Understanding the Conceptual Interplay Between Learners' Motivation and Patterns of Personal Meaning in the Mathematics Classroom: Results From Germany and Finland

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Abstract

The orientation of motivation plays an important role in the quality of students' functioning in mathematics learning. In particular, autonomous (i.e., self-determined) motivation has multiple positive effects on their learning processes, for instance as it relates to proactive coping with failure, taking responsibility for one's own learning, volitional actions, and most importantly, well-being in the classroom. Intrinsic motivation represents the hallmark of self-determination. However, not all students are intrinsically motivated; learners seek to fulfil their need for meaning in learning situations by referring to their concept of the personal relevance of studying mathematics. Recent research in the field of motivation has emphasized that teachers' external efforts can affect the learner's autonomous motivation by highlighting the student's sense of the personal relevance of mathematics. The conceptual relationship between the personal relevance attach to studying and their quality of motivation is not clarified yet. While studies have identified motivational profiles so as to identify configurations of behavioural regulations, profiles of personal relevance have not received substantial attention yet. That is why the present investigation examines what patterns of personal relevance (termed as individual relevance systems) move learners to study mathematics and how the learner's individually constructed personal relevance is interconnected with their motivation in the mathematics classroom.

To this effect, two theoretical lenses, self-determination theory (SDT) and social constructivism, are adopted to jointly study why individuals show a certain motivational orientation in line with what kind of individual relevance system they construct in mathematics as a means of orienting themselves towards the social environment in the classroom. In this connection, the motivational construct of personal meaning reflects the personal relevance learners attach to studying mathematics in an academic context. Theoretically, personal meaning encompasses a biological and a social ontological nature, so this construct can be conceptualized as straddling the boundary of both theoretical lenses, SDT and social constructivism. To understand the interaction between both perspectives, I studied the interplay between data from German and Finnish ninth graders (aged 14–16) to contrast the results and refine the theoretical relationships between both affective constructs.

Research findings were gathered within a sequential-dependent research design: The initial empirical work that was carried out using quantitative statistical analyses only captured personal meaning as an individual phenomenon, as it focused on a classical survey method (self-report data). To take into account the social ontological nature of personal meaning, I subsequently considered the approach of networking theories and applied the strategy of coordinating.

I condense the main findings of this present dissertation into two central messages: Firstly, the individual relevance systems of learners found in Germany (GER) and Finland (FIN) can be differentiated into four kinds of mathematics-learning orientation in view of emotional-social integration—learners' starting internalization (either balanced (GER) or autonomous motivation (FIN)); enjoyment in mathematics learning (learners experience well-being and integration—autonomous motivation); self-improvement (learners feel freedom of

action—autonomous motivation), and external pressure (learners follow obligations—either balanced (FIN) or less controlled motivation (GER)).

The empirical evidence suggests that basic psychological needs can be stimulated by the construction (or fail to be constructed in the event of non-construction) of the three specific personal meanings EXPERIENCE OF AUTONOMY, COMPETENCE, and SOCIAL RELATEDNESS through the learner's reaction to their social environment in the mathematics classroom (known as the inner feedback mechanism). The satisfaction of the respective basic psychological needs for autonomy, competence, and relatedness seems to retroact to the learner's social space in class. So, the basic psychological needs are the factors in personal meaning that have a strengthening or weakening effect on biological (the link between SDT and personal meaning) and social regulatory mechanism (the link between social constructivism and personal meaning). Through this inner feedback mechanism, the biological regulatory mechanism regulates the learner's self-determination and well-being in the classroom (autonomous or controlled motivation), and, in the classroom setting, the social regulatory mechanism regulates the direction of the learner's type of individual relevance system that is produced in line with the biological regulation (geared towards or away from the relevance in individual and mathematics-related personal meanings). The mediating roles of teacher and cultural amplifiers reshape the learner's social regulatory mechanism, and this in turn explain why similar patterns of personal meaning result in different biological regulations across countries.

Secondly, the biological and social regulatory mechanisms are coordinated within the boundary object of personal meaning, and the indicators found within the coordination of the biological and social regulatory mechanisms can be modelled as a hypothetical feedback mechanism to explain the genesis of motivation in the mathematics classroom. The social regulatory mechanism seems to trigger the biological regulatory mechanism, and that in turn seems to feedforward to their social regulatory mechanism. As a result of this linkage, it is not possible to separate the social and biological regulatory mechanisms or view them as independent of each other. I conclude that the boundary object of personal meaning has major significance for the genesis of motivation in the mathematics classroom.

