we analyze the inertia that is inherent to the gravitational field of a mass M. For it, we apply the energy momentum relation of a physical object, whereby the object has an energy E or a dynamical mass or mass $M = E/c^2$, whereby the object has a momentum p, and whereby the object has a zero or nonzero rest mass m_0:

$$E^2 = p^2 \cdot c^2 + m_0^2 \cdot c^4 \tag{3.88}$$

The right hand side represents the inertial properties of the object, as the rest mass m_0 represents inertia directly, whereas the momentum p represents inertia that becomes obvious e. g. when the object is absorbed by another rest mass $m_{0,2}$, which is accelerated as a consequence. The left hand side describes the energy, which becomes obvious in energetic processes such as described in section (3.5.2). The above Eq. (3.88) clearly shows that the inertia of the object depends on the square of the energy only. A negative energy $E_{obj} < 0$ of an object represents the same inertia as the absolute value of that energy $|E_{obj}| > 0$. Note that these relations hold in a HUF, as the HUF excludes additional energies that might occur in local frames. Note further that most observers measure the inertial properties of an object, instead of the object's energy with respect to a HUF, since most measurements are performed in a laboratory or in a part of space that is relatively small compared to the volume within the light horizon.

According to the fact that the inertia of an object corresponds to the absolute value of the energy E of the object in a HUF, we define an inertial energy density of the field by the absolute value of the energy density as follows:

Definition 3 Inertial energy density of the field

The inertial energy density $u_{f,In}$ of the field is its absolute value. Correspondingly, the inertial density $\rho_{f,In}$ of the field is its absolute value as well:

$$u_{f,In} = |u_f| = \frac{\vec{G}^{*2}}{8\pi \cdot G} \quad \text{and} \tag{3.89}$$

$$\rho_{f,In} = |\rho_f| = \frac{\vec{G}^{*2}}{8\pi \cdot G \cdot c^2} \tag{3.90}$$

We summarize our findings in the following theorem.

Theorem 3 Energy and inertia of the field

(1) The gravitational energy is inherent to modifications of space such as curvature or additionally formed volume or a gravitational field.

(2) In a HUF, a gravitational field $\vec{G}^*$ *has the energy density* u_f *as follows:*

$$u_f = -\frac{\vec{G}^{*2}}{8\pi \cdot G} = \rho_f \cdot c^2 \tag{3.91}$$

(3) In a HUF, a gravitational field $\vec{G}^*$ *has the inertial energy density* $u_{f,In}$ *as follows:*

$$u_{f,In} = \frac{\vec{G}^{*2}}{8\pi \cdot G} = \rho_{f,In} \cdot c^2 \tag{3.92}$$

(4) If an observer measures the energy density of an object in a laboratory, at Earth, in the Solar System, in the Milky Way, in Laniakea, see e. g. Tully et al. (2014), Carmesin (2021c), or in a part of space that is relatively small compared to the volume within the light horizon, then that observer measures $u_{f,In}$*, as long as the result is not transformed to a homogeneous universe frame, HUF, or to an equivalent frame.*

(5) In a HUF with a field generating mass M*, or in a FMF, at a distance* r *from a mass* M*, a gravitational field* $G^*(r)$ *occurs as follows, see e. g. Carmesin (2021d):*

$$|\vec{G}^*| = \frac{G \cdot M}{r^2} \tag{3.93}$$

3.5.4 An essential density

The density $\rho_{f,In}$ of the field describes an essential energy, whenever there occurs the phenomenon of gravity. Correspondingly, that energy must be included in the combined DEQ (3.68). Accordingly, we represent a possible additional density ρ_{add} and the density of the gravitational field $\rho_{f,In}$ in that combined equation, whereby ρ_{add} can represent a density of matter or a density of radiation, for instance:

$$\dot{\hat{\varepsilon}}_{jj}^2 = 8\pi G \cdot (\rho_{add} + \rho_{f,In}) \tag{3.94}$$

3.6 Rate gravity scalar RGS & 4-vector RGV

The combined Eq. (3.94) contains a scalar in spacetime. In order to identify it, we insert equation (3.92) into Eq. (3.94):

$$\dot{\hat{\varepsilon}}_{jj}^2 = 8\pi G \cdot \rho_{add} + G^{*2}/c^2 \qquad \text{or} \tag{3.95}$$

$$\dot{\hat{\varepsilon}}_{jj}^2 - G^{*2}/c^2 = 8\pi G \cdot \rho_{add} \tag{3.96}$$

3.6.1 Formation of vacuum in vacuum

In this section, we analyze the formation of vacuum without any additional density ρ_{add}, see Eq. (3.96). For that process, we say vacuum forms in vacuum. In that case, the rate takes the following form:

$$\dot{\hat{\varepsilon}}_{jj}^2 - G^{*2}/c^2 = 0 \tag{3.97}$$

As vacuum forms in vacuum, the component of the field G_j^* is the same as that of the rate $\dot{\hat{\varepsilon}}_{jj}$:

$$\dot{\hat{\varepsilon}}_{jj}^2 - G_j^{*2}/c^2 = 0 \tag{3.98}$$

We remind that the corresponding inhomogeneous equation is as follows:

$$\dot{\hat{\varepsilon}}_{jj}^2 - (G_j^*/c)^2 = 8\pi G \cdot \rho_{add} \tag{3.99}$$

3.6.2 Isotropic formation of vacuum in vacuum

In this section, we derive the isotropic formation of vacuum in vacuum. For it, we apply the sum $\Sigma_{j=1}^3$ to Eq. (3.98):

$$\Sigma_{j=1}^3 \dot{\hat{\varepsilon}}_{jj}^2 - \Sigma_{j=1}^3 G_j^{*2}/c^2 = 0 \tag{3.100}$$

We identify the change of volume $\dot{\hat{\varepsilon}}$ or $\dot{\varepsilon}$:

$$\dot{\varepsilon}^2 = \dot{\hat{\varepsilon}}^2 = \Sigma_{j=1}^3 \dot{\hat{\varepsilon}}_{jj}^2 \qquad \text{thus} \tag{3.101}$$

$$\dot{\hat{\varepsilon}}^2 - \Sigma_{j=1}^3 G_j^{*2}/c^2 = 0 \tag{3.102}$$

We remind that the corresponding inhomogeneous equation is as follows:

$$\Sigma_{j=1}^3 \dot{\hat{\varepsilon}}_{jj}^2 - \Sigma_{j=1}^3 G_j^{*2}/c^2 = \Sigma_{j=1}^3 8\pi G \cdot \rho_{add} \qquad \text{thus} \tag{3.103}$$

$$\dot{\varepsilon}^2 - G^{*2}/c^2 = 24\pi G \cdot \rho_{add} \tag{3.104}$$

We identify the left hand side of Eq. (3.102) by a scalar RGS in spacetime:

$$RGS = \dot{\hat{\varepsilon}}^2 - \Sigma_i G_i^{*2}/c^2 \tag{3.105}$$

We call the scalar RGS *rate gravity scalar*, as $\dot{\hat{\varepsilon}}$ is a rate of formation of volume, while $\Sigma_i G_i^{*2}/c^2$ represents gravity in terms of the gravitational field. Moreover, it is a scalar product or a scalar. Accordingly, we call the corresponding four vector *rate gravity vector* RGV. Additionally, the gravitational field can be expressed by a potential.

$$G_j^* = -\partial_j \phi \tag{3.106}$$

$$RGV_i = \begin{pmatrix} \dot{\varepsilon} \\ G_1^*/c \\ G_2^*/c \\ G_3^*/c \end{pmatrix} = \begin{pmatrix} \partial_t \varepsilon \\ -\partial_{r_1}\phi/c \\ -\partial_{r_2}\phi/c \\ -\partial_{r_3}\phi/c \end{pmatrix} \tag{3.107}$$

Using the metric tensor of flat spacetime,

$$\eta_{i,k} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \eta_i^k \tag{3.108}$$

we express the RGS in terms of the RGV:

$$RGS = \Sigma_{i=0}^3 \Sigma_{k=0}^3 RGV_i \cdot \eta_{i,k} \cdot RGV_k \tag{3.109}$$

Theorem 4 Invariant formation of vacuum

For the case of formation of vacuum without any additional density ρ_{add}, the RGS in the DEQ

$$\boxed{RGS = \dot{\varepsilon}^2 - G^{*2}/c^2 = 0} \tag{3.110}$$

is an invariant for the following reasons:

(1) The only possible accelerations in a HUF are particular accelerations. A field $\vec{G}^*$ of a particular acceleration can be measured by a local observer.

(2) A possible absolute velocity cannot be measured. The DEQ $RGS = 0$ is invariant with respect to a Lorentz transformation, as the RGS is a relativistic square of a four vector, the rate gravity four-vector:

$$RGV_i = \begin{pmatrix} \dot{\varepsilon} \\ G_1^*/c \\ G_2^*/c \\ G_3^*/c \end{pmatrix} = \begin{pmatrix} \partial_t \varepsilon \\ -\partial_{r_1}\phi/c \\ -\partial_{r_2}\phi/c \\ -\partial_{r_3}\phi/c \end{pmatrix} \quad \text{thus} \tag{3.111}$$

$$RGS = \Sigma_{i=0}^3 \Sigma_{k=0}^3 RGV_i \cdot \eta_{i,k} \cdot RGV_k \tag{3.112}$$

Accordingly, the RGS is a Lorentz scalar.

(3) Corresponding inhomogeneous DEQs are as follows:

$$\dot{\hat{\varepsilon}}_{jj}^2 - \left(\frac{G_j^*}{c}\right)^2 = 8\pi G\rho_{add} = (\partial_t \hat{\varepsilon}_{jj})^2 - \left(\frac{\partial_j \phi}{c}\right)^2 \tag{3.113}$$

$$\dot{\hat{\varepsilon}}^2 - \left(\frac{G^*}{c}\right)^2 = 24\pi G\rho_{add} = (\partial_t \hat{\varepsilon})^2 - \Sigma_{j=0}^3 \left(\frac{\partial_j \phi}{c}\right)^2 \tag{3.114}$$

Note that the sign of the rate is physically determined as follows: If the average of the particular radial accelerations is positive, then additional vacuum must be formed so that the universe expands (Carmesin (2020b), Carmesin (2020a)).

3.7 Rate gravity waves, RGW

In this section, we analyze solutions of the DEQs in theorem (4). For simplicity, we abbreviate $\hat{\varepsilon}_{jj}$ by $\hat{\varepsilon}_j$:

$$RGS = 8\pi G \cdot \rho_{add} \tag{3.115}$$

$$\text{with } \; RGS = \dot{\hat{\varepsilon}}_j^2 - (\partial_j \phi)^2/c^2 \tag{3.116}$$

3.7.1 Solutions in the vacuum

Firstly, we analyze the above DEQ for the case of zero additional density ρ_{add}. So we analyze solutions in the vacuum. Accordingly, we set the RGS in Eq. (3.116) equal to zero:

$$RGS = \dot{\hat{\varepsilon}}_j^2 - \left(\partial_{x_j} \phi / c\right)^2 = 0 \tag{3.117}$$

As the rate $\dot{\varepsilon}_j$ represents a tensor, in general, we represent it with a hat, $\dot{\hat{\varepsilon}}_j$. Similarly, the amplitudes of the corresponding waves represent a tensor, in general, and so they are marked with a hat as well, see e. g. Eq. (3.118). The following waves are possible solutions of the above DEQ:

$$\hat{\varepsilon}_j = \hat{\varepsilon}_{j,\omega} \cdot \exp(-i \cdot \omega \cdot t + i \cdot k_j \cdot r_j) + \hat{\varepsilon}_{j,const.} \tag{3.118}$$

$$\hat{\phi}_j = \hat{\phi}_{j,\omega} \cdot \exp(-i \cdot \omega \cdot t + i \cdot k_j \cdot r_j) + \hat{\phi}_{j,const.} \tag{3.119}$$

Hereby, we apply the usual sign convention of quantum physics in the exponent, see e. g. (Kumar, 2018, Eq. 3.2.11), (Ballentine, 1998, Eq. 4.26). We insert these solutions into the DEQ (3.117):

$$\hat{\varepsilon}_{j,\omega}^2 \cdot \omega^2 = \frac{k_j^2}{c^2} \cdot \hat{\phi}_{j,\omega}^2 \tag{3.120}$$

Thus the velocity of propagation of a wave in direction of the coordinates r_j or k_j is as follows:

$$v_{prop} = \frac{\lambda}{T} = \frac{\omega}{k_j} \tag{3.121}$$

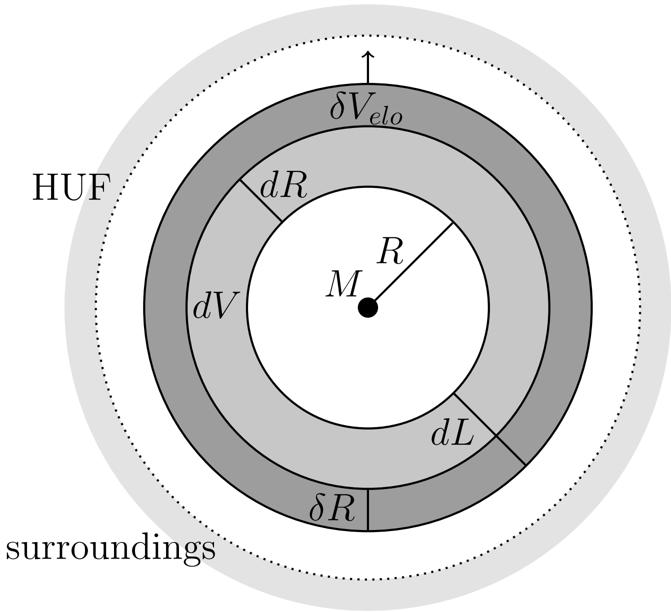


Figure 3.7: In the Schwarzschild metric, a length dR is elongated by an additional length δR, corresponding to an additional volume δV_{elo}.

We apply this result to (Eq. 3.120):

$$\hat{\phi}_{j,\omega} = \hat{\varepsilon}_{j,\omega} \cdot c \cdot v_{prop} \tag{3.122}$$

So we can express the wave in terms of a single amplitude $\hat{\varepsilon}_{j,\omega}$. Thus the waves are as follows, see equations (3.118, 3.119).

$$\hat{\varepsilon}_j(t, r_j) = \hat{\varepsilon}_{j,\omega} \cdot e^{-i\cdot\omega\cdot t+i\cdot k_j\cdot r_j} + \hat{\varepsilon}_{j,const.} \tag{3.123}$$

$$\hat{\phi}_j(t, r_j) = \hat{\varepsilon}_{j,\omega} \cdot c \cdot v_{prop} \cdot e^{-i\cdot\omega\cdot t+i\cdot k_j\cdot r_j} + \hat{\phi}_{j,const.} \tag{3.124}$$

$$\hat{\phi}_j(t, r_j) = \hat{\varepsilon}_{j,\omega}(t, r_j) \cdot c \cdot v_{prop} + \hat{\phi}_{j,const.} \tag{3.125}$$

For the case of waves with zero average, we neglect the constant:

$$\hat{\varepsilon}_j(t, r_j) = \hat{\varepsilon}_{j,\omega} \cdot \exp(-i \cdot \omega \cdot t + i \cdot k_j \cdot r_j) \tag{3.126}$$

$$\hat{\phi}_j(t, r_j) = \varepsilon_{j,\omega}(t, r_j) \cdot c \cdot v_{prop} \tag{3.127}$$

The DEQ of the RGWs (3.117) has the oscillatory solutions in equations (3.118, 3.119), and it makes possible exponentially growing solutions, in addition. Such solutions can exhibit the so-called reheating problem, Broy (2016). Accordingly, such solutions are not analyzed in this book.

3.7.2 DEQ for stationary fields

The DEQ of the RGWs (3.117) describes the relation between a field G_j^* and a rate $\dot{\hat{\varepsilon}}_j$. Physically, there are two essential cases:

1. If there is no additional source, then the field and the rate cause each other, and an oscillatory or an exponential solution occur.
2. If the field is caused by an additional source such as a mass or dynamic mass M_q, then the field causes the rate according to the DEQ of the RGWs (3.117).

In the presence of a source, the field $G_j^*(R)$ at a distance R from M_q is determined according to Gaussian gravity as follows, see section (2.3):

$$G_j^*(R) = \frac{G \cdot M_q}{R^2} \tag{3.128}$$

In order to derive the corresponding rate of unidirectional formation of vacuum, we apply the DEQ of RGWs (3.117):

$$\dot{\hat{\varepsilon}}_j = G_j^*(R)/c = \frac{G \cdot M_q}{R^2 \cdot c} \tag{3.129}$$

We summarize our results as follows:

Theorem 5 Properties of RGWs

The RGWs (Eqs. 3.123, 3.124 and 3.125)

$$\varepsilon_j(t, r_j) = \hat{\varepsilon}_{j,\omega} \cdot e^{-i\cdot\omega\cdot t + i\cdot k_j \cdot r_j} + \hat{\varepsilon}_{j,const.} \tag{3.130}$$

$$\phi_j(t, r_j) = \hat{\varepsilon}_{j,\omega} \cdot c \cdot v_{prop} \cdot e^{-i\cdot\omega\cdot t + i\cdot k_j \cdot r_j} + \hat{\phi}_{j,const.} \tag{3.131}$$

$$\phi_j(t, r_j) = \varepsilon_{j,\omega}(t, r_j) \cdot c \cdot v_{prop} + \hat{\phi}_{j,const.} \tag{3.132}$$

have the following **properties***:*

(1) Some RGWs are **plane waves or** *discrete or continuous* **linear combinations** *of these. These linear combinations include waves with various symmetries, as the plane waves establish a* **complete orthonormal basis** *of a Fourier transform*

including Fourier integrals, see e. g. Sakurai and Napolitano (1994) or Teschl (2014), (Ballentine, 1998, p. 17-22).

(2) In general, the amplitudes $\hat{\varepsilon}_{j,\omega}$ and $\hat{\phi}_{j,\omega}$ are tensors.

(3) The RGWs **propagate** *at a velocity v_{prop} with $v_{prop} = c$ or $v_{prop} < c$. If an RGW describes the propagation of vacuum, then its velocity is $v_{prop} = c$, as otherwise an object with $m_0 > 0$ could exhibit velocity $v < c$ relative to the vacuum, in contrast to SR.*

(4) In general, the RGWs represent solutions of the inhomogeneous DEQ in theorem (4). Accordingly, the rates $\dot{\hat{\varepsilon}}$ can also describe the **formation of vacuum** *with a nonzero time average.*

(4a) The RGWs describe the formation of vacuum in the vicinity of a mass M_q, whereby there occurs a stationary additional volume as follows:

$$\dot{\hat{\varepsilon}}_j = G_j^*(R)/c = \frac{G \cdot M_q}{R^2 \cdot c} \qquad \text{with} \;\; \rho_{f,In} = \frac{G^{*2}}{8\pi G c^2} \tag{3.133}$$

$$\dot{\hat{\varepsilon}}_j^2 = 8\pi G \rho_{f,In} \tag{3.134}$$

(4b) The RGWs describe the formation vacuum during the expansion of space and at a density ρ as follows:

$$\dot{\hat{\varepsilon}}_j^2 = 8\pi G \rho \qquad \text{and} \tag{3.135}$$

$$3\dot{\hat{\varepsilon}}_j^2 = 24\pi G \rho = \dot{\varepsilon}^2 = \left(\frac{\delta V}{dV \delta t}\right)^2 \tag{3.136}$$

3.8 Emergence of quanta

In this section, we investigate the structure of the vacuum solutions in section (3.7.1). Based on these solutions, we derive the quantization in nature.

In order to derive the velocity v_{prop} of propagation or phase velocity v_{phase}, we insert Eq. (3.122) into equation (3.120):

$$\hat{\varepsilon}_{j,\omega} \cdot \omega = \frac{k_j}{c} \cdot \hat{\varepsilon}_{j,\omega} \cdot c \cdot v_{prop} \tag{3.137}$$

We solve for the velocity of propagation:

$$\omega / k_j = v_{prop} = v_{phase} \tag{3.138}$$

3.8.1 Quantization derived

In this section, we analyze RGWs that propagate at the velocity $v_{prop} = c$. So Eq. (3.138) implies the following relation:

$$\frac{\omega}{k_j} = c \tag{3.139}$$

Each wave that propagates at the velocity of light c, and that is emitted during a finite interval of time from a finite source, has the following properties:

(1) The wave forms a wave packet, as it essentially has a finite extension in space and time.

(2) The wave packet has an energy E and a momentum p, as it essentially has a finite extension in space and time.

(3) As the wave packet propagates at c, its energy E and its momentum p obey the the following relation:

$$\frac{E}{p} = c \tag{3.140}$$

(4) As the wave packet propagates at c, its circular frequency ω and its wave number k obey the following relation:

$$\frac{\omega}{k} = c \tag{3.141}$$

(5) So the two above fractions are equal:

$$\frac{E}{p} = \frac{\omega}{k} = c \tag{3.142}$$

(6) As ω is nonzero, we can divide by ω and multiply by p. So the following fractions are equal:

$$\frac{E}{\omega} = \frac{p}{k} = \frac{p \cdot c}{\omega} \neq 0 \tag{3.143}$$

(7) In particular, the first two fractions do not depend on time, as E and p are conserved according to the laws of conservation of energy and momentum, and as ω and k of the RGW do not change as a function of time:

$$\frac{p}{k} = K(k) \tag{3.144}$$

$$\frac{E}{\omega} = K(k) \tag{3.145}$$

$$K(k) = constant(k) \tag{3.146}$$

Hereby, $constant(k) = K(k)$ is the constant of quantization. It could be a function of the wave number k, most generally.

(8) The energy E of the wave packet is constant and proportional to ω, so the energy of the wave packet is quantized. Similarly, the momentum p of the wave packet is constant and proportional to k, so the momentum of the wave packet is quantized. Thereby, that quantization are as follows:

$$E = K(k) \cdot \omega \qquad \text{and} \tag{3.147}$$

$$p = K(k) \cdot k \qquad \text{with} \tag{3.148}$$

That constant has been measured. It is the Planck constant h divided by 2π. It is called the reduced Planck constant (see 7.1):

$$K(k) = \hbar \tag{3.149}$$

However, we should first prove that $K(k)$ does not depend on k, see section (3.8.2).

3.8.2 Universality of Planck's constant derived

In this section, we show that $K(k)$ does not depend on k. For it, we analyze the standard deviations or uncertainties inherent to the wave functions.

These standard deviations are characterized by an uncertainty relation as follows:

$$\Delta x \cdot \Delta p \geq \frac{K(k)}{2} = \frac{\text{quantization factor}}{2} \quad \text{with} \tag{3.150}$$

$$\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2} \tag{3.151}$$

Hereby, Δx is the standard deviation of x and Δp is the standard deviation of p.

However, there is a universal uncertainty relation, which holds for wave functions (in the corresponding Hilbert space, see section (3.12)), it is a mathematical fact, see for instance Carmesin et al. (2020), (Sakurai and Napolitano, 1994, p. 56-57):

$$\Delta x \cdot \Delta k \geq \frac{1}{2} \quad \text{with} \tag{3.152}$$

$$k\psi = -i\partial_x \psi(x, k) \quad \text{and} \tag{3.153}$$

$$\Delta k = \sqrt{\langle k^2 \rangle - \langle k \rangle^2} \tag{3.154}$$

Hereby, Δx is the standard deviation of x and Δk is the standard deviation of k.

In particular, the product of the uncertainties Δx and Δk has a minimum, whereby that minimum does not depend on k (in a usual mathematical normalization, that minimum has the value 1/2). That mathematical result about the (Hilbert space of) wave functions does hold for the physical wave functions as well, as it is a mathematical fact. Thus $\frac{K(k)}{2}$ in Eq. (3.150) must be a constant. This shows that $K(k)$ does not depend on k, q. e. d.

3.8.3 Schrödinger equation derived

In this section, we show that the DEQ of the RGW (3.117) is the Schrödinger equation. For it, we solve that equation for $\dot{\hat{\varepsilon}}_j$. Thereby, we choose different signs of the square roots (so we obtain positive energy):

$$\partial_t \hat{\varepsilon}_j(t, r_j) = -\partial_j \hat{\phi}(t, r_j)/c \tag{3.155}$$

In order to find the wave equation, we apply the solution in Eq. (3.127),

$$\hat{\phi}_j(t, r_j) = \hat{\varepsilon}_{j,\omega}(t, r_j) \cdot c^2, \tag{3.156}$$

so we derive:

$$\partial_t \hat{\varepsilon}_j(t, r_j) = -\partial_j \hat{\varepsilon}_j(t, r_j) \cdot c \tag{3.157}$$

For comparison, the Schödinger equation is as follows, see section (1.3.2.3):

$$i\hbar \partial_t \psi(t, r_j) = -i \cdot \hbar \cdot c \cdot \partial_{r_j} \psi(t, r_j) \tag{3.158}$$

In fact, the above Eq. (3.157) is already mathematically equivalent to the Schrödinger equation. However, the square of the wave function should be proportional to the energy density $u_{f,In}$, as the energy density $u_{f,In}(\vec{R}, t)$ is proportional to the probability of finding the object at $(\vec{R}, t)$, see section (1.3.2.1). Moreover, the wave function should have the physical dimension or unit $[\psi] = 1$. For that purpose, we apply the time derivative to Eq. (3.157), and we multiply with a normalization factor of time t_n. That factor t_n is determined so that the wave function ψ has an amplitude corresponding to the respective physical situation under investigation. In particular, the sum or integral of all probabilities or probability densities is normalized to one:

$$\partial_t \dot{\hat{\varepsilon}}_j(t, r_j) \cdot t_n = -\partial_{r_j} \dot{\hat{\varepsilon}}_j(t, r_j) \cdot t_n \cdot c \tag{3.159}$$

In order to show that the DEQ of the RGW is equivalent to the Schödinger equation, we multiply Eq. (3.159) by $i\hbar$:

$$\boxed{i\hbar\partial_t\dot{\hat{\varepsilon}}_j(t,r_j)\cdot t_n = -i\hbar\partial_{r_j}\dot{\hat{\varepsilon}}_j(t,r_j)\cdot t_n\cdot c} \qquad (3.160)$$

We conclude that the DEQ of the RGW (3.160) is equivalent to the Schödinger equation (3.158), whereby we identify the normalized unidirectional rate $\dot{\hat{\varepsilon}}_j(t,r_j)\cdot t_n$ with the normalized wave function $\psi(t,r_j)\cdot f_n$, see figure (3.3), whereby f_n denotes a normalization factor of a wave function ψ:

$$\boxed{\dot{\hat{\varepsilon}}_j(t,r_j)\cdot t_n = \psi(t,r_j)\cdot f_n} \qquad (3.161)$$

In order to make the Schödinger equation (3.160) even more obvious, we apply the momentum operator $\hat{p}_j = -i\hbar\partial_{r_j}$, the operator of kinetic energy $\hat{E}_{kin} = \hat{p}_{r_j}\cdot c = -i\hbar\partial_{r_j}\cdot c$ and the operator of energy $\hat{E} = i\hbar\partial_t$, see equations (1.2, 1.6):

$$\hat{E}\dot{\hat{\varepsilon}}_j(t,r_j)\cdot t_n = \hat{p}_j\dot{\hat{\varepsilon}}_j(t,r_j)\cdot t_n\cdot c = \hat{E}_{kin}\dot{\hat{\varepsilon}}_j(t,r_j)\cdot t_n \quad (3.162)$$

3.8.4 Objects with $v_{prop} < c$

An object with a velocity $v_{prop} < c$ has a rest mass m_0. According to SR, the energy momentum relation holds:

$$E^2 = p^2c^2 + m_0^2\cdot c^4 \qquad (3.163)$$

In order to obtain the Schödinger equation, we apply the root:

$$E = \sqrt{p^2c^2 + m_0^2\cdot c^4} \qquad (3.164)$$

In many applications, the non-relativistic approximation of the above root is applied. Usually, the linear order in $p/(m_0c)$ is used:

$$E \hat{=} m_0\cdot c^2 + \frac{p^2}{2m_0} \qquad (3.165)$$

It is convenient to use the kinetic energy $E_{kin,non-relativistic} = E - m_0 \cdot c^2$:

$$E_{kin,non-relativistic} \hat{=} \frac{p^2}{2m_0} \tag{3.166}$$

In order to obtain the Schödinger equation, we apply the corresponding operators. In particular, we use Eq. (3.166), we insert the operator $\hat{p}$ for the momentum p (Eq. 1.2), we insert the operator $\hat{E}$ for the energy $E_{kin,non-relativistic}$ (1.6). Moreover, we multiply by the wave function:

$$\boxed{i\hbar\partial_t\psi(t, r_j) = -\frac{\hbar^2}{2m_0}\partial^2_{r_j}\psi(t, r_j)} \tag{3.167}$$

This is the non-relativistic Schödinger equation (1.11), whereby we identify the normalized unidirectional rate $\dot{\varepsilon}_j(t, r_j) \cdot t_n$ with the normalized wave function $\psi \cdot f_n$:

$$\dot{\varepsilon}_j(t, r_j) \cdot t_n = \psi(t, r_j) \cdot f_n \tag{3.168}$$

We summarize our results as follows:

Theorem 6 Emergence of quanta

(1) Each wave that propagates at the velocity of light $v_{prop} = c$, and that is emitted at a finite interval of time and from a finite source, has the following properties:

(1.1) The wave forms a wave packet with an energy E, a momentum p, a circular frequency ω and a wave number k.

(1.2) The wave packet is quantized as follows:

$$E = K \cdot \omega \qquad \text{and} \tag{3.169}$$

$$p = K \cdot k \qquad \text{with} \tag{3.170}$$

$$K = \text{universal constant of quantization} \tag{3.171}$$

Hereby, the universal constant of quantization K does not depend on E or ω, K has been measured, and K is Planck's *constant h divided by* 2π*, so K is the reduced Planck constant* $\hbar = \frac{h}{2\pi} = K$*, see table (7.1).*

(1.3) If that wave is a rate gravity wave, RGW, it obeys the Schödinger equation, SEQ. Hereby, the normalized wave function is equal to the normalized rate of the unidirectional relative change of the volume of vacuum, see figure (3.3):

$$\dot{\hat{\varepsilon}}_j(t, r_j) \cdot t_n = \psi(t, r_j) \cdot f_n \tag{3.172}$$

$$i\hbar\partial_t \dot{\hat{\varepsilon}}_j(t, r_j) \cdot t_n = -i\hbar\partial_{r_j} \dot{\hat{\varepsilon}}_j(t, r_j) \cdot t_n \cdot c \tag{3.173}$$

(2) Each RGW that propagates at the velocity of light $v_{prop} < c$*, and that is emitted at a finite interval of time and from a finite source, has the following properties:*

(2.1) The RGWs are quantized. From the above one dimensional SEQ, the three dimensional SEQ is constructed as usual, see e. g. Sakurai and Napolitano (1994), Ballentine (1998), Kumar (2018).

(2.2) The RGW obeys the Schödinger equation, SEQ. Hereby, the normalized wave function is equal to the normalized rate of the unidirectional relative change of the volume, see figure (3.3). For $v/c << 1$*, the SEQ is as follows:*

$$\dot{\varepsilon}_j(t, r_j) \cdot t_n = \psi(t, r_j) \cdot f_n \tag{3.174}$$

$$i\hbar\partial_t \psi(t, r_j) = -\frac{\hbar^2}{2m_0} \cdot \partial^2_{r_j} \psi(t, r_j) \tag{3.175}$$

Hereby, m_0 *is the rest mass of the described quantum object.*

All results derived in this theorem are based on the spacetime-quadruple.

3.9 Derivation of the SM

Using the spacetime-quadruple, SQ, we derived two essential results: In section (3.1), we obtained the energy factor $\epsilon(R)$

of the Schwarzschild-metric, SM. In the previous section (3.8), we realized the emergence of the quantization in nature. In this section, we apply these two findings in order to derive the Schwarzschild metric g_{ij}, SM.

Note that the Schwarzschild metric g_{ij} has been derived by using the energy factor $\epsilon(R)$ and quanta (photons), see e. g. Carmesin (2021d). Thereby we did not yet derive the quanta on the basis of the SQ, whereas in this book, we derive $\epsilon(R)$, quanta and the SM (see g_{ij} in Eq. 3.192) from the SQ.

Blue shift: For it we consider a photon that is placed in a HUF, and that starts at $r \to \infty$, and that has a corresponding periodic time T_∞ (see left rectangle in Fig. 3.8), and that falls vertically towards a field-generating mass M at $r = 0$. Thus the energy of the photon is:

$$E_{HUF}(r \to \infty) = \frac{h}{T_\infty} \tag{3.176}$$

The field generating mass M generates the field G^*. The energy in the field is described by the position factor $\epsilon(r)$.

For instance, if the position r of the photon decreases, then its energy decreases by that position factor $\epsilon(r)$ and is simultaneously multiplied by the inverse factor $\frac{1}{\epsilon(r)}$, so that the energy remains invariant.

If the photon is observed in a local frame (LUF) at the radius r, then the observer has the same position factor as the photon, and so the measurement apparatus only takes care of the inverse position factor $\frac{1}{\epsilon(r)}$ by measuring the energy $\frac{h}{T_\infty} \cdot \frac{1}{\epsilon(r)}$ of the photon in the local frame. As a consequence, the photon appears to have the short periodic time:

$$T(r) = T_\infty \cdot \epsilon(r) \tag{3.177}$$

This corresponds to a blue shift (see central rectangle in Fig. 3.8), and the energy of the photon in the local frame is as fol-

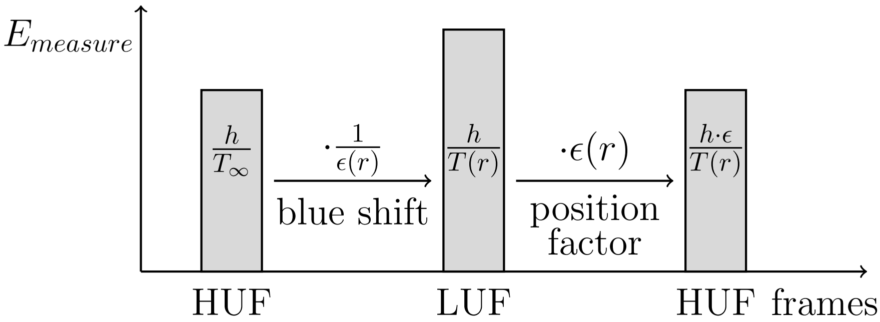


Figure 3.8: Photon propagating down towards a mass M: Measured energy $E_{measure}$ in the HUF and LUF.

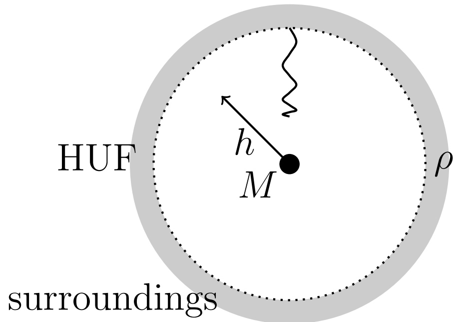


Figure 3.9: A **field generating mass frame, FMF**, is embedded in a HUF. The height h characterizes the field. A photon is falling down, thereby its wavelength decreases.

lows:

$$E_{blue\ shift} = \frac{h}{T(r)} = \frac{h}{T_\infty \cdot \epsilon(r)} = \frac{E_{HUF}(r \to \infty)}{\epsilon(r)} \tag{3.178}$$

Many observations are carried out in such a local frame (see central rectangle in Fig. 3.8).

Definition 4 Local universe frame, LUF

A frame that is accelerated or that experiences a field or a curvature of spacetime or that is falling freely is called **local universe frame, LUF**.

For instance, a LUF may be embedded in a HUF, and it may contain a field generating mass, or it may be falling freely.

In general, the energy in such a local frame is obtained from the corresponding energy in the HUF by multiplication with the inverse position factor, $\frac{1}{\epsilon(r)}$.

$$\boxed{E_{LUF} = \frac{E_{HUF}(r \to \infty)}{\epsilon(r)} \quad \text{for description via } \epsilon(r)} \tag{3.179}$$

Alternatively, the local frame can be described by a potential energy $E_{pot}(r)$ instead of a position factor $\epsilon(r)$. In that case, the energy in the LUF is obtained from the corresponding energy in the HUF by subtracting the potential energy $E_{pot}(r)$:

$$E_{LUF} = E_{HUF}(r \to \infty) - E_{pot}(r) \quad \text{descr. via } E_{pot} \tag{3.180}$$

Conversely, the energy in the HUF is obtained from the energy in the LUF by multiplication with the position factor, see right rectangle in Fig. (3.8) and Eq. (3.179). In the case of a description with a potential energy, the energy in the HUF is obtained from the energy in the LUF by subtracting the potential energy, see Eq. (3.179).

Gravitational time dilation: The periodic time $T(r)$ of photons is used for time measurement, e.g. in atomic clocks (Bundesanstalt (2007), Lombardi et al. (2007)). Accordingly, the periodic time changes the time interval $dt(r)$ by the same factor:

$$dt(r) = dt_\infty \cdot \epsilon(r) \quad \text{in the LUF} \tag{3.181}$$

Altogether, the time elapses at a decreased rate near M. This effect is called **gravitational time dilation**.

Gravitational radial elongation: An observer in the HUF at $r \to \infty$ measures a radial length L_{LUF} in a LUF at finite r. For it, the observer sends a light signal to a mirror in a LUF, detects the reflected signal, and measures the **t**ime **o**f **f**light $t_{tof,HUF}$

with a clock in the HUF. The observer evaluates the length (as the light propagates the path twice, there is a factor 1/2):

$$L_{HUF} = t_{tof,HUF} \cdot c \cdot \frac{1}{2} \tag{3.182}$$

We apply $t_{tof,HUF} = t_{tof,LUF}/\epsilon(r)$:

$$L_{HUF} = t_{tof,LUF} \cdot c \cdot \frac{1}{2} \cdot \frac{1}{\epsilon(r)} \tag{3.183}$$

We identify $t_{tof,LUF} \cdot c \cdot \frac{1}{2}$ by L_{LUF}:

$$L_{HUF} = L_{LUF} \cdot \frac{1}{\epsilon(r)} > L_{LUF} \tag{3.184}$$

Altogether, the radial length increases near M. We identify this effect as a **gravitational radial elongation**.