The dissertation concludes with an in-depth discussion of how its findings relate to those of earlier studies, and of the potential for its future application in fields other than the specific area of study, i.e., mathematics-related motivation.

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I Dedicate this Thesis to
all Learners who are on the Way to Becoming Self-Determined
and to the Teachers Supporting Them on Their Individual Journeys.

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1. Introduction

Education is the kindling of a flame, not the filling of a vessel.

Socrates, 470 BC

If, according to Socrates, education kindles the flame of knowledge, motivation can be seen as the torch that arouses and sustains that flame within students (Liu et al., 2016, p. 1) in (e.g.,) mathematics-learning situations. Understanding each student's motivation to study mathematics is not easy to manage in the classroom context. Like a snowflake, each learner is unique, and thus they[1] bring their individual motivations with them into the mathematics classroom. Although motivating students to study mathematics is a matter of much debate, teachers may find it overwhelming due to their busy workloads (Reeve, 2016). Therefore, it is sometimes necessary for teachers to apply pressure to students to ensure they complete their work—regardless of whether the student is interested in the topic for its own sake, finds no personal importance in it, or is even bored by it (Reeve, 2016; Vansteenkiste et al., 2018). This appears to teachers to be a necessary strategy if they are to meet the requirements of the mathematics curriculum and handle the increasing amount of responsibilities they hold in that classroom, in the face of a lack of resources. Most importantly, teachers use pressure and control to ensure they do not drop out of the profession, as well as for their own personal well-being, in order to function sustainably over time, both in and out of the classroom. Hence, teachers do not intentionally seek to establish a controlling environment or to avoid teaching styles that would motivate their students, such as recognizing learners' perspectives while teaching (Reeve & Cheon, 2021). Exploring *what* personal relevance drives learners to study mathematics not only helps students to experience a positive learning process (Ryan & Deci, 2017) but also helps educational professionals to make their daily routine smoother and to foster students' motivation to learn mathematics in the classroom.

First of all, what does it mean to be motivated? And why is it important, for a variety of reasons, for learners to be autonomously motivated in the classroom instead of feeling controlled? Learners who are motivated in the mathematics classroom are *moved* to direct their behaviour towards a certain action (Hannula, 2004; Ryan & Deci, 2000a). The extent of motivation can vary, both in terms of quantity and in terms of the learner's quality of functioning. Quantitative motivational processes, or *achievement-related behaviour*, concern *how much* a student is motivated; however, *qualitative motivation* refers to the *why* of orientations towards the *what*, or content, of goal pursuits (Deci & Ryan, 2000; Ryan et al., 1996; Ryan & Deci, 2000a). To illustrate this, a student may be extremely motivated to study mathematical contents out of passion and interest or a desire to use the learned mathematical skills in their future education. Alternatively, a learner may also be motivated to solve difficult mathematical tasks to impress important figures in their life (e.g., teachers, peers, or parents), or they may value the skills for other advantages they bring with them (Ryan & Deci, 2000a).

[1] I use "they" as a generic third-person singular pronoun to ensure bias-free language within this study's context and to refer to all individuals as human beings (American Psychological Association, 2020).

Both motivational processes, quantity and quality, matter. However, when examining these motivations at the level of quantity, there is no substantial difference: that is, all actions can appear to be highly motivated regardless of the nature of that motivation (Ryan & Deci, 2000a). In contrast, the quality aspect of motivation differs in terms of its composition (i.e., autonomous or controlled) and its underlying focus of orientation (e.g., goals, Ryan et al., 1996; Ryan & Deci, 2000a), and this in turn affects the quality of mathematics learning.

For this reason, this study focuses on learners' *quality of motivation* when learning mathematics. Thinking of the energizing nutrients of quality of motivation within educational settings, several scholars (to name but a few: Boekaerts, 1999; Hannula, 2006; Nuttin, 1984; Ryan & Deci, 2020) emphasize the functional significance of *basic psychological needs for autonomy*, *competence*, and *relatedness* in terms of students' learning processes and optimizing their motivation. These basic psychological needs are interpreted as the roots (Hannula, 2004) and the driving force of autonomous-oriented motivation in learning and its sustainability (Krapp, 2005). Students engage more energetically in learning processes only if they subjectively experience an overall balance of basic-psychological-needs satisfaction (Krapp, 2005).