Metric tensor: Tensor formulations of GR are very popular (see for instance Einstein (1915), Stephani (1980), Carmesin (1996), Moore (2013)). Accordingly, we express the above results in terms of the metric tensor, additionally. A line element ds in spacetime is expressed as follows:

$$ds^2 = \Sigma_{i=0,j=0}^{i=3,j=3} \; g_{ij} \; dx_i \cdot dx_j \tag{3.185}$$

For the case of a change $dx_j = c \cdot dt_\infty = dx_i$, we get:

$$dt(r)^2 = |g_{tt}| \cdot dt_\infty^2 \;\; \text{or} \;\; |g_{tt}| = \epsilon(r)^2 = 1 - \frac{R_S}{r} \tag{3.186}$$

For the case of a change $dx_j = dr_\infty = dx_i$, we get:

$$dR(r)^2 = g_{rr} \cdot dR_\infty^2 \;\; \text{or} \;\; g_{rr} = \frac{1}{\epsilon(r)^2} = \frac{1}{1 - \frac{R_S}{r}} \tag{3.187}$$

According to the isotropic field near M, the metric factors for the angular polar coordinates are zero, as there is no gravity in the horizontal direction. We apply three dimensional polar coordinates. So the angular element of the metric tensor $g_{\theta\theta}$ is proportional to r^2:

$$g_{\theta\theta} = r^2 \tag{3.188}$$

Similarly, the angular element of the metric tensor $g_{\phi\phi}$ is proportional to $r^2 \cdot \sin^2\theta$:

$$g_{\phi\phi} = r^2 \cdot \sin^2\theta \tag{3.189}$$

For the same reason, all non-diagonal elements are zero.

$$g_{i,j} = 0 \quad \text{for} \quad i \neq j \tag{3.190}$$

According to a convention, the element g_{tt} is supplemented by a factor -1 (Straumann (2013), Stephani (1980), Carmesin (1996)). We use this convention, as it maximizes the number of positive signs in the tensor η_{ij} or g_{ij} in the limit $r \to \infty$, see Eq. (3.108). Of course, the sign is not determined at all by physical reasons. We present the derived tensor elements by the vector notation in the following Eq. below. We summarize our derivation:

Theorem 7 The spacetime-quadruple implies the SM

The Schwarzschild metric, SM, can be derived from the SR as follows:

(1) The energy function $E(r, v)$ in the **field generating mass frame, FMF** *is derived from the gravitational field, the PFF and the SR*[3]*:*

$$E(r, v) = m_0 \cdot c^2 \cdot \frac{\sqrt{1 - R_S/r}}{\sqrt{1 - v^2/c^2}} \tag{3.191}$$

(2) The elements of the metric tensor g_{ij} are derived by analyz-

[3]It will be derived later that physical states fulfill the relation $R > R_S$.

ing a photon in the **local universe frame, LUF***:*

$$g_{i,j} = \begin{pmatrix} -\left(1-\frac{R_S}{r}\right) & 0 & 0 & 0 \\ 0 & \frac{1}{1-\frac{R_S}{r}} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \cdot \sin^2\theta \end{pmatrix} \tag{3.192}$$

.

3.10 Particle wave transformation, PWT

So far, we investigated the SQ as follows:

Firstly, we introduced a **general object** with a mass or dynamical mass M, see chapter (2).

Secondly, we described the effect of such an object on the region outside the object by the Schwarzschild metric, SM, see sections (3.1 , 3.9). In this manner, we described the object in terms of a **particle**. More generally, the object can be described by the Kerr metric of a spinning black hole, see Kerr (1963). Or the object can be described by the Kerr-Newman metric of a charged and spinning black hole, see Newman and Janis (1965) or (Straumann, 2013, chapter 8). Even more generally, the object can be described by any mass or dynamical mass M, the surroundings of which can be characterized by a curvature of space or of spacetime or by a gravitational field.

Thirdly, we described the general object by an unidirectional rate gravity wave, RGW, see sections (3.2, 3.3, 3.4, 3.5, 3.6, 3.7). In this manner, we described the object in terms of a **wave**. Hereby the rate $\dot{\varepsilon}$ of the rate gravity wave corresponds to the **wave function** ψ of quantum physics, see section (3.8).

By performing the procedure outlined above, we transformed a description of the general object in terms of a particle to a description of the same object in terms of a wave. We call that transformation **particle wave transformation, PWT**.

Obviously, that transformation describes the **duality of particles and waves observed in nature**. Accordingly, the PWT should be characterized and analyzed, in order to understand nature more deeply.

In this section, we elaborate the PWT in precise detail. For it, we apply results obtained in (Carmesin, 2021d, sections 1.9, 1.13 and 2.5).

3.10.1 Effect of a general object upon its surroundings

In this section, we summarize the effects of a general object with mass or dynamical mass M upon its surroundings. Hereby, the object is described as a particle.

Of course, the particle exhibits an effect upon the surroundings as a **direct consequence of gravity**. If the particle has a charge, for instance an electric charge, a hypercharge or a color-charge in the sense of the strong interaction, see e. g. Tanabashi et al. (2018), Griffiths (2008), then the particle exhibits additional effects upon its surroundings. We do not analyze such charges here, as they can easily be added later, see e. g. Landau and Lifschitz (1971), Tanabashi et al. (2018), Griffiths (2008), Carmesin (2021e).

The direct gravitational effect of the particle upon its surroundings can be described by the SM, see section (3.9).

3.10.2 Elongation δR corresponding to curvature

In the SM, see e. g. Schwarzschild (1916) or Carmesin (2021d)) or section (3.9), there occurs a curvature, corresponding to an elongation in the radial direction, see figure (3.7). Accordingly, the coordinate distance dR at a coordinate R in figure (3.7) is elongated to the following length dL:

$$dL = \frac{dR}{\sqrt{1 - R_S/R}} \quad \text{with} \quad R_S = \frac{2Gm}{c^2} \tag{3.193}$$

Thus the additional distance in figure (3.7) is as follows:

$$\delta R = dL - dR \tag{3.194}$$

3.10.3 Vacuum δV_{elo} corresponding to elongation δR

The additional vacuum with the volume δV or δV_{elo} in figure (3.7) is the volume of a shell with thickness δR and radius R:

$$\delta V_{elo} = 4\pi \cdot R^2 \cdot \delta R \tag{3.195}$$

We insert equations (3.193, 3.194):

$$\delta V_{elo} = 4\pi \cdot R^2 \cdot dR \cdot \left(\frac{1}{\sqrt{1 - R_S/R}} - 1 \right) \tag{3.196}$$

We apply the volume $4\pi \cdot R^2 \cdot dR = dV$ of the shell:

$$\delta V_{elo}(R) = dV \cdot \left(\frac{1}{\sqrt{1 - R_S/R}} - 1 \right) \tag{3.197}$$

In particular, in the case of a small ratio R_S/R and in linear order in that ratio, we derive the following linear approximation, see (Carmesin, 2021d, Eq. 1.46):

$$\delta V_{elo}(R) \doteq dV \cdot \frac{1}{2} \cdot \frac{R_S}{R} \qquad \text{(linear approximation)} \tag{3.198}$$

3.10.4 Locally formed vacuum δV_{LFV} in the SM

In the surroundings of M, the locally formed vacuum can be described with help of a field G^*, see chapter (2). We showed already, that this field G^* corresponds to a density $\rho_{f,In}$, which in turn causes the formation of vacuum, see sections (3.5, 3.7). As the field G^* and the density $\rho_{f,In}$ are local quantities in the vicinity of M, we call the formed vacuum a **locally formed vacuum** δV_{LFV}. In this section, we derive the amount of that locally formed vacuum δV_{LFV} that is generated at a distance R of the mass M in figure (3.7).

The vacuum propagates at the velocity c, see section (2.6). According to the symmetry, the vacuum exhibits a net propagation in a radial direction. Thus the volume $\delta V_{elo}(R)$ of elongation is represented by the volume of the net propagation of the vacuum. If the volume is the same at two radii R and $R + \delta R$, then no volume is formed in the difference δR. Accordingly, the formed volume δV_{LFV} is the difference of the volumes of elongation as follows:

$$\underline{\delta} V_{LFV} = \delta V_{elo}(R + \delta R) - \delta V_{elo}(R) \tag{3.199}$$

We apply Eq. (3.197) to the above equation:

$$\underline{\delta} V_{LFV} = dV \cdot \left(\frac{1}{\sqrt{1 - \frac{R_S}{R}}} - \frac{1}{\sqrt{1 - \frac{R_S}{R+\delta R}}} \right) \tag{3.200}$$

In the limit δR to zero, that term can be expressed by using a derivative as follows:

$$\lim_{\delta R \to 0} \frac{\delta V_{LFV}}{\delta R} = -dV \cdot \frac{\partial}{\partial R} \frac{1}{\sqrt{1 - \frac{R_S}{R}}} \tag{3.201}$$

We evaluate the derivative:

$$\lim_{\delta R \to 0} \frac{\delta V_{LFV}}{\delta R} = \frac{dV \cdot R_S}{2R^2} \cdot \frac{1}{\sqrt{1 - \frac{R_S}{R}}^3} \tag{3.202}$$

We apply the notation of the Leibniz calculus, see e. g. Leibniz (1684). Accordingly, we interpret $\frac{\delta V_{LFV}}{\delta R}$ as a derivative, or we interpret δR as an infinitesimal quantity. So the limit in the above equation is not noted explicitly:

$$\frac{\delta V_{LFV}}{\delta R} = \frac{dV \cdot R_S}{2R^2} \cdot \frac{1}{\sqrt{1 - \frac{R_S}{R}}^3} \tag{3.203}$$

We apply the term of the volume dV of the shell with thickness dR and radius R in figure (3.7), $dV = dR \cdot 4\pi \cdot R^2$:

$$\frac{\delta V_{LFV}}{\delta R} = 2\pi \cdot R_S \cdot dR \cdot \frac{1}{\sqrt{1 - \frac{R_S}{R}}^3} \tag{3.204}$$

3.10.5 Rate of locally formed vacuum, LFV

The vacuum propagates at the velocity c of light, see section (2.6). Moreover, the volume $\underline{\delta} V_{LFV}$ of the locally formed vacuum, flows radially, see section (3.10.5). Thus the volume $\underline{\delta} V_{LFV}$ of the LFV propagates the radial distance δR in the time interval $\delta t = \delta R/c$:

$$\delta t = \delta R/c \tag{3.205}$$

During that microscopic time interval δt, the locally formed vacuum propagates through the volume $\underline{\delta} V_{LFV}$ that it forms, so that new volume is formed during the time δt of propagation.

We apply that relation to Eq. (3.204), and we multiply by c:

$$\frac{\underline{\delta} V_{LFV}}{\delta t} = \frac{dV \cdot R_S \cdot c}{2R^2} \cdot \frac{1}{\sqrt{1 - \frac{R_S}{R}}^3} \tag{3.206}$$

3.10.6 Relative rate of LFV

In order to derive the relative rate $\frac{\underline{\delta} V_{LFV}}{\delta t \cdot dV}$ of the volume $\underline{\delta} V_{LFV}$ of the LFV, we divide Eq. (3.206) by dV:

$$\frac{\underline{\delta} V_{LFV}}{\delta t \cdot dV} = \frac{R_S \cdot c}{2R^2} \cdot \frac{1}{\sqrt{1 - \frac{R_S}{R}}^3} \tag{3.207}$$

Remind that we denote the formed volume per volume and time by $\underline{\delta V}$, see section (3.2.2.1). As the volume $\underline{\delta} V_{LFV}$ of the LFV forms in the radial direction, we identify the rate by the rate

$\dot{\varepsilon}_{j,LFV}$ of the unidirectional vacuum, see figure (3.3) and equations (3.42,3.43,3.44):

$$\frac{\delta V_{LFV}}{\delta t \cdot dV} = \dot{\varepsilon}_{j,LFV} \tag{3.208}$$

We insert Eq. (3.207) and obtain the rate as follows:

$$\dot{\varepsilon}_{j,LFV} = \frac{R_S \cdot c}{2R^2} \cdot \frac{1}{\sqrt{1-\frac{R_S}{R}}^3} \tag{3.209}$$

We apply the Schwarzschild radius $R_S = \frac{2GM}{c^2}$:

$$\dot{\varepsilon}_{j,LFV} = \frac{G \cdot M}{R^2 \cdot c} \cdot \frac{1}{\sqrt{1-\frac{R_S}{R}}^3} \tag{3.210}$$

We identify the field in the SM, $G^* = \frac{G \cdot M}{R^2}$:

$$\boxed{\dot{\varepsilon}_{j,LFV} = \frac{G^*}{c} \cdot \frac{1}{\sqrt{1-\frac{R_S}{R}}^3}} \tag{3.211}$$

Here, we express G^* by a potential ϕ via $G^* = -\partial_{r_j}\phi(R)$:

$$\boxed{\dot{\varepsilon}_{j,LFV} = \frac{-\partial_{r_j}\phi(R)}{c} \cdot \frac{1}{\sqrt{1-\frac{R_S}{R}}^3}} \tag{3.212}$$

Hereby, we may express $\frac{R_S}{R}$ by $\frac{2}{c^2} \cdot \frac{G \cdot M}{R} = \frac{2}{c^2}\phi(R)$ in a similar manner. Remind that the fields G^* caused from sources outside a HUF add up to zero in a HUF, so the Eq. (3.212) does also hold globally in the universe. The above equation represents a wave equation. We call it a **generalized SEQ**.

3.10.7 Far distance limit: usual and universal QP

In this section, we analyze an especially simple case of the DEQ (3.211). It is the limit R_S/R towards infinity, we call it the **far**

distance limit. Moreover, we apply $G^* = -\partial_{x_j}\phi$:

$$\lim_{R_S/R\to 0} \dot{\varepsilon}_{j,LFV} = \frac{G^*}{c} = -\frac{\partial_{x_j}\phi}{c} \tag{3.213}$$

Physically, this limit corresponds to an observer that is at some distance from our general object. We denote the above limit of the rate by $\dot{\varepsilon}_j$:

$$\dot{\varepsilon}_j := \lim_{R_S/R\to 0} \dot{\varepsilon}_{j,LFV} \tag{3.214}$$

So we derive the DEQ as follows, see Eq. (3.213):

$$\dot{\varepsilon}_j = -\frac{\partial_{x_j}\phi}{c} \tag{3.215}$$

Using that equation, we derived the Schrödinger equation, SEQ, see section (3.8.3):

$$\partial_t \varepsilon_j(t, r_j) = -\partial_j \varepsilon_j(t, r_j) \cdot c \qquad \text{or} \tag{3.216}$$

$$i\hbar\partial_t\psi(t, r_j) = -i \cdot \hbar \cdot c \cdot \partial_{r_j}\psi(t, r_j) \qquad \text{or} \tag{3.217}$$

$$i\hbar\partial_t\psi(t, r_j) = \hat{p} \cdot c \cdot \psi(t, r_j) = \hat{E}_{kin} \cdot \psi(t, r_j) \tag{3.218}$$

Theorem 8 Particle wave transformation, PWT

I. Representations

The general object with mass or dynamical mass M has a particle description, PD, of M. The PD provides the curvature of space or of spacetime or the gravitational field in the vicinity of M. Examples are the SM, the Kerr metric or the Kerr-Newman metric:

$$PD(M) \quad \text{is curvature} = \text{function}(M, \vec{R}) \qquad \text{or} \tag{3.219}$$

$$PD(M) \quad \text{is } G^* = \text{function}(M, \vec{R}) \tag{3.220}$$

The general object with mass or dynamical mass M has a wave description, WD, of M. The WD provides the SEQ, and the *generalized WD, the WD_g, provides the generalized SEQ, the SEQ_g describing M:*

$$WD(M) \quad \text{is} \quad SEQ(M) \quad \text{or} \tag{3.221}$$

$$WD_g(M) \quad \text{is} \quad SEQ_g(M) \tag{3.222}$$

II. PWT

Starting from the $PD(M)$, we derive the $WD(M)$ via the following steps.

(1) We derive the rate $\dot{\varepsilon}$ as a function of the particle description, $rate(PD(M))$. So we obtain an equation of the following form, see for instance Eq. (3.210):

$$\dot{\varepsilon}_{j,LFV} = term_1(M, G^*) \tag{3.223}$$

In terms of the steps of transformation, we denote the above equation as follows:

$$transformation_1[PD(M)] \quad \text{is} \quad rate(PD(M)) \tag{3.224}$$

(1a) For it, we derive the rate $\dot{\varepsilon}(PD(M))$ from the curvature as described above, if the curvature is provided by the PD.

(1b) Or we apply $\dot{\varepsilon}(PD(M)) = \pm G^*/c$, if the field is provided by the PD.

(2) In $term_1$ in Eq. (3.223), we express M (or M within R_S) by the field G^*.

Thereby, the evaluation of the field may be achieved by the application of Gaussian gravity, see chapter (2) or e. g. Carmesin (2021d). Note that GG is applicable most microscopically.

In terms of the steps of transformation, we denote the transformed equation as follows:

$$transformation_2[PD(M)] \quad \text{is} \quad field(rate(PD(M))) \tag{3.225}$$

(3) We express the field in terms of a potential:

$$G^* = -\partial_{\vec{r}}\phi(\vec{r}) \tag{3.226}$$

Thus we obtain a wave equation of the following form:

$$\dot{\varepsilon}_{j,LFV} = term_2(\partial_{\vec{r}}\phi(\vec{r})) \tag{3.227}$$

That wave equation represents a **generalized SEQ**, *see for instance Eq. (3.212).*

In terms of the steps of transformation, we denote the transformed equation as follows:

$$transformation_3[PD(M)] \;\text{ is }\; potential(field(rate(PD(M)))) \tag{3.228}$$

(4) We apply the far distance limit to Eq. (3.227):

$$\lim_{\frac{R_S}{R}\to 0} \dot{\varepsilon}_{j,LFV} = \lim_{\frac{R_S}{R}\to 0} term_2(\partial_{\vec{r}}\phi(\vec{r})) \tag{3.229}$$

Thus only the leading order term in R_S/R remains. So the SEQ is obtained, see e. g. equations (3.216, 3.217, 3.218).

In terms of the steps of transformation, we denote the transformed equation as follows:

$$transformation_4[PD(M)] \quad \text{is} \tag{3.230}$$

$$\lim_{\frac{R_S}{R}\to 0} potential(field(rate(PD(M)))) \tag{3.231}$$

Altogether, the steps (1), (2), (3) and (4) constitute the fourth transformation, $transformation_4$, it is the particle wave transformation, PWT:

$$WD(M) \;\text{ is }\; PWT(PD(M)) \;\text{ with} \tag{3.232}$$

$$PWT(PD(M)) \;\text{ is }\; \lim_{\frac{R_S}{R}\to 0} potential(field(rate(PD(M)))) \tag{3.233}$$

Since the resulting equation is the SEQ, we obtain the following result of the PWT:

$$SEQ(M) \;\text{ is }\; PWT(PD(M)) \tag{3.234}$$

III. Generalized PWT

The generalized SEQ, SEQ_g, is obtained by the generalized particle wave transformation, PWT_g, as follows:

$$SEQ_g(M) \text{ is } PWT_g(PD(M)) \text{ and} \tag{3.235}$$
$$PWT_g(PD(M)) \text{ is } potential(field(rate(PD(M)))) \tag{3.236}$$

The generalized PWT has an inverse as follows:

$$PWT_g^{-1}(SEQ_g(M)) \tag{3.237}$$
$$\text{is } rate^{-1}(field^{-1}(potential^{-1}(SEQ_g(M)))) \quad \text{or} \tag{3.238}$$
$$PWT_g^{-1}(WD_g(M)) \tag{3.239}$$
$$\text{is } rate^{-1}(field^{-1}(potential^{-1}(WD_g(M)))) \tag{3.240}$$

Thereby, the generalized SEQ represents the generalized wave description of M, $WD_g(M)$.

IV. Particle wave duality

A general object with a mass or dynamical mass M has two descriptions, the $PD(M)$ and the $WD_g(M)$. Both descriptions can be transformed into each other by application of the $PWT_g(PD(M))$ and by the $PWT_g^{-1}(WD_g(M))$. These two mutually transformable descriptions represent the observed wave particle duality, whereby these descriptions have been derived from the SQ. So the observed wave particle duality is a property of the SQ.

3.11 Probabilistic nature of quantum physics

In this section, we show that the particle wave transformation implies that a general object described by a mass or dynamical mass M is observed according to probabilities.

The object can be described as a wave, according to the PWT_g. During the propagation of the object as a wave, the

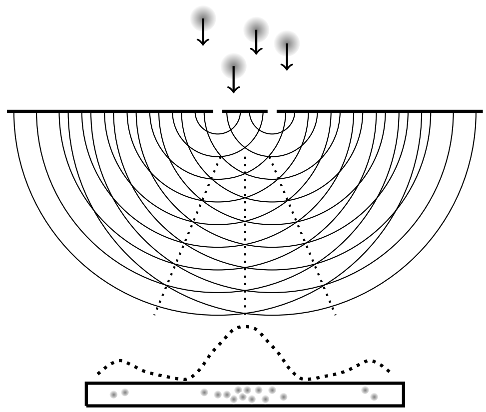


Figure 3.10: Double slit: Single photons arrive at a double slit. Behind the double slit, each photon interferes with itself. Accordingly, it is described in terms of wave fronts. At the screen, complete photons are detected by a camera and marked in the memory. In the memory, a diffraction pattern forms (dotted), based on many photons, each of which interfered with itself.

amplitude of the object decreases, since the waves distributes in space, in general.

However, the same object can be transformed to a particle description at any time according to the PWT_g^{-1}. For instance, a detector with an aperture A may detect the object, whereby the part of the wave entering the aperture has insufficient energy in order to constitute the particle, in general. As a consequence, the detector will detect the object at a probability that is proportional to the fraction of the energy of the wave that enters the detector.

Altogether, the object is of a probabilistic nature, as a consequence of the PWT. Since we derived the PWT from the SQ, the probabilistic nature of objects is a consequence of the SQ.

In the following, we use a double slit experiment as an example, and we show how the above probability is calculated.

3.11.1 Probability in a quantum system

In this section, we consider an object in chapter (2) that behaves different from a classical particle. So the object is described by a typical propagating wave or wave packet. During its propagation, such a wave packet distributes in space, and it can be diffracted at a double slit experiment, see e. g. Young (1802) or (Kumar, 2018, cover) or Fig. (3.10). So the object can correspond to a diffraction pattern, see figures (3.11, 3.10). However, though the intensity is distributed at the screen in figure (3.11), a single object can be observed at only one location x of the screen. Correspondingly, the pattern forms gradually in time, see figures (3.10, 3.12).

Thereby, the probability density $P(x)$ to find the object at a location x is proportional to the energy density $u_{f,In}$ of the particle or to the density $\rho_{f,In}$ of that particle, see proposition (3).

So the energy density $u_{f,In}$ is proportional to the square of the field G^{*2}, see proposition (3). Moreover, the field G^* is proportional to the unidirectional rate $\dot{\varepsilon}_j$, see Eq. (3.117) and figure (3.3). That unidirectional rate is proportional to the wave function, $\psi \propto \dot{\varepsilon}_j$. Altogether, the probability density $P(x)$ is proportional to the square of the wave function, and as we allow for complex functions, as usual, $P(x)$ is proportional to the absolute square of the wave function:

$$P(x) \propto |\psi(x)|^2 \tag{3.241}$$

Hereby, the normalization factors t_n and f_n in equations (3.243, 3.245, 3.248) are chosen so that the integral or sum or integral and sum of all probabilities are one:

$$P(x)dx = |\psi(x) \cdot f_n|^2 dx \qquad \text{with} \tag{3.242}$$

$$\int P(x)dx = \int |\psi(x) \cdot f_n|^2 dx = 1 \qquad \text{for continuous } x \tag{3.243}$$

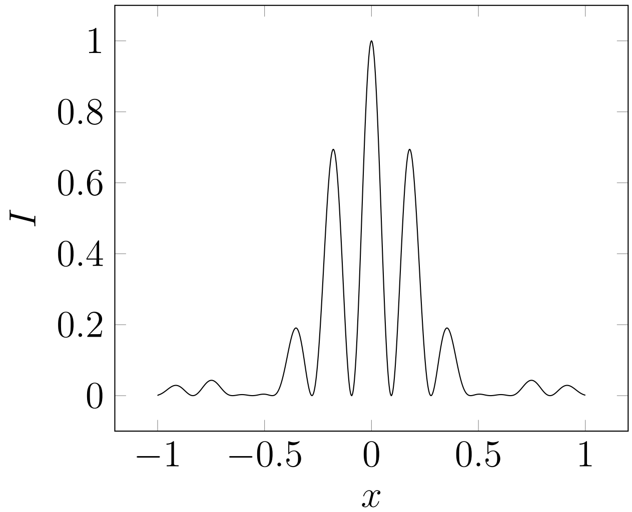


Figure 3.11: Intensity I as a function of the coordinate x at a screen of a double slit experiment.

$$P(x_j) = |\psi(x_j) \cdot f_n|^2 \qquad \text{with} \qquad (3.244)$$
$$\Sigma_j P(x_j) = \Sigma_j |\psi(x_j) \cdot f_n|^2 = 1 \qquad \text{for discrete } x \qquad (3.245)$$

$$P(x_j) = |\psi(x_j) \cdot f_n|^2 \qquad \text{or} \qquad (3.246)$$
$$P(x)dx = |\psi(x) \cdot f_n|^2 dx \qquad \text{with} \qquad (3.247)$$
$$1 = \Sigma_j P(x_j) + \int P(x)dx \qquad \text{in general} \qquad (3.248)$$

These results show that the occurrence of objects according to a probability distribution is already inherent to the SQ. Moreover, it is clear that the inverse PWT cannot be applied to a typical object with a spreading wave function.

3.11.2 Particle wave duality

The same very general object can be represented by a particle. Moreover, the PWT transforms that representation to a wave representation. So the object is at the same time a particle and a wave. This is the essence of the particle wave duality, see for instance (Kumar, 2018, p. 7, 33).

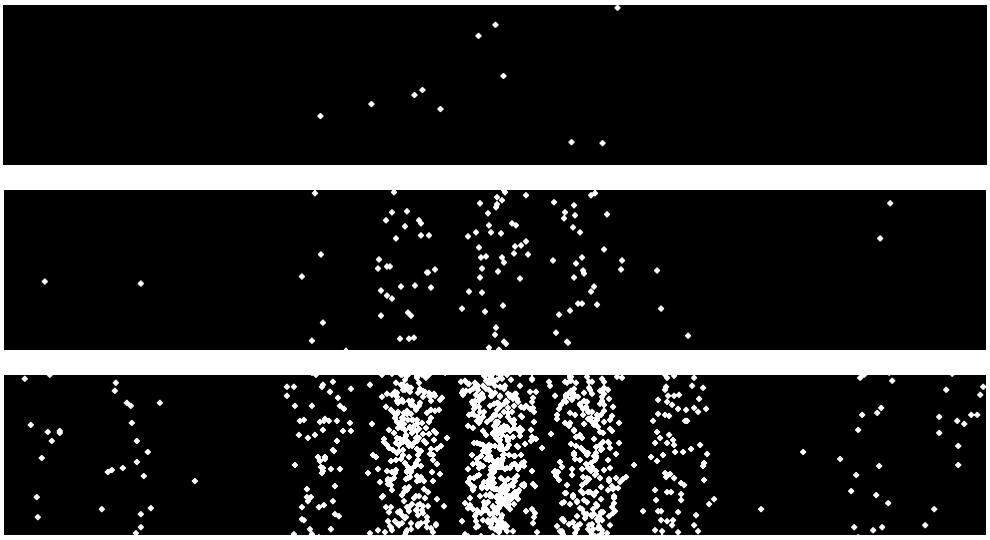


Figure 3.12: Patterns of the double slit experiment after 1 s, 10 s and 100 s.

3.11.3 Universality of QP

In this section, we characterize a universal property of QP.

In a double slit experiment, see figure (3.10), there occurs a diffraction pattern, see figure (3.11) as a function of time, see figure (3.12), independent of the type of object that propagates through the double slit. For instance, the double slit experiment has been performed with photons, with electrons, see Tonomura et al. (1988), with neutrons, see Zeilinger et al. (1988), with helium atoms, see Grisenti et al. (2000), and with large molecules, see Nairz et al. (2003). In all cases, the diffraction pattern can be derived from the wave length of the respective objects as a whole, whereas the constituents of the objects do not change the positions of the diffraction maxima. These findings show that the diffraction pattern is universal in the sense that it does not depend on the used objects (photons, neutron, atoms, molecules).

Similarly, the **universal gas law** describes the **ideal gas**, and that law provides a relation between the pressure p, volume

V, absolute temperature T and number of particles N,

$$p \cdot V = N \cdot k_B \cdot T \tag{3.249}$$

Hereby k_B denotes the Boltzmann constant, see table (7.1). While the universal gas law becomes exact in the **thermodynamic limit** N to infinity, the universality of the usual QP is achieved in the **far distance limit** R_S/R to zero.

The universality of the usual QP arises as follows: The far distance limit simplifies the DEQ of the RGWs. Thereby, the DEQ becomes equivalent to the SEQ, and this SEQ holds for all objects in the same manner. In this manner, all properties of QP that arise from the SEQ become universal in the far distance limit of the DEQ of the RGWs.

3.11.4 Generalized QP

The usual QP is achieved in the far distance limit, see section (3.11.3). New physics is expected, if we do not apply that limit. Similarly, the real gas, see van der Waals (1873), differs from the ideal gas, and the real gas describes new physics, such as phase transitions.

In this case, the quantum physics beyond the usual and universal QP is described by the DEQ (3.211):

$$\dot{\varepsilon}_{j,LFV} = \frac{G^*}{c} \cdot \frac{1}{\sqrt{1 - \frac{R_S}{R}}^3} \quad \text{or} \tag{3.250}$$

$$\dot{\varepsilon}_{j,LFV} = \frac{-\partial_j \phi(R)}{c} \cdot \frac{1}{\sqrt{1 - \frac{R_S}{R}}^3} \quad \text{and} \tag{3.251}$$

$$\lim_{\frac{R_S}{R} \to 0} \dot{\varepsilon}_{j,LFV} = \frac{-\partial_j \phi(R)}{c} \quad \text{is the SEQ} \tag{3.252}$$

Accordingly, the above Eq. (3.251) represents the **generalized SEQ**. Such general QP is beyond the scope of the present book and is to be analyzed in the future.

Such general QP is also achieved, when details of the microscopic structure are analyzed. For instance, the elementary charge and the mass of the Higgs boson have been derived by a microscopic analysis based on quantum gravity and provided new physics, see Carmesin (2021a), Carmesin (2021e).

Theorem 9 Probabilistic nature

In general, the amplitude of the wave function $\dot{\varepsilon} \cdot t_n$ decreases during the propagation in space. If that amplitude is too small in order to form a particle, then the corresponding particle occurs with a probability P that is proportional to the inertial energy density $u_{f,In}$, which is proportional to the absolute square of the wave function, $P \propto u_{f,In} \propto |\dot{\varepsilon} \cdot t_n|^2$.

3.12 Derivation of the quantum postulates

In this section, we derive the quantum postulates from the SQ. Thereby, the postulates have been designed by a guess with a subsequent elaboration, see figure (3.13) or (Grawert, 1977, p. 148). Accordingly, the postulates are not determined in a unique manner. Correspondingly, I use postulates present in the literature.

3.12.1 P1: Quantum states form a Hilbert space

In this section, we derive the following postulate by (Kumar, 2018, p. 168):

'The state of a quantum mechanical system, at a given instant of time, is described by a vector $|\Psi(t)\rangle$, in the abstract Hilbert space $\mathcal{H}$ of the system.'

3.12.1.1 Derivation

Firstly, we identify the states in the SQ:

Applications such as

diagnostics, e. g. electron microscope, MRT
band structure, e. g. electronics, computer
atomic orbitals, e. g. chemistry, pharmaceutics
entanglement, e. g. cryptography, computer
coherence, e. g. laser, laser-tools

↑

postulates of QP

guess with elaboration ↑ ↓ explanation

many experiments such as the double slit experiment

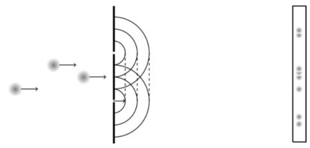


Figure 3.13: On the discovery of QP: Many experiments are subsumed by postulates of QP so that the postulates explain QP.

In the present SQ, a **state of a quantum mechanical system** is generated by the particle wave transformation. So it is a state in the far distance limit.

Moreover, the state is described by the normalized rate as a function of location and time $\dot{\varepsilon}(\vec{r},t)\cdot t_n$, see section (3.7), which is equal to the normalized wave function in quantum physics see section (3.8):

$$\dot{\varepsilon}(\vec{r},t)\cdot t_n = \psi(\vec{r},t)\cdot f_n \tag{3.253}$$

Secondly, we show that these states of the present SQ form a Hilbert space $\mathcal{H}$:

The states $\psi(\vec{r},t)$ form a **linear space**, as they are solutions of the linear DEQ of the RGWs, see section (3.7).

The states $\psi(\vec{r},t)$ form a **complete space**, as they include all linear combinations of states $\psi(\vec{r},t)$, including Fourier integrals. These form a complete Hilbert space $\mathcal{H}$, see e. g. (Teschl, 2014, p. 47) or (Sakurai and Napolitano, 1994, p. 57).

Thereby, two wave functions, $\psi_1(\vec{r}, t)$ and $\psi_2(\vec{r}, t)$, form a scalar product as follows:

$$\langle \psi_1 | \psi_2 \rangle = \int d^3 r \psi_1^*(\vec{r}, t) \cdot \psi_2(\vec{r}, t) \tag{3.254}$$

Altogether, the states of the SQ in the far distance limit form a Hilbert space $\mathcal{H}$, q. e. d.

Remark about physical completeness: Note that the physical system is usually NOT defined in a physically complete manner by its wave function $\psi(\vec{r}, t)$ or state $|\Psi(t)\rangle$. We consider a counter example: In a double slit experiment with helium atoms, the wave function $\psi(\vec{r}, t)$ of the atoms describes the pattern completely, but that wave function $\psi(\vec{r}, t)$ does NOT describe the electrons within the atoms at all. Note that essentially more complete descriptions are obtained by the SQ, see e. g. Carmesin (2021d), Carmesin (2021a), Carmesin (2021e).

Remark about complex-valued wave functions: Note that Fourier sums and Fourier integrals can alternatively be achieved in terms of real valued sine- and cosine-functions, see for instance Carmesin (2021d). So it would be possible to describe the measurable quantities based on real valued wave functions. However, that would not be useful: Similarly, it would be possible to describe the universe in a Cartesian coordinate system without negative numbers, if the origin is placed far outside the observable universe so that all points of the observable universe exhibit positive coordinates - but it is more useful to place the origin of the coordinate system into the center of the observable universe, of course.

We summarize, that the description by complex valued functions is a matter of convenience, not a matter of physical necessity.

3.12.2 P2: Observables correspond to operators

In this section, we derive the following postulate by (Kumar, 2018, p. 169):

'A measurable physical quantity A (called an observable or dynamical physical quantity), is represented by a linear and hermitian operator $\hat{A}$ acting in the Hilbert space of state vectors.'

3.12.2.1 Derivation

We showed in S. (3.12.1) that the SQ in the far distance limit provides wave functions or states that form a Hilbert space $\mathcal{H}$.

Here we derive the correspondence of observables A and hermitian operator $\hat{A}$ acting in $\mathcal{H}$.

An **observable**, A, such as the energy, represents the corresponding possible values of a measurement of a quantum state. Most generally, the values of a measurement form a partially discrete and partially continuous set of values $M_{A\,discrete\,continuous}$.

Similarly, a **hermitian operator** $\hat{A}$ acting in $\mathcal{H}$ has, most generally, a set of partially discrete and partially continuous eigenvalues $M_{\hat{A}\,discrete\,continuous}$, see e. g. (Teschl, 2014, THM 3.6 or spectral theorem).

So the SQ in the far distance limit provides a Hilbert space $\mathcal{H}$.

For hermitian operator $\hat{A}$ acting on that Hilbert space $\mathcal{H}$ provides a set $M_{\hat{A}\,discrete\,continuous}$ of eigenvalues.

That set of eigenvalues $M_{\hat{A}\,discrete\,continuous}$ corresponds to a set $M_{A\,discrete\,continuous}$ of possible values of a measurement at the RGWs of the SQ in the far distance limit.

Altogether, the SQ in the far distance limit provides the correspondence of observables A and operators $\hat{A}$ described by the postulate, q. e. d.

3.12.3 P3: Possible outcomes of a measurement are eigenvalues

In this section, we derive the following postulate by (Kumar, 2018, p. 169):

'The measurement of an observable A in a given state may be represented formally by the action of an operator $\hat{A}$ on the state vector $|\Psi(t)\rangle$. The only possible outcome of such a measurement is one of the eigenvalues, $\{a_j\}, j = 1, 2, 3, \ldots, of \hat{A}$.'

3.12.3.1 Derivation

For each observable A of the SQ in the far distance limit, there is a set $M_{A\,discrete\,continuous}$ of possible outcomes of a measurement at the corresponding states, see section (3.12.2).

That set $M_{A\,discrete\,continuous}$ of possible outcomes corresponds to a set $M_{\hat{A}\,discrete\,continuous}$ of the eigenvalues of the corresponding operator $\hat{A}$ acting in $\mathcal{H}$ that represents the RGWs, see section (3.12.2).