The implementation of learning processes can be qualitatively improved if learners engage with (e.g.,) mathematics for its own sake; namely, with autonomous motivation. Autonomously motivated learning situations are connected to various robust positive outcomes such as more self-efficacy (Wang et al., 2016), investing more effort in learning mathematics (León et al., 2015; Ryan & Deci, 2020), perseverance, positive emotions, and high performance (Deci & Ryan, 1991; Ryan & Deci, 2017, 2020; Vansteenkiste et al., 2018). Moreover, learning activities that are built on autonomous motivation can more easily be absorbed into those core values which make up the students' "identity as scientists" (Skinner et al., 2017, p. 2436). Focusing on STEM courses, Skinner et al. (2017) found that if learners' classroom experiences fulfilled their basic psychological needs (i.e., were associated with autonomous motivation), students were more engaged and performed to a higher level; they also saw themselves as scientists and positively identified with the field of STEM science. In contrast, controlled motivation is associated with the frustration of basic psychological needs (Haerens et al., 2015), making the need for self-esteem more salient as a result (Ryan & Deci, 2017), and has a detrimental effect on intrinsically motivated behaviour (Ryan & Deci, 2017), as well as leading to ill-being (Stenling et al., 2017). Furthermore, students' learning is suboptimal, particularly when learning activities are complex or demand a conceptual and creative sequence of operations (Benware & Deci, 1984), which is often the case in mathematics. In particular, controlled motivation is associated with a lack of maintenance of learning when the student is not being observed; this unsustainable approach makes dropout more likely (Deci & Ryan, 2000; Pelletier et al., 2001; Ryan & Deci, 2017; Vallerand & Bissonnette, 1992).

Particularly during adolescence, dropping out of mathematics education has an impact on students' cognitive and brain development. In fact, scholars at the University of Oxford discovered that dropping out of mathematics education, specifically at the age of 16, was detrimental for British learners' neurological and cognitive development (Zacharopoulos et al., 2021). In some countries, students can drop out of mathematics voluntarily. The study showed that learners who stopped studying the subject had a lower concentration of GABA (gamma-Aminobutyric acid), a neurochemical that is necessary for brain plasticity within an area of the brain that is important for cognitive functioning, for example logic, mathematics ability, memory, problem-solving, and mathematical reasoning (Zacharopoulos et al., 2021). Previously, when they were still engaged in mathematics, no neurochemical changes had been observed (Zacharopoulos et al., 2021). Zacharopolous et al. did not merely stress the high relevance of mathematics education for students' brain development and cognitive functioning; they also underlined the reciprocal interplay between biology and mathematics education that takes place during adolescence (Zacharopoulos et al., 2021, p. 8).

These results show the importance of autonomously motivated learning, particularly of mathematics, for students' learning processes and personal development. As schematically pointed out by Hannula (2012), this empirical evidence underlined the interplay between (autonomous) motivation, (positive) emotion, and (development and positive impact on) cognition. Vygotsky made this point succinctly when he stated that affect is never external to the learner's intellect (Vygotsky, 1962, p. 8; 1987, p. 50). In terms of this issue, I agree with Vygotsky (1987), who argued that

> The inevitable consequence of the isolation of these functions [intellect and affect] has been the transformation of thinking into an autonomous stream. Thinking itself became the thinker of thoughts. Thinking was divorced from the full vitality of life, from the motives, interests, and inclinations of the thinking individual. (…) By isolating thinking from affect at the outset, we effectively cut ourselves off from any potential for a causal explanation of thinking. A deterministic analysis of thinking presupposes that we identify its motive force, that we identify the needs, interests, incentives and tendencies that direct the movement of thought in one direction or another. (p. 50)

On the basis of Vygotsky's argument, Hickey (1997) stated that motivation as an affective construct should not be distinguished from cognitive processes. That is the case because motivation as an individual construct is not separable from cognitive activities that take place while learning. The importance of mathematics learning foregrounds the question: How can educators foster learners' autonomously motivated learning in mathematics? Specifically, it is necessary to find solutions with which to engage students when access to mathematics education in school is limited (e.g., due to the COVID-19 pandemic crisis of 2020/2021), in order to balance the impacts of this on adolescents' (and children's) neurological and cognitive development.

In the case of mathematics education, there is relatively little existing research focusing on subject-related motivation compared to the amount of research in general education. For instance, Middleton and Spanias (1999) emphasized the necessity of clarifying why learners are motivated to study mathematics and of exploring the interrelations of motivational factors that play a determining role in learners' "social and cognitive worlds" (pp. 83–84) if researchers are to get a full picture of the phenomenon, that is, of motivation in mathematics education. In this relation, Hannula (2006) suggested that enhancing the theoretical knowledge of the interplay between what motivates students to study mathematics and how their motivation to learn is regulated—that is, whether it is autonomous or controlled—may help scholars to understand learners' behaviour in the mathematics classroom. However, students typically have multiple motives within the educational context (Ryan & Deci, 2017, 2020). Recently, researchers have investigated the patterns of various types of motivation (motivational profiles) in mathematics education (e.g., Wang et al., 2017).