Thus each possible outcome of the measurement corresponds to an eigenvalue, q. e. d.

3.12.4 P4: Probabilistic outcomes of a measurement

In this section, we derive the following postulate by (Kumar, 2018, p. 169, 170):

If a measurement of an observable A is made in a state $|\Psi(t)\rangle$ of the quantum mechanical system, then the following holds:

(1) The probability of obtaining one of the non-degenerate discrete eigenvalues a_j of the corresponding operator $\hat{A}$ is given by

$$P(a_j) = \frac{|\langle\phi_j|\Psi\rangle|^2}{\langle\Psi|\Psi\rangle}, \tag{3.255}$$

where $|\phi_j\rangle$ is the eigenfunction of $\hat{A}$ with the eigenvalue a_j. If the state vector is normalized to unity, $P(a_j) = |\langle\phi_j|\Psi\rangle|^2$.

(2) If the eigenvalue a_j is m-fold degenerate, this probability is given by

$$P(a_j) = \frac{\Sigma_{i=1}^{m}|\langle\phi_j^i|\Psi\rangle|^2}{\langle\Psi|\Psi\rangle}, \tag{3.256}$$

(3) If the operator $\hat{A}$ possesses a continuous eigenspectrum $\{a\}$, the probability that the result of a measurement will yield a value between a and $a + da$ is given by

$$P(a) = \frac{|\langle\phi(a)|\Psi\rangle|^2}{\langle\Psi|\Psi\rangle}da = \frac{|\langle\phi(a)|\Psi\rangle|^2}{\int_{-\infty}^{\infty}|\Psi(a')|^2 da'}da \tag{3.257}$$

3.12.4.1 Derivation

Firstly, we note that this postulate considers only operators $\hat{A}$ that have either a discrete spectrum of eigenvectors or a continuous spectrum of eigenvectors. However, most generally, an operator $\hat{A}$ has a mixed spectrum $M_{\hat{A}\ discrete\ continuous}$. In the following, we consider operators $\hat{A}$ that have either a discrete or a continuous spectrum of eigenvectors, as the generalization is straight forward.

Secondly, the SQ in the far distance limit provides results of measurements in a **probabilistic manner**, see section (3.11.1).

Thirdly, in all three cases (1), (2) and (3), the probability $P(a_j)$ or $P(a)da$ of the outcome a_j or $[a; a+da]$ is proportional to the inertial energy density $u_{f,In}$ of the RGW, according to the law of energy conservation:

$$P(a_j) \propto u_{f,In}(a_j) \tag{3.258}$$

$$P(a) \propto u_{f,In}(a) \tag{3.259}$$

Fourthly, we derive the probabilities for the three cases, (1), (2) and (3), one at a time.

Derivation for case (1):

As a first step, we decompose the considered state $|\Psi\rangle$ into components $|\phi_j\rangle$, whereby these components are the eigenvectors of the operator $\hat{A}$:

$$|\Psi\rangle = |\Psi\rangle \qquad | \cdot 1 = \Sigma_k |\phi_k\rangle\langle\phi_k| \tag{3.260}$$

$$|\Psi\rangle = \Sigma_k |\phi_k\rangle\langle\phi_k|\Psi\rangle \tag{3.261}$$

For a k-th component $|\Psi_k\rangle$ of $|\Psi\rangle$, we identify the amplitude $\langle\phi_k|\Psi\rangle$ and the normalized eigenvector $|\phi_k\rangle$:

$$|\Psi_k\rangle = |\phi_k\rangle\langle\phi_k|\Psi\rangle \tag{3.262}$$

The corresponding inertial energy density of the field $u_{f,In,k}$ is proportional to the square of the corresponding field G_k^*:

$$G_k^* = |\phi_k\rangle \cdot \langle\phi_k|G^*(r)\rangle = |\phi_k\rangle \cdot \int dr^3 \phi_k^{cc}(\vec{r}) \cdot G^*(\vec{r}) \quad \text{and} \tag{3.263}$$

$$u_{f,In,k} \propto |G_k^*|^2 = G_k^{*,cc} \cdot G_k^* \tag{3.264}$$

Hereby, we denote the conjugate complex by a superscript ^{cc}. For instance, we mark the conjugate complex of $\phi_k(\vec{r})$ by $\phi_k^{cc}(\vec{r})$.

Moreover, G_k^* can be expressed by the corresponding rate as follows:

$$G_k^* = c \cdot \dot{\varepsilon}_k \tag{3.265}$$

Furthermore, that rate $\dot{\varepsilon}_k$ is proportional to the corresponding wave function:

$$\dot{\varepsilon}_k \propto |\Psi_k\rangle \qquad \text{and} \tag{3.266}$$

$$\dot{\varepsilon}_k^{cc} \propto \langle\Psi_k| \tag{3.267}$$

Consequently, the inertial energy density $u_{f,In,k}$ is proportional to the absolute square of the corresponding wave functions:

$$u_{f,In,k} \propto \dot{\varepsilon}_k^{cc} \cdot \dot{\varepsilon}_k \propto \langle\Psi_k|\Psi_k\rangle \tag{3.268}$$

Corresponding to Eq. (3.258), the probability $P(a_k)$ is proportional to the inertial energy density $u_{f,In,k}$:

$$P(a_k) \propto u_{f,In,k} \propto \langle \Psi_k | \Psi_k \rangle \tag{3.269}$$

In order to analyze the dependence on ϕ_k, we apply Eq. (3.262):

$$P(a_k) \propto \langle \Psi_k | \Psi_k \rangle = \langle \phi_k | \langle \Psi | \phi_k \rangle \cdot | \phi_k \rangle \langle \phi_k | \Psi \rangle \tag{3.270}$$

We use the normalization $\langle \phi_k | \phi_k \rangle = 1$ and the relation $\langle \Psi | \phi_k \rangle = (\langle \phi_k | \Psi \rangle)^*$ as well as the equality $\langle \Psi | \phi_k \rangle \cdot \langle \phi_k | \Psi \rangle = |\langle \phi_k | \Psi \rangle|^2$:

$$P(a_k) \propto |\langle \phi_k | \Psi \rangle|^2 \qquad \text{or} \tag{3.271}$$

$$P(a_k) = \cdot |\langle \phi_k | f_n \Psi \rangle|^2 \tag{3.272}$$

Hereby, we used the normalization factor f_n. It is determined next:

$$1 = \Sigma_k P(a_k) = f_n^2 \cdot \Sigma_k |\langle \phi_k | \Psi \rangle|^2 \qquad \text{or} \tag{3.273}$$

$$1 = f_n^2 \cdot \Sigma_k \langle \Psi | \phi_k \rangle \cdot \langle \phi_k | \Psi \rangle \qquad | Id = \Sigma_k | \phi_k \rangle \langle \phi_k | \tag{3.274}$$

$$1 = f_n^2 \cdot \langle \Psi | \Psi \rangle \qquad \text{we solve} \tag{3.275}$$

$$f_n^2 = 1 / \langle \Psi | \Psi \rangle \tag{3.276}$$

Hereby, we used the identity operator $Id = \Sigma_k | \phi_k \rangle \langle \phi_k |$. We apply the term for f_n^2 to Eq. (3.272):

$$P(a_k) = \frac{|\langle \phi_k | \Psi \rangle|^2}{\langle \Psi | \Psi \rangle} \tag{3.277}$$

As this probability holds for each eigenvalue a_k, it holds also for a_j, and so it proves the first case, (1), of the postulate, see Eq. (3.255), q. e. d.

The other two cases (2) and (3) can be worked out in a similar manner. Accordingly, we do not elaborate these cases in the present book.

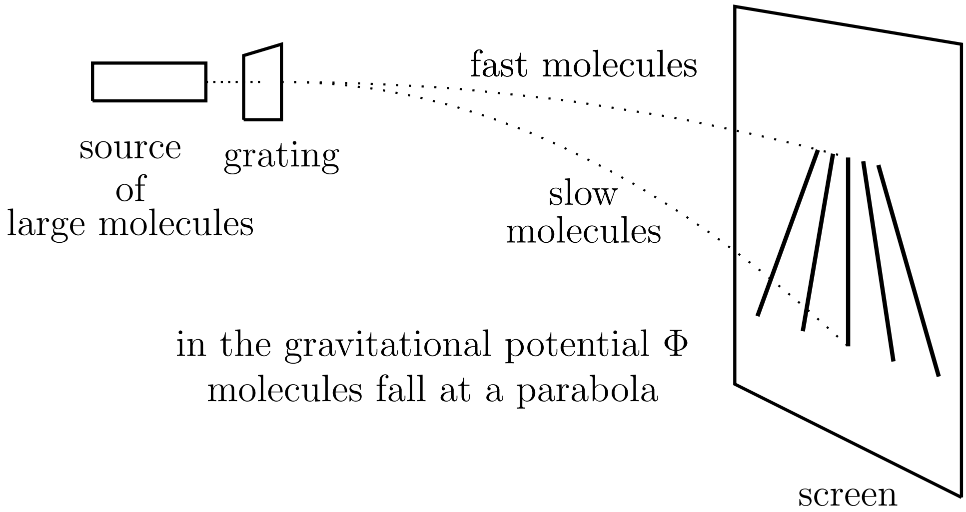


Figure 3.14: Large molecules fall in a potential Φ and are diffracted at a grating.

3.12.5 P5: Time evolution

In this section, we derive the following postulate by (Kumar, 2018, p. 170):

'The time evolution of the state vector is governed by the time-dependent Schrödinger equation:

$$i\hbar\partial_t|\psi\rangle = \hat{H}|\psi\rangle, \tag{3.278}$$

where $\hat{H}$ is the Hamilton operator corresponding to the total energy of the system.'

3.12.5.1 Derivation

Using the SQ and the far distance limit, we derived the time dependent Schrödinger equation for the relativistic and for the classical system and for the case without a potential energy term, see sections (3.8.3, 3.8.4).

Next we add an additional energy. For it, we consider an example: Brand et al. (2019) performed a diffraction experiment with large molecules with the mass $m = 514\text{u} = 8.53 \cdot 10^{-25}$ kg.

Thereby, the molecules had velocities v ranging from 150 m/s to 350 m/s. So the molecules were falling significantly in the gravitational potential Φ, see figure (3.14). Accordingly, each molecule has an additional gravitational energy at a height h as follows:

$$E_{add}(h) = \Phi(h) \cdot h \quad \text{with} \tag{3.279}$$

$$\Phi(h) = -m \cdot g \quad \text{with } g = 9.81\,\frac{\mathrm{m}}{\mathrm{s}^2} \tag{3.280}$$

In the framework of the present SQ, that energy should be added to the energy of the RGW of a molecule. In order to describe an energy, we have to choose a frame or a representation first. Here we choose the SEQ as a representation of the RGW, see equation (3.278). So the energy is added to $\hat{H}$. According to the small velocity, the kinetic energy of a molecule is non-relativistic as follows:

$$E_{kin} = \tfrac{1}{2}m \cdot v^2 = \tfrac{p^2}{2m} \quad \text{so} \tag{3.281}$$

$$\hat{E}_{kin} = \tfrac{\hat{p}^2}{2m} = -\tfrac{\hbar^2}{2m}\Delta \quad \text{with the Laplace operator} \Delta \tag{3.282}$$

The operator $\hat{H}$ of the energy of a molecule is the sum of the kinetic energy and the gravitational energy:

$$\hat{H} = -\tfrac{\hbar^2}{2m}\Delta + \Phi(h) \cdot h \tag{3.283}$$

The examples illustrate how an additional energy can be added to a RGW.

According to the principle of conservation of energy, the energy of a RGW can be supplemented by an additional energy. So the SQ provides the SEQ with the respective kinetic energy, as derived in section (3.8), and with a possible additional energy, q. e. d.

Theorem 10 Derivation of postulates of QP

The postulates of quantum physics, QP, can be derived on the basis of the spacetime-quadruple, SQ.

As the SQ represents gravity combined with relativity, QP is a consequence thereof.

Thus QP is inherent to gravity combined with relativity.

Hence the postulates of QP are **derived rules of QP** *now.*

3.13 Quantization based on GR

In this section we show how quantum systems can be derived on the basis of GR.

For it, we start with the Einstein field equation, EFE, see e. g. Einstein (1915), Stephani (1980), Carmesin (1996), Hobson et al. (2006):

$$G_{ij} = -\frac{8\pi G}{c^4} \cdot T_{ij} \quad \text{with the Einstein} - \text{tensor} \tag{3.284}$$

$$G_{ij} = R_{ij} - \frac{1}{2} G_{ij} R \quad \text{with the Ricci} - \text{tensor} \tag{3.285}$$

$$R_{ij} = R^k_{ijk} \quad \text{and the Ricci} - \text{scalar} \tag{3.286}$$

$$R = g^{ij} R_{ij} \tag{3.287}$$

Hereby, the curvature tensor acts upon a vector v_a like a commutator $[\nabla_c, \nabla_b] = \nabla_c \nabla_b - \nabla_b \nabla_c$ of covariant derivatives, see e. g. (Hobson et al., 2006, Eq. 7.12, section 3.12):

$$R^d_{abc} \cdot v_d = [\nabla_c, \nabla_b] \cdot v_a \quad \text{with covariant derivatives} \tag{3.288}$$

$$\partial_b \vec{v} = (\nabla_b v^a) \vec{e}_a \tag{3.289}$$

Using the EFE, quantum systems can be derived as follows.

3.13.1 Identification of an object

In this section, we summarize conditions for the application of the PWT, see section (3.10): It is necessary to describe an object representing a mass or dynamical mass M.

In the case of a **localized object**, see section (2.1), the surroundings of that object should be described by a curvature or a gravitational field. Examples are the SM, the Kerr metric or the Newman-Kerr metric, see Schwarzschild (1916), Kerr (1963), Newman and Janis (1965).

In the case of a **homogeneous density of objects**, see section (2.1), the surroundings of these objects can be characterized in terms of a diagonal change tensor $\hat{\varepsilon}_{ij,diagonal}$, see proposition (5).

In general, the surroundings of an object can be characterized in terms of a change tensor, $\hat{\varepsilon}_{ij}$, and in terms of the direction of propagation of the corresponding rate gravity wave.

Hereby, the change tensor describes the vacuum in the terms of one microscopic location only, whereas the curvature used in the EFE requires at least three locations, see figure (2.5). In this sense, the EFE is mesoscopic. Moreover, the curvature of the EFE is explained by the formed vacuum. In spite of the mesoscopic nature of the EFE, the EFE may be supplemented by microscopic objects, and then the PWT can be applied to such microscopic objects. In this manner, the PWT can be applied to mesoscopic theories, if microscopic objects can be defined in addition to the mesoscopic theory.

3.13.2 Application of the PWT

If an object can be defined for the case of the EFE, see section (3.13.1), then the PWT can be applied. As a result, the object is described by the SEQ, and the postulates of quantum physics apply, see section (3.12).

Moreover, we can apply the PWT_g, in order to derive the generalized Schrödinger equation, SEQ_g, in order to obtain a description beyond the usual QP.

Chapter 4

Formation of Vacuum

In this chapter, we analyze the formation of the vacuum that forms the present day space. In particular, we derive the density ρ_Λ of the vacuum, also called dark energy. For it, we apply the RGWs. Thereby, we could apply a second quantization of the RGWs, see (Carmesin, 2021d, chapter 6), or we could apply a quantization derived at the Planck scale, see Carmesin (2018b), Carmesin (2018a), (Carmesin, 2021d, sections 8.5, 8.6). However, we will show here that the density ρ_Λ can be derived more directly by using a **semiclassical description of the RGWs**. In the following, we denote the RGWs of the vacuum by RGW_Λ.

A quantum description of the formation of the vacuum has been elaborated earlier, see for instance Carmesin (2018b) or Carmesin (2018a), Carmesin (2019b), Carmesin (2019a) or also Carmesin (2020b), Carmesin (2021a), Carmesin (2021c), for relation to geometry see Carmesin (2021b).

4.1 Basics of the derivation

4.1.1 Vacuum only

In this book, we analyze the vacuum in an especially pure and ideal case: we derive the density ρ_Λ of the dark energy in a universe that consists of dark energy only, without any content such as matter or radiation.

We showed elsewhere, this ideal case can be generalized directly to a realistic universe that is filled with vacuum, radiation and matter, see e. g. Carmesin (2018b), Carmesin (2018a), Carmesin (2021a). Hereby, we achieve precise accordance with observation, whereby we do not apply any fit, see e. g. Carmesin (2021a), Carmesin (2021c). Thereby, the mechanism of the formation of vacuum presented here is used and confirmed by observation, Planck-Collaboration (2020).

4.1.2 Homogeneous and constant vacuum

As there is no radiation or matter in the vacuum modeled here, the system is homogeneous. In particular, there is no increase of structure in the present model of the vacuum. Accordingly, we derive the constant and homogeneous density $\rho_{\Lambda,c.,h.}$ of the vacuum.

4.1.3 Separation of space and time

In the constant and homogeneous density $\rho_{\Lambda,c.,h.}$ of the vacuum modeled here, the time increases at a homogeneous and constant rate. Accordingly, we can investigate space and time separately[1].

4.2 Dark energy in a homogeneous universe

In this section, we derive the **constant and homogeneous density** $\rho_{\Lambda,c.,h.}$ of the dark energy.

4.2.1 Source by present vacuum

The RGW_Λ constitute the vacuum and space. **These** RGW_Λ **present sources** that form additional vacuum. Such a source

[1] Note that the use of a single parameter of time t is especially realistic in the present case of a universal length scale such as the Hubble radius R_H. Remind that the corresponding dynamics of the scale factor is described by a single DEQ with a single time parameter t, see Eq. (3.48) and Balbi (2013).

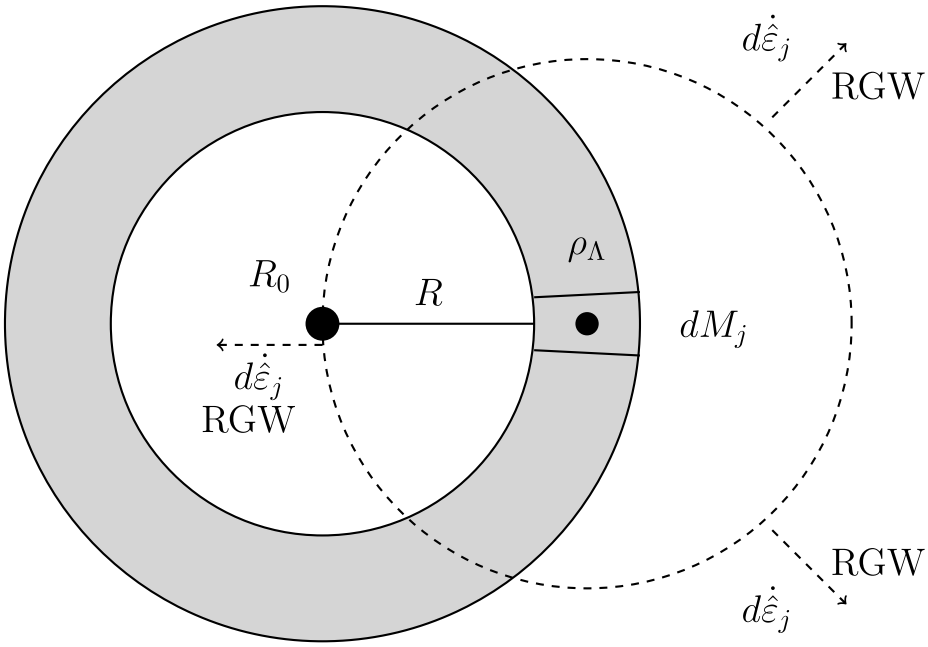


Figure 4.1: The density ρ_Λ in an area at a distance R from R_0 has a $j-th$ dynamic mass dM_j. It generates rates $d\dot{\hat{\varepsilon}}_j$ propagating to all directions in an isotropic manner.

can be described by a $j-th$ dynamical mass dM_j, see figure (4.1). Accordingly, the rate of unidirectional formed vacuum is as follows, see Eq. (3.129):

$$d\dot{\hat{\varepsilon}}_j = dG^*(R)/c = \frac{G \cdot dM_j}{R^2 \cdot c} \tag{4.1}$$

4.2.2 RGWs propagating towards R_0

A $j-th$ mass causes RGWs that propagate to all directions. In this section, we integrate that part of the RGWs that propagate to an observer at a location R_0, see figure (4.1).

4.2.2.1 Rates $d\dot{\varepsilon}_j(R)$ of formed volume

In order to derive the density $\rho_{\Lambda,c.,h.}$ at an observer at a place R_0, we integrate all RGW_Λ that propagate to that observer, see figure (4.1). The dynamic mass dM_j of RGW_Λ in figure (4.1) forms vacuum according to Eq. (4.1):

$$d\dot{\hat{\varepsilon}}_j(R) = \frac{1}{c} \cdot \frac{G \cdot dM_j}{R^2} \tag{4.2}$$

Each mass dM_j in that shell generates a rate $d\dot{\hat{\varepsilon}}_j$ that is proportional to that mass, and these rates are scalars. Hence the sum of the rates $d\dot{\varepsilon}(R) = \Sigma_{j,\, R_j \in \text{shell}}\, d\dot{\hat{\varepsilon}}_j$ is equal to the rate of the sum of the masses $dM(R) = \Sigma_{j,\, R_j \in \text{shell}}\, dM_j$. So we get:

$$d\dot{\varepsilon}(R) = \frac{1}{c} \cdot \frac{G \cdot dM(R)}{R^2} \tag{4.3}$$

That mass $dM(R)$ is equal to the product of the density $\rho_{\Lambda,c.,h.}$ and the volume $dV = 4\pi \cdot R^2 \cdot dR$ of the shell:

$$dM(R) = \rho_{\Lambda,c.,h.} \cdot 4\pi \cdot R^2 \cdot dR \tag{4.4}$$

We insert the mass in Eq. (4.4) into Eq. (4.3):

$$d\dot{\varepsilon}(R) = \frac{1}{c} \cdot \frac{G \cdot \rho_{\Lambda,c.,h.} \cdot 4\pi \cdot R^2 \cdot dR}{R^2} \tag{4.5}$$

We cancel R^2. So we get:

$$d\dot{\varepsilon}(R) = \frac{G \cdot \rho_{\Lambda,c.,h.} \cdot 4\pi}{c} \cdot dR \tag{4.6}$$

4.2.2.2 Invariance of additional rates $d\dot{\varepsilon}(R)$

The above Eq. (4.6) shows that each shell around R_0 with thickness dR causes the same additional rate $d\dot{\varepsilon}(R)$, irrespective of the radius R of the shell.

4.2.2.3 Integration of $d\dot{\varepsilon}(R)$

In order to integrate $d\dot{\varepsilon}(R)$, we analyze the properties of the RGW_Λ:

1. We model the constant density $\rho_{\Lambda,c.,h.}$ corresponding to the present day universe.

2. So $\rho_{\Lambda,c.,h.}$ is characterized by the Hubble constant H_0.

3. The RGW_Λ do not propagate 'inside' the vacuum, instead the RGW_Λ constitute vacuum propagating at c isotropically.

4. So the RGW_Λ do not experience a redshift or the expansion of the space. Instead they cause the expansion.

5. Thus the duration of the propagation is the Hubble time $t_H = 1/H_0$.

6. The RGW_Λ propagate at the velocity c, see section (2.6).

7. Hence the RGW_Λ propagate the distance $c \cdot t_H = R_H$, the Hubble radius.

8. So the upper limit of the integration is R_H.

9. The lower limit of integration is a length near the Planck length, that length is negligible at a very good approximation.

So we derive:

$$\int_0^{\dot{\varepsilon}} d\dot{\varepsilon}(R) = \frac{4\pi \cdot G}{c} \cdot \int_0^{R_H} \rho_{\Lambda,c.,h.} dR \tag{4.7}$$

We evaluate the integrals. So we derive the rate $\dot{\varepsilon}_{to\ R_0}$ caused at R_0 during the Hubble time t_H by the RGWs arriving at R_0:

$$\boxed{\dot{\varepsilon}_{to\ R_0} = \frac{4\pi \cdot G \cdot R_H}{c} \cdot \rho_{\Lambda,c.,h.} = 4\pi \cdot G \cdot t_H \cdot \rho_{\Lambda,c.,h.}} \tag{4.8}$$

4.2.3 Derivation of the formed vacuum with dV

In this section, we analyze how a present day volume dV at the location R_0 in figure (4.1) was physically formed by vacuum arriving at R_0 during the time t_H according to the rate of the arriving vacuum $\dot{\varepsilon}_{to\ R_0}$.

For it, we express the rate $\dot{\varepsilon}_{to\ R_0}$ in Eq. (4.8) in terms of its definition:

$$\dot{\varepsilon}_{to\ R_0} = \frac{\underline{\delta} V}{\delta t \cdot dV} \tag{4.9}$$

As the rate is constant, see Eq. (4.8), we can derive the volume $\underline{\delta} V$ of the vacuum arriving at R_0 during a time δt. For it, we solve for $\underline{\delta} V$:

$$\underline{\delta} V(\delta t) = \dot{\varepsilon}_{to\ R_0} \cdot \delta t \cdot dV \tag{4.10}$$

In particular, during the time $\delta t = t_H$, the following volume arrived at R_0:

$$\underline{\delta} V(t_H) = \dot{\varepsilon}_{to\ R_0} \cdot t_H \cdot dV \tag{4.11}$$

Physically, the volume $\underline{\delta} V(t_H)$ of the vacuum that arrived at R_0 per volume dV at R_0 is exactly the volume dV:

$$\underline{\delta} V(t_H) = dV \tag{4.12}$$

We insert the amount of arrived vacuum $\underline{\delta} V(t_H)$ in Eq. (4.11) in the above equality (4.12):

$$\dot{\varepsilon}_{to\ R_0} \cdot t_H \cdot dV = dV \tag{4.13}$$

We insert the rate $\dot{\varepsilon}_{to\ R_0}$, see Eq. (4.8), and we divide by dV:

$$4\pi \cdot G \cdot t_H \cdot \rho_{\Lambda,c.,h.} \cdot t_H = 1 \tag{4.14}$$

We solve for $\rho_{\Lambda,c.,h.}$:

$$\boxed{\rho_{\Lambda,c.,h.} = \frac{1}{4\pi \cdot G \cdot t_H^2}} \tag{4.15}$$

4.2.4 Density parameter Ω_Λ

We derive the density parameter Ω_Λ, for the case of the approximation of constant $\rho_{\Lambda,c.,h.}$. Hereby, Ω_Λ is defined as the ratio of $\rho_{\Lambda,c.,h.}$ and the critical density ρ_{cr,t_0}:

$$\Omega_{\Lambda,c.,h.} = \frac{\rho_{\Lambda,c.,h.}}{\rho_{cr,t_0}} \tag{4.16}$$

Thereby, the critical density ρ_{cr,t_0} is defined as the present day density, at which the curvature parameter k in the FLE is zero, whereby $k = 0$ is the realistic value, see Planck-Collaboration (2020), (Carmesin, 2021d, theorem 32(6)) and Eq. (3.20):

$$H_0^2 = \frac{8\pi G \cdot \rho_{cr,t_0}}{3} - k \cdot \frac{c^2}{r^2} \qquad \text{with } \; k = 0, \; \text{ so} \tag{4.17}$$

$$\rho_{cr,t_0} = \frac{3H_0^2}{8\pi G} \tag{4.18}$$

For it we use the Hubble constant $H_0 = \frac{c}{R_H} = \frac{1}{t_H}$. So we get:

$$\Omega_{\Lambda,c.,h.} = \frac{\rho_{\Lambda,c.,h.}}{\rho_{cr,t_0}} = \frac{1}{4\pi \cdot G \cdot t_H^2} \cdot \frac{8\pi G}{3H_0^2} = \frac{2}{3} \tag{4.19}$$

4.2.5 Comparison with observation

In this section, we compare with an observation $\Omega_{\Lambda,obs}$. Hereby, each observation of $\Omega_{\Lambda,obs}$ uses a physical object that was emitted at some time t_{em}. As a matter of fact, the observation depends slightly on that time t_{em}, see e. g. Carmesin (2018a), Carmesin (2021a), Carmesin (2021c). Accordingly, we choose a time t_{em} that represents a relatively homogeneous universe. Such a time corresponds to the early universe[2].

Accordingly, we compare with an observation at high redshift z. Correspondingly, we compare with an observation based on the CMB. In particular, the observations of the CMB by the Planck satellite provide temperature power spectra and a corresponding value $\Omega_{\Lambda,obs}$ of the density parameter as follows, see (Planck-Collaboration, 2020, table 2)

$$\Omega_{\Lambda,obs} = 0.679 \pm 0.013 \tag{4.20}$$

So the theoretical value is within the error of measurement, so it is in precise accordance with observation.

[2]Note that a homogeneous density of radiation does hardly affect the observation, see e. g. (Carmesin, 2021d, section 7.5), Carmesin (2021a), Carmesin (2021c)

Theorem 11 Formation and density of the vacuum

(1) The formation of the present day vacuum is explained by the formation of vacuum since the Big Bang until today according to the rate $\dot{\hat{\varepsilon}}$.

(2) In the ideal case of a universe filled of vacuum only, the formation of the vacuum in (1) provides the density $\rho_{\Lambda,c.,h.} = \frac{1}{4\pi \cdot G \cdot t_H^2}$ *and the density parameter* $\Omega_{\Lambda,c.,h.} = \frac{2}{3}$, *whereby no fit is applied. This result is in precise accordance with observation.*

(3) In a realistic universe filled with vacuum, radiation and matter, the formation of vacuum is explained by the same process as in (1), whereby the heterogeneity of matter and radiation causes a slight modification. The resulting density ρ_Λ *is in precise accordance with observation, whereby no fit is applied, see (Carmesin, 2021d, section 7.5), Carmesin (2021a), Carmesin (2021c).*

Chapter 5

Explanation of Quantum Physics

Weinberg (2017) wrote about quantum mechanics or quantum physics: *'Today, despite of the great successes of quantum mechanics, arguments continue about its meaning, and its future.'*

Indeed, the meaning of quantum physics should be clarified. And in fact, our derivation of the postulates of quantum physics on the basis of the SQ presents a rich source for the derivation of explanations of quantum physics. In this chapter, we elaborate such explanations of QP.

5.1 Direct understanding based motions

So far, QP has been derived and understood by the indirect method of an experimentally based guess with a subsequent elaboration, see figure (3.13) or Sakurai and Napolitano (1994), Ballentine (1998), Kumar (2018).

In contrast, we start with a direct understanding of motions here, see chapter (2) or Brahe and Kepler (1627), Kepler (1619), Galileo (1638), Newton (1686), Einstein (1905), de Sitter (1913), Einstein (1915). For it, we start with the SQ. It is based on motions, see chapter (2). In this manner, the spacetime-quadruple, SQ, can be directly understood.

$$\text{motions} \rightarrow \text{SQ} \tag{5.1}$$

Based on the SQ, the rate of formation of the vacuum $\dot{\varepsilon}$ can be derived and understood directly, see chapter (3):

$$\text{SQ} \rightarrow \text{dynamics of } \dot{\varepsilon} \tag{5.2}$$

Based on the rate of formation of the vacuum $\dot{\varepsilon}$, the quantization, the wave function, the Schrödinger equation and the postulates of quantum physics, QP, can be derived and understood directly, see chapter (3):

$$\text{dynamics of } \dot{\varepsilon} \rightarrow \text{quantization} \qquad \text{and} \tag{5.3}$$

$$\text{dynamics of } \dot{\varepsilon} \rightarrow \text{postulates of QP} \tag{5.4}$$

In this manner, the spacetime-quadruple, SQ, implies the quantization in nature and the postulates of QP. As the SQ is based on motions, the nature of quantization can be understood directly.

5.2 Clarifications of QP

Feynman (1967) wrote *'I think I can safely say that no one understands quantum mechanics'*. Accordingly, a clarification is necessary. In this section, we apply our derivations in chapters (3, 4), in order to clarify the postulates and traditional concepts of QP.

5.2.1 Quantization

The ubiquitous formation of vacuum according to a rate $\dot{\varepsilon}$ gives rise to rate gravity waves, RGWs, described by a linear differential equation, see theorem (4). Wave packets of such waves form quanta with a universal constant of quantization, see theorem (6). Thus the fact of quantization is derived from the SQ as follows:

$$\text{motions} \rightarrow \text{SQ} \rightarrow \text{dynamics of } \dot{\varepsilon} \rightarrow \text{quantization} \tag{5.5}$$

5.2.2 Quantization of a general object

A general object with a mass or dynamical mass M modifies its surroundings. At a mesoscopic level, that modification is described by curvature, on a microscopic level, that modification is described by the formation of vacuum according to a rate $\dot{\varepsilon}$. Additionally, many observations measure the surroundings of an object instead of the internal structure of an object. A typical example is the observation of phenomena at a double slit experiment, see figures (3.10, 3.12, 3.14). Accordingly, such behavior of a general object is described by the formation of vacuum according to a rate $\dot{\varepsilon}$.

Moreover, essential elements of the internal structure of a general object are also described by the formation of vacuum according to a rate $\dot{\varepsilon}$. Examples are the formation of mass, see Carmesin (2021a), and the formation of the elementary charge and electromagnetism, see Carmesin (2021e).

Altogether, a general object is described by the formation of vacuum according to a rate $\dot{\varepsilon}$ at larger length scales and at small length scales. So the quantization of general objects is explained by the space-time quadruple:

$$\text{general object} \rightarrow \text{dynamics of } \dot{\varepsilon}(\text{SQ}) \rightarrow \text{quantization} \tag{5.6}$$

5.2.3 Wave function

A quantum state of an object is described by a wave function, see e. g. Kumar (2018). According to Weinberg (2017) (section 1), a wave function is essentially a list of numbers. We clarify that a wave function of a general object is the normalized rate $\dot{\varepsilon} \cdot t_n$ of vacuum formed by that object. Thereby all possible linear combinations are included, as the differential equation of the rate $\dot{\varepsilon} \cdot t_n$ is linear[1], see theorems (4, 5).

5.2.4 Time evolution of the wave function

In QP, the time evolution of the wave function is described by the Schrödinger equation, SEQ, in a deterministic manner, see e. g. Schrödinger (1926a), Kumar (2018), Weinberg (2017)[2]. However, the traditional theory of quantum physics does not explain, **why** the wave function is described by the Schrödinger equation, see e. g. Schrödinger (1926a), Sakurai and Napolitano (1994), Ballentine (1998), Kumar (2018), Weinberg (2017).

We clarify that the Schrödinger equation is the differential equation of the formation of vacuum according to the rate $\dot{\varepsilon} \cdot t_n$, which indeed describes the time evolution of the wave function $\dot{\varepsilon} \cdot t_n$, see theorem (10) or section (3.12):

$$\text{general object} \rightarrow \text{dynamics of } \dot{\varepsilon}(\text{SQ}) \rightarrow \text{SEQ} \qquad (5.7)$$

In particular, based on the spacetime-quadruple, it is clear that the time derivative in the SEQ is based on Galileo's equivalence principle, which is the ultimate basis of all dynamical equations in this book. So the dynamics can be rooted back to free fall and to the Galileo's gedankenexperiments or experiments at the tower at Pisa, see figure (2.2).

5.2.5 Probabilistic nature of quantum physics

The dynamics of quantum physics is described by two elements: the time evolution of the deterministic Schrödinger equation and the probabilistic behavior, as it is observed in measured quantities, for instance. That probabilistic behavior should be explained: E. g. Weinberg (2017) described two approaches, an

[1]Hereby, we investigate some inner degrees of freedom or quantum numbers for some essential quantities, see Carmesin (2020c), Carmesin (2020c).

[2]Of course, a representation may be changed to the Heisenberg picture, for instance, however, such a transformation cannot explain the source of the time evolution.

instrumentalist's approach and a realist's approach, he made clear that he is not convinced of these approaches and concluded *'O time, thou must untangle this, not I'*.

5.2.5.1 Particle wave transformation

We explain the combined dynamics of a general object by the following fact: Based on the spacetime-quadruple, SQ, we can transform a particle description, PD, to a generalized wave description, WD_g, and vice versa, see theorem (8):

$$\text{SQ} \rightarrow (PD \leftrightarrow WD_g) \tag{5.8}$$

In this manner, the SQ clarifies the two types of dynamics in QP.

Thereby, the traditional quantum mechanics applies an additional far distance limit upon the WD_g and thus arrives at the usual or traditional wave description WD. As a consequence, the transformation cannot be inverted:

$$\text{SQ} \rightarrow (PD \rightarrow \lim_{R_S/R \rightarrow 0} WD_g = WD) \tag{5.9}$$

Moreover, the particle wave transformation, PWT, in Eq. (5.8) represents the particle wave duality observed in nature and quantum physics, see e. g. (Kumar, 2018, p. 33). So the PWT clarifies the particle wave duality.

5.2.5.2 Clarification of results of measurements

In this section, we elaborate the basis of the fact that the only possible results of a measurement of a quantity A are the eigenvalues of the corresponding operator $\hat{A}$.

As the DEQ of the rate $\dot{\varepsilon} \cdot t_n = \psi \cdot f_n$ is linear, the SEQ, the wave functions form a linear vector space of functions. Since the absolute square of these functions is proportional to the energy density and to the probability of a location, the above space of functions is a Hilbert space $\mathcal{H}$.

In a Hilbert space $\mathcal{H}$, possible values that an operator generates are the eigenvalues, corresponding to the respective eigenfunctions. So the linear DEQ of the rates $\dot{\varepsilon} \cdot t_n$, or of the wave functions, is the basis of the Hilbert space in QP, which is the basis of the eigenvalues of operators $\hat{A}$ as possible results of measurements:

$$\text{linear SEQ} \rightarrow \psi \cdot f_n \in \mathcal{H} \rightarrow \text{measurement}(A) = \text{eigenvalue}(\hat{A}) \tag{5.10}$$

5.2.5.3 Clarification of uncertainty principles

In this section, we elaborate the basis of the Heisenberg uncertainty principle.