Although these results concerning profiling provide an insight into students' patterns of various types of motivation as regards mathematics learning, merely concentrating on why learners show a certain behaviour in the classroom is not enough to foster their motivation in mathematics. Mathematics learning is not interesting for all individuals. Specifically, due to the decrease of interest in mathematics with increasing age (e.g., Frenzel et al., 2010, 2012) it is nearly impossible for teachers to stimulate the intrinsic motivation in mathematics that comes with interest, curiosity, and enjoyment. However, students can also positively engage with mathematical activities by means of a learning environment that is supportive of their basic psychological needs and that highlights the personal relevance to those students (it tends to be more valued on an individual than on a social basis, Ryan & Deci, 2017, 2020; Vansteenkiste et al., 2018). In this connection, Bikner-Ahsbahs (2005) emphasized that mathematics learning can also become more interesting to students when they experience it as personally relevant. For this reason, it is necessary to focus on what kinds of personal relevance students construct when studying mathematics.

Concerning the relevance of mathematics, Niss stated that "although the social significance of mathematics seems to be ever increasing in scope and density, the place, rôle [*sic*], and function of mathematics are largely invisible to – and unrecognized by – the general public" (Niss, 1994, p. 371). Niss pointed out that despite the commonly accepted relevance of mathematics, mathematics remains subjectively irrelevant for the majority, which he calls a "relevance paradox" (Niss, 1994, p. 371). To highlight learners' personal relevance, researchers therefore need to explore what personal relevance students subjectively construct in mathematics-learning situations. Therefore, they need to understand the learners' individual "mental compasses": that is, the set of personal relevance that provides them with the orientations to study mathematics. The motivational construct *personal meaning* concerns the personal relevance students attach to mathematics (Vollstedt, 2011a; Vorhölter, 2009). In this connection, Vollstedt (2011a, 2011b) reconstructed 17 personal meanings that learners (aged 15–17) attached to mathematics learning within an educational context. Vollstedt and Vorhölter (2008) made the theoretical claim that each learner constructs a pattern of personal relevance (Vollstedt, 2011b; Vollstedt & Vorhölter, 2008; Vorhölter, 2009) that they assign to mathematics learning. If learners truly construct various patterns of personal meaning, the empirical evidence of those patterns of personal meaning, here termed *individual relevance systems*,[2] can help to clarify exactly what it is that provides them with a learning orientation in the mathematics classroom.

The cultural context and the mathematics curriculum as its outcome are indivisible from learners' mathematics-classroom environments and the teaching styles that provide the backdrop for motivational factors (Hickey, 1997; Ryan & Deci, 2020). Since each country's educational policy is unique, its principles will also vary in terms of how to support students' mathematics education (Zacharopoulos et al., 2021). Learners' participation in mathematics

[2] A clear distinction should be made here between this definition and that of Alfred Schutz (1899–1959).

education is determined by the various opportunities provided within the mathematics curricula. Accordingly, an investigation into different cultural contexts that have different foci within mathematics curricula would be fruitful, in order to see which individual relevance systems students construct. For this reason, this dissertation considers data from both German and Finnish ninth graders (aged 14–16; OECD, 2020a, 2020b). Nevertheless, an isolated study of students' individual relevance systems does not provide enough context either, and will need to be supplemented by further research.

To the best of my knowledge, so far, no well-defined conceptual framework has been developed that discusses the theoretical connection between learners' personal relevance in an academic context and the quality of motivation they experience in (mathematics) education. According to this *problematique*, this dissertation's main purpose is to link the theoretical perspectives on "why" students show a certain motivation with those concerning "what" personal relevance they construct in mathematics in order to orient themselves in their social settings of the mathematics classroom. Exploring learners' individual relevance systems in connection with their motivational behaviour has the potential to increase our theoretical knowledge of the interrelation and interplay between the "why" and the "what" of the development of motivation within mathematics education (Hannula, 2006; Middleton & Spanias, 1999). Not all types of personal relevance are supportive for mathematics learning or are close to the learner's sense of self. An exploration of the conceptual relation between personal relevance and motivation also helps to identify those individual relevance systems that promote students' autonomous motivation and those relevance systems which are less supportive. Accordingly, the overall objectives of this thesis are to explore the following questions:

- what kinds of individual relevance system do learners from Germany and Finland construct when learning mathematics?
- how are the individual relevance systems from Germany and Finland associated with motivation in mathematics?