The linear DEQ of the wave functions, the SEQ, implies the Hilbert space $\mathcal{H}$ of wave functions. An analysis of standard deviations $\Delta_A = \langle(\hat{A} - \langle\hat{A}\rangle)^2\rangle$ and $\Delta_B = \langle(\hat{B} - \langle\hat{B}\rangle)^2\rangle$ as well as commutators $[\hat{A}, \hat{B}]$ provides the corresponding uncertainty relations $\Delta_A^2 \cdot \Delta_B^2 \geq \frac{1}{2} \cdot |\langle[\hat{A}, \hat{B}]\rangle|$, see (Ballentine, 1998, section 8.4). Altogether, we clarify that the Heisenberg uncertainty principle is a consequence of the linear DEQ of the wave functions, the SEQ, and the probabilistic dynamics described by expectation values:

$$\text{SQ} \rightarrow \text{deterministic and probabilistic dynamics of QP} \tag{5.11}$$

$$\rightarrow \text{uncertainty relations} \tag{5.12}$$

5.2.5.4 Universal behavior of QP

Moreover, the far distance limit clarifies, why electrons, atoms and molecules exhibit the same diffraction patterns at a double slit experiment, see figures (3.10, 3.14). More generally, the far distance limit clarifies the typical universal behavior of different objects in quantum physics.

Even more generally, it is clear that a limit generates a universal behavior, since many functions have the same limit. Another very important physical example is the thermodynamic limit. In that limit, the particle number N of a set or ensemble tends to infinity. In that limit, the theory of thermodynamics can be explained by the more general theory of statistical physics, see e. g. Landau and Lifschitz (1980). For instance, an ideal gas can be described by the thermodynamic limit, and in that limit, the corresponding law holds for all gases, irrespective of the contained atoms or molecules, Landau and Lifschitz (1980).

Even the real gas can be described in the thermodynamic limit, see van der Waals (1873). Hereby, droplets or clusters are not analyzed in the thermodynamic limit.

5.2.5.5 Clarification of constant of quantization h

In this section, we clarify the constant of quantization, the Planck constant. Of course, it is always possible to change a system of units.

However, we analyze the units in a manner that is independent from a possible change of the system of units:

We call a constant fundamental, if that constant constitutes a physical structure that can not yet be explained by an underlying more microscopic structure. Hereby, we prefer microscopic structures rather than macroscopic structures, since macroscopic structures are usually formed from microscopic structures, but not vice versa.

Firstly, we derived that there is only one constant of quantization, see section (3.8).

Secondly, the value of that constant in SI-units must be measured, it is the Planck constant, see table (7.1).

Thirdly, the constant is based on a particular analysis of the SQ: the particle wave transformation, including the far distance limit. So the constant h describes a structure that is inherent to the SQ. Thus the constant h is not fundamental, as it does not describe fundamentally new physics in addition to the SQ.

Fourthly, the constant h is universal, as it describes an ubiquitous property of the SQ: the quantization.

Fifthly, the SQ has the following fundamental constants: the gravitational constant G and the velocity of light c. These constants are fundamental, as they characterize the physics of the SQ at a microscopic level.

Similarly, the Boltzmann constant k_B is not fundamental, as it describes the energy that corresponds to an entropy, which is a statistical quantity of a physical system. In the corresponding statistical analysis, an existing physics is analyzed, but no fundamentally new physics is added.

For comparison, the elementary charge e is a dependent constant of nature, as it can be derive from the SQ completely. A measurement is only necessary as a test, see Carmesin (2021e).

5.2.5.6 Generalized Schrödinger equation, SEQ_g

We clarify that the generalized wave description represents a generalized Schrödinger equation, SEQ_g, see section (3.10). So the SEQ_g corresponds to the case without the far distance limit and without the universal behavior of traditional QP:

$$\text{SQ} \rightarrow (PD \leftrightarrow WD_g \leftrightarrow SEQ_g) \tag{5.13}$$

5.2.5.7 Spacetime-quadruple, SQ, provides probabilities

The SQ provides the correct probabilities of quantum physics, see theorem (10) or section (3.12).

$$\text{SQ} \rightarrow \text{probabilities} \tag{5.14}$$

Hereby, we clarify that the probabilities of QP are proportional to the energy densities of the formed vacuum. Thereby the proportionality factor is obtained by the fact that the sum or integral of all probabilities is one. Altogether, the SQ provides the deterministic dynamics represented by the Schrödinger equation, as well as the probabilistic dynamics, represented by the probabilities based on the energy density of the formed vacuum:

$$\text{SQ} \rightarrow \text{deterministic and probobilistic dynamics of QP} \quad (5.15)$$

5.2.5.8 General object provides probability

One might wonder how even slight amounts of the formed vacuum of a general object can describe the quantization, the propagation of the wave function according to the Schrödinger equation, the full Hilbert space $\mathcal{H}$ as well as the correct probabilities. The answer is very simple: The amplitude of the wave function is provided by the normalization of the probability to one, so the amount of the formed vacuum is not essential. Accordingly, the fact that the Schrödinger equation is linear gives rise to the full Hilbert space $\mathcal{H}$, irrespective of the amount of the formed vacuum. Note that the amount has been derived in (Carmesin, 2021d, Eq. 2.18).

5.2.5.9 Tunneling

The SQ implies the postulates of quantum physics. So it implies tunneling, see e. g. (Kumar, 2018, section 3.8). Thus it implies the propagation of evanescent modes, Nimtz (2003), Hoffmann et al. (2021). An evanescent mode occurs according to the probabilities inherent to quantum physics, and such a mode may exhibit a corresponding velocity larger than c, Nimtz (2003). Such evanescent modes exhibit a nonlocal effect of quantum physics, thus it is a consequence of the SQ, of course. We emphasize that there is no contradiction to the used velocities $v \leq c$ of classical objects in the SQ, since evanescent modes are implications of the SQ.

5.2.5.10 Positive inertia at negative energy

In quantum physics, the positron has been described by a negative total energy, see e. g. Dirac (1928). Accordingly, matter could hardly be stable, as the ground state would tend to an energy at minus infinity, see e. g. (Kumar, 2018, p. 408). Here, the mass m_0 corresponds to the inertial energy $E = m_0 \cdot c^2$, which is positive for electrons and for positrons, see section (3.5.3).

5.2.5.11 Angular momentum

The quantization of the angular momentum provides an example for an especially counterintuitive property of quantum physics. The angular momentum is an observable J, so it corresponds to an operator $\hat{J}$, according to the SQ. Thus, $\hat{J}$ has a spectrum. The algebraic analysis shows that the eigenvalues are $J = 0$, $J = \hbar/2$, $J = \hbar$, $J = 3\hbar/2$, and so forth, see e. g. (Ballentine, 1998, Eq. 7.16).

For any rigid body, the rotation by an angle 2π around the z-axis reproduces the original state or orientation. However, if we apply that rotation by an angle 2π around the z-axis to a state $|\alpha\rangle$ with $J = \hbar/2$, the resulting state is $-|\alpha\rangle$, see e. g. (Sakurai and Napolitano, 1994, Eq. 3.2.15).

This example shows that counter-intuitive phenomena can occur as a result of the algebraic structure of the Hilbert space, which is caused by the spacetime-quadruple, SQ.

5.2.6 Postulates of QP

The SQ provides RGWs as solutions. Using these RGWs, the postulates of QP have been derived.

In this manner, the nature of QP is clarified: QP is a consequence of the SQ. Moreover, the wave function and its deterministic and probabilistic dynamics is explained by the dynamics of the vacuum.

$$\begin{aligned}
\text{SQ} &\rightarrow \text{dynamics of vacuum} && (5.16)\\
&\rightarrow \text{deterministic and probabilistic dynamics of QP} && (5.17)\\
&\rightarrow \text{postulates of QP} && (5.18)
\end{aligned}$$

5.2.7 Dynamics of vacuum

In this section, we elaborate the relation of the dynamics of the vacuum to other fields of physics.

As shown above, the dynamics of the vacuum is based on the SQ. Moreover, the dynamics of the vacuum implies QP.

5.2.7.1 Relation to GR

The curvature described in GR is explained by the dynamics of the vacuum, see sections (3.9, 3.10) or Carmesin (2021d), Carmesin (2021a), Carmesin (2021e):

$$\text{dynamics of vacuum} \rightarrow \text{curvature in}\, GR \tag{5.19}$$

5.2.7.2 Relation to quantum gravity

As the dynamics of the vacuum implies QP as well as the curvature in GR, the dynamics of the vacuum also implies quantum gravity:

$$\text{dynamics of vacuum} \rightarrow QP \qquad \text{so} \tag{5.20}$$

$$\text{dynamics of vacuum} \rightarrow \text{quantum gravity} \tag{5.21}$$

In particular, the Planck length and the Planck scale emerge, see section (7.5). So the singularity problem of GR is solved, see e. g. Kiefer (2003), Carmesin (2017), Carmesin (2018a), Carmesin (2019b).

5.2.7.3 Formation and density of vacuum

As shown in chapter (4), the dynamics of the vacuum alias dark energy explains the process of formation of the present day vacuum by the formation of vacuum since the Big Bang. Moreover, that dynamics of the vacuum explains the density of the vacuum $\rho_{\Lambda,c.,h.}$, in precise accordance with observation, derived without use of a fit:

$$\text{dynamics of vacuum} \rightarrow \text{formation of vacuum} \,\&\, \rho_{\Lambda,c.,h.} \tag{5.22}$$

Consequently, also the Planck length L_P and the Planck scale are inherent to the spacetime - quadruple and to the dynamics of the vacuum, see section (7.5). As the Planck length presents the smallest length L_P that can be observed by a single observation, the spacetime - quadruple implies already that smallest observable length L_P, though the concept of the spacetime - quadruple is completely continuous at its definition or introduction. Moreover, that smallest observable length makes clear that the mesoscopic structure of at least three L_P of GR is inherent to the combination of gravity and relativity.

5.2.7.4 Formation of space by vacuum

In this section, clarify how vacuum forms space, even though the dynamics of the vacuum is described with help of space.

For it, we make clear the categories: Space is a mathematical concept. Vacuum is a physical entity that can be observed in nature. For instance, the density ρ_Λ of the vacuum has been measured, see for instance Perlmutter et al. (1998), Riess et al. (2000), Spergel et al. (2007), Planck-Collaboration (2020), Riess et al. (2021), Blakeslee et al. (2021).

The answer is that the mathematical description of space does not at all restrict the formation of vacuum in nature. In particular, it is not necessary that you first solve a problem in the field of GR, before you are allowed to use a mathematical tool describing space.

Similarly, we can describe the propagation of RGWs with the help of mathematical tools that describe space, even before we present a solution for the formation of the present-day vacuum. This is possible, even though the present-day vacuum constitutes what we call space in everyday life[3]

[3]Moreover, the space we experience in everyday life appears static, whereas the vacuum is dynamic. This fact underlines the difference between the dynamic vacuum and the static space experienced in everyday life that is conceptualized by static mathematical models such maps or globes.

5.2.7.5 Graviton

Accordingly, the SQ also provides the graviton, the hypothetic particle of the gravitational interaction, see Blokhintsev and Galperin (1934). In fact, the essential properties of the graviton have indeed been explained by the SQ, see Carmesin (2021d).

5.2.8 Nonlocality

In this section, we elaborate the role of nonlocality in nature, GR, QP and the SQ.

5.2.8.1 A property of nature

So far, nonlocality was a mystery of nature. Einstein et al. (1935) and possibly Weinberg (2017) even presumed that GR would not be nonlocal.

However, we see that QP is inherent to the spacetime - quadruple. Moreover, QP is inherent to an object described by GR. Furthermore, QP is nonlocal. So the SQ and GR are not local.

Such presumptions do sometimes happen in science. For instance, more than 2000 years ago, mathematicians presumed that all numbers are rational. Later it turned out that the numbers $\sqrt{2}$ and π are not rational. Mathematicians made a difference between rational and irrational or real numbers, even though limits of rational numbers provide real numbers quite naturally.

Analogously, nonlocal phenomena can be derived quite naturally within GR. However, it is also possible to define GR in such a manner that all nonlocal phenomena are artificially excluded from GR. That would redefine the set of phenomena belonging to GR.

Of course, the SQ is more general than GR, as it has less presumptions such as continuity. As a consequence, the SQ can also describe discontinuities and phase transitions in space that can hardly be described by GR, see e. g. Carmesin (2021d). Additionally, the SQ can explain the curvature of GR in terms of the formed vacuum.

5.2.8.2 An explanation

Using the representation of wave functions by RGWs derived here, and using the formation of mass and charge from RGWs, see Carmesin (2021a), Carmesin (2021e), nonlocality can be explained as follows:

We consider a mass or dynamical mass m or a property of it that is observed at a location $\vec{r}_{obs}$ in a nonlocal manner[4].

The observed m is formed at $\vec{r}_{obs}$ from vacuum (see Carmesin (2021a), Carmesin (2021e)), which is already at $\vec{r}_{obs}$, in particular, the vacuum forming m at $\vec{r}_{obs}$ does not need any transport or propagation through the three dimensional space from $\vec{r}_{em}$ to $\vec{r}_{obs}$.

While this explanation makes clear how a transport of energy or mass can be minimized in principle, it does not provide a model of the observed instantaneous transport of correlations. A more advanced model including phase transitions provides possible answers to the question of the transport of correlations, see Carmesin (2021d).

5.2.8.3 A solution of the EPR paradox

Einstein et al. (1935) presumed that GR would not be nonlocal, while they pointed out that QP is nonlocal. The corresponding difference is the essence of the EPR paradox.

Here we derived QP by considering an object described by GR. So GR is not free of QP, thus nonlocality is inherent to GR. Hence the presumption is wrong. Thence there is no paradox[5].

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[4]That is, the mass m or a corresponding wave or quantum was emitted at a location $\vec{r}_{em}$ at a time t_{em}, whereby a light signal emitted at $(t_{em}, \vec{r}_{em})$ arrives at $\vec{r}_{obs}$ after the arrival of m.

[5]As noted in section (5.2.8.1), GR could be made free of nonlocality and of QP, if one

5.2.9 Summary of clarifications

Feynman (1967) wrote *'I think I can safely say that no one understands quantum mechanics'*. I think I can safely say that you can explain the essential features and postulates of quantum mechanics on the basis of gravity and relativity, after reading this book.

5.3 Extensions

The theory of QP can be extended directly into many established fields. So these fields can now be explained on the basis of motions in space and time as well. We consider some examples next.

5.3.1 Many particles

If the QP of a j-th particle is described by a corresponding Hilbert space $\mathcal{H}_j$, then N particles are described by the product space:

$$\mathcal{H} = \Pi_j^N \mathcal{H}_j \tag{5.23}$$

5.3.2 Internal states and transitions

In general, particles have internal states. Examples are orbitals in atoms, see Einstein (1905), Bohr (1913), Schrödinger (1926a), Schrödinger (1926b), or the isospin states of a neutron, see e. g. Fermi (1933), Weinberg (1967), Weinberg (1996), Tanabashi et al. (2018). Such states can be modeled in the framework of the SEQ.

5.3.3 Reactions among elementary particles

The reactions from one elementary particle to another is usually modeled via transitions. For instance, an up quark changes to a

would artificially restrict a future version of GR to local phenomena.

downquark by changing its isospin, see e. g. Weinberg (1967), Weinberg (1996), Tanabashi et al. (2018).

As transitions can be modeled in terms of QP, also the transformations and reactions of elementary particles can be modeled by QP. So they are based on the SQ as well, and they can be described by the PWT.

Moreover, the spacetime quadruple is naturally generalized to dimensions $D \geq 3$. Thereby, there occur phase transitions in a natural manner. With help of these phase transitions, the formation of mass and of the elementary charge have been modeled, in precise accordance with observation, whereby no fit has been applied, see Carmesin (2021a), Carmesin (2021e).

5.3.4 Transformations within quantum physics

Transformations within QP have always been helpful in the QP, see e. g. Jordan (1935), Holstein and Primakoff (1940), Bogoliubov (1958). In this context, the present PWT is especially interesting, as it provides a transformation from the spacetime-quadruple to QP.

Chapter 6

Discussion

Problem: Ballentine (1998) wrote: 'Einsteins's *locality* postulate, which is the key to Bell's theorem, is strongly motivated by special relativity. Thus the conflict between quantum mechanics and locality suggests a deep incompatibility between quantum mechanics and relativity.'

Accordingly, many researchers have been asking for a clarification of the relation of relativity and quantum mechanics. For instance, in a letter to Born, Einstein wrote in 1926, see Weinberg (2017), Pais (1982): 'Quantum mechanics is very impressive. But an inner voice tells me that it is not the real thing. The theory produces a good deal but hardly brings us closer to the secret of the Old One. I am at all events convinced the *He* does not play dice.' Similarly, Feynman (1967) wrote: 'I think I can safely say that no one understands quantum mechanics'. Even recently, Weinberg (2017) wrote about the apparent conflict between quantum mechanics and relativity: 'O time, thou must untangle this, not I'.

Solution: Indeed, in this book we clarify that apparent 'deep incompatibility between quantum mechanics and relativity': We show that quantum mechanics is a natural structure that is inherent to relativity combined with gravity. Consequently, we resolve the proposed apparent 'incompatibility'. For it, we investigate the combination of relativity and gravity at a microscopic level by analyzing the dynamics of the formation and propagation of vacuum. We emphasize that the vacuum is a very real physical quantity, the density of which has been measured by very different methods, see e. g. Perlmutter et al. (1998), Riess et al. (2000), Spergel et al. (2007), Planck-Collaboration (2020) or by various methods mentioned in Carmesin (2021c). Moreover, the energy of the vacuum amounts to more than 65 % of the total energy of the universe, see e. g. Carmesin (2021c).

We derive the dynamics of the vacuum. With it, we discover that the Schrödinger equation is equivalent to the dynamics of the vacuum that each physical object generates as its contribution to the expansion of space since the Big Bang. Moreover, we derive the postulates of quantum physics from that dynamics of the vacuum.

Basis of the solution: Of course, our derived dynamics of the vacuum must have a very clear and deeply founded basis, in order to be considered as a basis for the solution of the above mentioned 'a deep incompatibility between quantum mechanics and relativity'. For this purpose, we derived that dynamics of the vacuum on the basis of widely accepted principles, summarized in the spacetime-quadruple, SQ. We emphasize that these principles are based on observed motions of planets and stars, see figures (1.3, 2.2, 2.3, 3.8) or Brahe and Kepler (1627), Kepler (1619), Galileo (1638) Rømer (1676), Newton (1686), Einstein (1905), Einstein (1915), so you can easily comprehend these principles.

Tests of the dynamics of the vacuum: Of course, our derived dynamics of the vacuum must be tested by observations that are independent from quantum physics.

For this purpose, we applied our dynamics of the vacuum in order to derive the density of the vacuum, for the ideal case of a universe consisting of vacuum only, see in chapter (4). Hereby, we achieved precise accordance with observation, whereby we do not apply any fit.

Moreover, we used our dynamics of the vacuum in order to derive the density ρ_Λ of the vacuum, for the realistic case of a universe filled with vacuum, radiation and matter, see (Carmesin, 2021d, sections 6.6 and 7.5). Hereby, we achieved precise accordance with observation, whereby we do not apply any fit, see e. g. (Carmesin, 2021d, section 7.5), Carmesin (2021a), Carmesin (2021c).

Furthermore, we utilized our dynamics of the vacuum in order to derive the density of the vacuum, for the realistic case of a universe filled with vacuum, radiation as well as matter, including the case of the very early universe. Thereby, the case of the very early universe can no longer be described in the framework of general relativity, which marks an essential incompleteness of general relativity, see e. g. (Carmesin, 2020c, figure 5.10), (Carmesin, 2020b, figure 5.7). The solutions of that case provide a spectrum of the states of the vacuum consisting of a mixture of several energies, it is called a polychromatic vacuum, see e. g. Carmesin (2018b), Carmesin (2018a), Carmesin (2019b), Carmesin (2021d). Of course, such a mixture is analogous to the white sunlight in the atmosphere. And indeed, the spectrum of the vacuum varies as a function of time, similarly as the spectrum of the sunlight, which contains a relatively large amount of red light at sunrise and at sunset. Hereby, we achieved precise accordance with observation, including a whole function $\rho_\Lambda(t)$, whereby we do not apply any fit, see e. g. (Carmesin, 2021d, section 7.5), Carmesin (2021a), Carmesin (2021c).

Moreover, the spectrum of the vacuum provided in the very early universe is relevant also today as it is restricted by the causal horizon that is set by the light horizon. Furthermore, that spectrum forms the basis of the possible states of the vacuum, and these states explain the formation of mass, see Carmesin (2021a), and of the elementary charge, see Carmesin (2021e). Hereby, we achieved precise accordance with observation, whereby we do not apply any fit.

Altogether, our dynamics has already been tested by many and various observations, including the density of the vacuum, including a whole function $\rho_\Lambda(t)$, the sum of the mass of the neutrinos as well as the mass of the Higgs boson, see Carmesin (2021a), and the elementary charge, see Carmesin (2021e). In all cases, we achieved precise accordance with observation, of course, we do not apply any fit thereby.

Clarifications of quantum physics: Based in our theory of the vacuum, we clarified the basis of the quantization in nature, of the wave function, of the Schrödinger equation, of the Hilbert space in quantum physics, of the Heisenberg uncertainty relation, of the probabilistic properties of nature, of the postulates of quantum physics and of nonlocality in nature, see chapter (5).

Clarifications of general relativity: Based in our theory of the vacuum, we clarified the basis of the curvature of spacetime or of space and time.

Clarification of the relation of quanta, vacuum and relativity: We showed that the dynamics of the vacuum implies both, quantization including the corresponding theory of quantum physics as well as the curvature used in general relativity. In particular, in the framework of the theory of the vacuum, there is no natural 'incompatibility' between quantum physics and the curvature of general relativity.

Instead, the quantization emerges in the theory of the vacuum quite naturally. Correspondingly, quantum physics and its postulates can be derived from the dynamics of the vacuum.

Similarly, the curvature of spacetime emerges in the theory of the vacuum quite naturally. Correspondingly, the curvature of spacetime can be derived from the dynamics of the vacuum.

However, on the level of the present-day version of the theories of general relativity and of quantum physics, the above mentioned scientists articulate an 'incompatibility' between these theories. Thereby Weinberg (2017) clearly expects a clarification or a step in which the problem is 'untangled'. The dynamics of the vacuum does 'untangle' that problem of 'incompatibility' felt by several scientists, by explaining both, quantization and curvature of spacetime.

Altogether, there are two great theories, quantum physics and general relativity. The present-day versions of these two theories exhibit an 'incompatibility'. On the level of our theory of the vacuum, quantization and curvature of spacetime are both explained, so the essential properties of both theories are explained, and so the 'incompatibility' between both theories vanishes.

Chapter 7

Appendix

7.1 Constants of nature

In this section we present useful constants of nature.

quantity	observed value
G	$6.674\,30(15)\ \frac{\mathrm{m}^3}{\mathrm{kg}\cdot\mathrm{s}^2}$
c	$299\,792\,458\ \frac{\mathrm{m}}{\mathrm{s}}$, exact
h	$6.626\,070\,150(69)\ \cdot 10^{-34}$ Js
k_B	$1.380\,649\,03(51)\ \cdot 10^{-23}\ \frac{\mathrm{J}}{\mathrm{K}}$
ϵ_0	$8.854\,187\,817\ \cdot 10^{-12}\ \frac{\mathrm{F}}{\mathrm{m}}$, exact

Table 7.1: Constants of nature (Newell et al. (2018), Tanabashi et al. (2018)).

7.2 Abbreviations

In this section we present used abbreviations.

abbreviation	full text	reference
DEQ	differential equation	S. (3.1.1)
∂_{r_j} or ∂_j	partial derivative with respect to r_j	
EEP	Einstein equivalence principle	S. (2.2.2)
GG	Gaussian gravity	S. (2.3)
GR	general relativity	S. (2.5)
LFV	locally formed vacuum	S. (3.10.5)
PFF	principles of free fall	S. (2.2)
PWT	particle wave transformation	S. (3.10)
QP	quantum physics	S. (1.1)
RGS	rate gravity scalar	S. (3.6)
RGV	rate gravity vector	S. (3.6)
RGW	rate gravity wave	S. (3.7)
SM	Schwarzschild metric	S. (3.9)
SR	special relativity	S. (2.4)
SEQ	Schrödinger equation	S. (1.3.2.1)
SQ	spacetime-quadruple	Eq. (2.1)

Table 7.2: Abbreviations

7.3 Observed values

In this section we present useful results of observations.

quantity	observed value
H_0 in $\frac{\mathrm{km}}{\mathrm{s \cdot Mpc}}$	67.36 ± 0.54 (0.8 %)
Ω_Λ	0.6847 ± 0.0073 (1.1 %)
Ω_K	$-0.011^{+0.0013}_{-0.0012}$
z_{eq}	3402 ± 26
Ω_m	0.3153 ± 0.0073
Ω_r	$9.265^{+0.288}_{-0.283} \cdot 10^{-5}$ (3.1 %)
σ_8	0.8111 ± 0.006(7.4%)
ρ_{cr,t_0} in $\frac{kg}{m^3}$	$8.660^{+0.137}_{-0.137} \cdot 10^{-27}$ (1.6 %)
$\tilde{\rho}_{cr,t_0}$	$7.037 \cdot 10^{-123}$
$\tilde{\rho}_{v,t_0}$	$4.8181 \cdot 10^{-123}$
Ω_b	0.0493 ± 0.00032
Ω_c	0.2645 ± 0.0048
R_{lh}	$4.1412 \cdot 10^{26}$ m (Carmesin (2019b))

Table 7.3: Data obtained on the basis of the CMB by the Planck satellite ((Planck-Collaboration, 2020, p. 15 and 38)) by using the modes TT, TE, EE, the low energy and the lensing results. Quantities with a tilde are presented in natural units alias Planck units (see subsection 7.4). Hereby 1 Mpc = $3.0856776 \cdot 10^{19}$ km.

7.4 Natural units

Planck units or natural units have been introduced by Planck (1899). We mark quantities in natural units by a tilde (s. Tab. 7.4, Carmesin (2019b)).

physical entity	Symbol	Term	in SI-Units
Planck length	L_P	$\sqrt{\frac{\hbar G}{c^3}}$	$1.616 \cdot 10^{-35}$ m
Planck time	t_P	$\frac{L_P}{c}$	$5.391 \cdot 10^{-44}$ s
Planck energy	E_P	$\sqrt{\frac{\hbar \cdot c^5}{G}}$	$1.956 \cdot 10^{9}$ J
Planck mass	M_P	$\sqrt{\frac{\hbar \cdot c}{G}}$	$2.176 \cdot 10^{-8}$ kg
Planck volume	$V_{D,P}$	L_P^D	
Planck volume, ball	$\bar{V}_{D,P}$	$V_D \cdot L_P^D$	
Planck density	ρ_P	$\frac{c^5}{G^2\hbar}$	$5.155 \cdot 10^{96} \frac{\text{kg}}{\text{m}^3}$
Planck density, ball	$\bar{\rho}_P$	$\frac{3c^5}{4\pi G^2\hbar}$	$1.2307 \cdot 10^{96} \frac{\text{kg}}{\text{m}^3}$
Planck density, ball	$\bar{\rho}_{D,P}$	$\frac{M_P}{\bar{V}_{D,P}}$	
Planck temperature	T_P	$T_P = \frac{E_P}{k_B}$	
scaled volume	$\tilde{V}_D$	$\frac{V_D}{\bar{V}_{D,P}}$	
scaled density	$\tilde{\rho}_D$	$\frac{\tilde{M}}{\tilde{r}^D} = \frac{\tilde{E}}{\tilde{r}^D}$	$\rho_D = \tilde{\rho}_D \cdot \bar{\rho}_{D,P}$
scaled length	$\tilde{x}$	L_P	$x = \tilde{x} \cdot L_P$
Planck charge	q_P	$M_P\sqrt{G 4\pi\varepsilon_0}$	11,71 e

Table 7.4: Planck - units.

In the following sections, we analyze the smallest possible physical objects in space and time. We call these objects the elements of spacetime, EST.

7.5 Definition of Planck scale quantities

Planck Planck (1899) introduced the *Planck units*. These can be based on three universal constants of nature. These basic *universal constants*, the *gravitational constant* G, the *velocity* *of light* c and the *Planck constant* h, can be defined as follows Tanabashi et al. (2018):

$$G = 6.674\,08(31) \cdot 10^{-11}\,\frac{\mathrm{m}^3}{\mathrm{kg} \cdot \mathrm{s}^2} \tag{7.1}$$

$$c = 299\,792\,458\,\frac{\mathrm{m}}{\mathrm{s}} \tag{7.2}$$

$$h = 6.626\,070\,15 \cdot 10^{-34}\,\mathrm{Js} \tag{7.3}$$

Based on the Planck constant h, the *reduced Planck constant* $\hbar$ is defined as shown next:

$$\hbar = \frac{h}{2\pi} \tag{7.4}$$

Two basic Planck units are the Planck length and the Planck mass Tanabashi et al. (2018):

The *Planck length* is defined as follows:

$$L_P = \sqrt{\frac{\hbar G}{c^3}} \tag{7.5}$$

The *Planck mass* is defined as shown below:

$$M_P = \sqrt{\frac{\hbar c}{G}} \tag{7.6}$$

Using the two basic Planck units, further Planck units are derived according to the corresponding definitions of the respective physical quantities, see below.

The *Planck volume* is defined as shown next:

$$V_P = L_P^3 \tag{7.7}$$

The *Planck energy* is defined as presented here:

$$E_P = M_P \cdot c^2 \tag{7.8}$$

The *Planck density* is defined via the next formula:

$$\rho_P = \frac{M_P}{L_P^3} \tag{7.9}$$

The *Planck time* is defined as follows:

$$t_P = L_P/c \tag{7.10}$$

As a notation, we mark scaled physical quantities by a tilde, see table (7.4).

7.6 Horizons of observation

In this section, we analyze the limits of observation.

We define a *physical object* as an object that can be observed. We regard an *event* as something that can be observed and that takes place at an observable spacetime.

As a proposition, *Newtonian gravity* Newton (1686) implies that a physical object with a mass m has a *smallest observable radius*. That radius is called *Schwarzschild radius*, whereby the following relation holds:

$$R_S = \frac{2G \cdot m}{c^2} \tag{7.11}$$

This result has been discovered by Michell Michell (1784) and Laplace Laplace (1796).

The Schwarzschild radius R_S can be interpreted as the *event horizon* of a black hole with a mass m.

As a proposition, *quantum physics* implies that a physical object with a momentum p_x that exhibits an uncertainty Δp_x has a *smallest observable uncertainty* Δx. Hereby the Heisenberg uncertainty relation holds:

$$\Delta p_x \cdot \Delta x \geq \frac{\hbar}{2} \tag{7.12}$$

Heisenberg Heisenberg (1927) was essentially involved in the discovery of this result, see e. g. Ballentine (1998).

A physical object obeys both above limits of observation that are represented in equations (7.11) and (7.12). The combination of these two limits has been visualized in a diagram showing the uncertainty Δx as a function of the energy E, see figure (7.1). The objects that can be observed according to the Schwarzschild radius (equation 7.11) are above the straight line and marked by a horizontally hatched area. The objects that can be observed according to the uncertainty relation (equation 7.12) are above the hyperbola and marked by a vertically hatched area. The objects that can be observed correspond to the intersection of the horizontally hatched area and the vertically hatched area and are marked by the cross hatch area. That cross hatched area exhibits a point with smallest length (marked by a filled circle). That point is the intersection of the straight line (equation 7.11) and the hyperbola (equation 7.12). The corresponding length can be derived, and the result is as follows: As a proposition, see e. g. (Carmesin, 2021a, proposition 4), the smallest observable uncertainty of a physical object is the Planck length L_P, and the corresponding energy is one half of the Planck energy $E_P/2$.

$$\Delta x \geq L_P \tag{7.13}$$

$$E(L_P) = E_P/2 =: \bar{E}_P \tag{7.14}$$

$$M(L_P) = M_P/2 =: \bar{M}_P \tag{7.15}$$

Hereby, we defined the corrected Planck energy $\bar{E}_P$ by $E_P/2$ as well as the corrected Planck mass $\bar{M}_P$ by $M_P/2$.

As a further proposition, see e. g. (Carmesin, 2021a, proposition 5), the largest possible density is equal to the corrected Planck mass divided by the volume of a ball with the radius L_P:

$$\rho \leq \frac{1}{2} \cdot \frac{3}{4\pi} \cdot \rho_P =: \bar{\bar{\rho}}_P \tag{7.16}$$

Hereby, we defined the corrected Planck density $\bar{\bar{\rho}}_P$ by $3\rho_P/(8\pi)$.

As an additional proposition, there is a minimal observable length $\tilde{x}_{min}$ as a function of the energy $E = p_x \cdot c$ corresponding

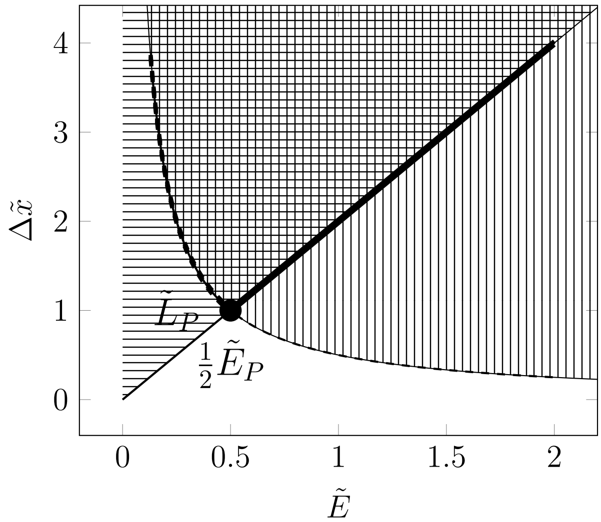

Figure 7.1: Shortest observable uncertainty (dot): Observable states outside the event horizon of a possible black hole (horizontally hatched area). Sufficient uncertainty according to the Heisenberg uncertainty relation (vertically hatched area). Observable objects (including sufficient uncertainty) are marked by the cross hatched area.

to the momentum p_x as follows:

$$\tilde{x}_{min}(\tilde{E}) = \begin{cases} \frac{1}{2\tilde{E}} & \text{for } \tilde{E} \leq 1/2 \\ 2 \cdot \tilde{E} & \text{otherwise} \end{cases} \tag{7.17}$$

For an illustration see figure (7.1).

7.7 Glossary

Abbreviation: S. (section), C. (chapter), DEF. (definition), PROP. (proposition), THM. (theorem), Eq. (equation).

Big Bang: Start of time evolution of visible space

CMB, Cosmic Microwave Background: Radiation emitted at $z \approx 1090$. (Tab. 7.3)

cosmological constant: Λ corresponds to the dark energy with its density ρ_Λ (Tab. 7.3).

curvature parameter: the curvature parameter k describes the global curvature of space, see e. g. Carmesin (2021d)

dark energy: Energy of the cosmological density of the vacuum ρ_Λ (Tab. 7.3).

density, critical: ρ_{cr,t_0} or ρ_{cr} (Tab. 7.3 or e. g. Carmesin (2021d))

density parameter: $\Omega_j = \rho_j/\rho_{cr,t_0}$ (Tab. 7.3)

density, vacuum: $\rho_\Lambda = \Omega_\Lambda \cdot \rho_{cr,t_0}$ (Tab. 7.3)

dynamical mass: $M = \frac{E}{c^2}$

frame: Each observation apparatus is localized in spacetime. That localization establishes a frame.

gravitational field: G^* (C. 3)

horizon: Global limit of visibility (C. 4)

Hubble - parameter: $H = \frac{\dot{a}}{a}$ (C. 3)

Hubble - constant: $H_0 = H(t_0)$ Hubble parameter at t_0 (C. 3)

light horizon: $R_{lh} = 4.142 \cdot 10^{26}$ m (Tab. 7.3)

rate gravity four-vector, RGV: C. (3)

rate gravity scalar, RGS: C. (3)

RGW, rate gravity wave: Carmesin (2021d) or C. (3)

rate of the formation of vacuum: (S. 3)

Schwarzschild radius R_S: At this radius the escape velocity is equal to c

spacetime: Combination of space and time (C. 3

vacuum: The vacuum has a volume, a density and the velocity c. (C. 4 or Carmesin (2021d))

7.8 SR fully based on a thought experiment

To each star in Fig. (1.3), we add an orbiting satellite with a radio transmitter emitting radio waves. The corresponding frequency is f_1, for the 1^{st} transmitter, and f_2, for the 2^{nd} transmitter. In the region $R_{binary-Earth}$ between the binary and Earth, the frequencies are modified according to the Doppler effect, so that f_1 becomes f_1', and f_2 becomes f_2'.

These transmitters are controlled so that in $R_{binary-Earth}$, the waves have the same frequencies $f_1' = f_2'$, phases and directions (of polarization and of propagation). That is possible, as f_1 and f_2 can be chosen small compared to the frequencies of electronics and since radio waves can be described by classical waves.

Altogether, in $R_{binary-Earth}$, the two waves have the same frequencies, phases and directions. Hence the two waves form common fields $\vec{E}$ and $\vec{B}$ in $R_{binary-Earth}$. Thence the two waves or the common wave have the same velocity of propagation in $R_{binary-Earth}$. We call this fact the **principle of free propagation, PFP: If two waves propagate in a homogeneous region and have the same physical quantity constituting the amplitude, the same frequencies, phases and directions (of polarization and propagation), then these waves exhibit the same velocity of propagation.** So the velocity of the radio waves is invariant, irrespective of the motion of the radio transmitters. Thus c is invariant. This implies SR, see e. g. Carmesin (2020b). Indeed, thought experiments provide the PFF, PFP and GG.

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