Moreover, the comparison of German and Finnish data could help to contrast and refine our understanding of the relation between learners' individual relevance systems and the quality of their motivation. Based on this conceptual understanding, the interplay between personal relevance and motivation may provide a theoretical explanation for the genesis of motivation in the mathematics classroom. Therefore, the two questions above lead us to the main question with which this study is concerned, namely:

- how does learners' motivation in mathematics classroom emerge from the interplay between learners' individual relevance systems and motivation?

An insight into this interplay may help teachers to address the specific motivational factors that increase the personal relevance of mathematics for learners and that are also associated with autonomous motivation or positive learning processes. Furthermore, such an insight might also help educators to avoid teaching styles that have a negative impact on learners' autonomous motivation and the satisfaction of their basic psychological needs. Accordingly, this study seeks to contribute to an in-depth understanding of students' motivation in mathematics education by focusing on learners' individual voices and on the need to provide them with conditions in mathematics lessons that support the learning process as well as their personal development.

Based on these aims, the present dissertation follows a sequential-dependent research design (Schoonenboom & Johnson, 2017) in order to understand the conceptual interplay between German and Finnish ninth graders' levels of personal relevance and quality of motivation, and the genesis of motivation in mathematics. The following chapter, Chapter 2, introduces the theoretical framework of this study. In relation to this study's research interest, two different theoretical perspectives are adopted – self-determination theory (SDT) and social constructivism—through which to discuss learners' motivation in an academic context in connection with the development of students' individual relevance systems. Therefore, each theoretical lens is introduced separately to highlight both perspectives' strengths and limitations as regards the aims of this study. Based on existing theoretical as well as empirical knowledge, an interconnection between both theories is assumed within the boundary object (Akkerman & Bakker, 2011) of personal meaning as it seems to be grounded on a theoretical conceptualization that synergizes both learners' individual (basic-psychological-needs experiences) and social experiences (the influence of interpersonal "conversations" in the classroom) into their personal relevance construction in mathematics.

Chapters 3 and 4 encompass the methodological considerations of this thesis. Here, I explore and discuss how to methodologically examine personal meaning within the context of SDT and social constructivism. In Chapter 3 I discuss the philosophical background of the quantitative research methods that are used to identify the learners' individual relevance systems and the corresponding motivational propensities in two different cultural contexts. Since the applied quantitative methods show certain limitations in terms of capturing personal meaning in the context of social constructivism, Chapter 4 points out ways in which the approach of networking theories can help to balance the weakness of the quantitative work.

Chapter 5 reports the empirical results collected in Germany and Finland using quantitative research methods, and Chapter 6 provides a discussion of this empirical work. Chapter 7 refers to the results according to the coordination analysis method based on the empirical data. Subsequently, I reflect theoretically on the coordination of the results in order to propose a theoretical model that might explain the genesis of motivation in the mathematics classroom.

In Chapter 8, I discuss the main findings of this dissertation, and I then examine the evidence from both a retrospective and a prospective viewpoint. Besides a review of my study's limitations, I look at its implications for future research and make some suggestions for the optimization of both mathematics teaching in the classroom and educational policy.

2. Theoretical Framework

As a decisive element of research, theory provides the language to articulate all observations that count to the nature of the research object under investigation (Mason & Waywood, 1996; Schoenfeld, 2002). In this spirit, this present study's theoretical framework aims to provide a robust basis on which to elucidate in a cross-cultural context *why* individual learners show a certain motivational orientation in line with *what* individual relevance systems they construct in mathematics to orient themselves in relation to the social environment in the classroom.

For this purpose, I initially elaborate on the theoretical perspective of self-determination theory (SDT; see section 2.1). SDT is a general theory that can (also) help to understand students' motivation to learn (e.g.,) mathematics.[3] In relation to the present research interest, this theory's strength is in explaining learners' *intrinsic* and *extrinsic motivational orientations* in mathematics-learning settings as an effect of students' individual perceptions of *basic-psychological-needs* satisfaction. Within SDT, scholars have articulated that autonomously motivated learning in the classroom is more likely when students perceive a personal relevance in their learning content. That requires us to answer the following questions: What personal relevance do learners attach to learning mathematics? Moreover, how do learners construct their individual relevance systems in the context of classroom interactions that orient them, as their mental compass, to engage (or not) in mathematics learning? To find answers to these questions, I present the social constructivist perspective (see section 2.2) and the motivational construct *personal meaning*, reflecting learners' perception of personal relevance when engaging with mathematics in an educational setting. Then, by critically reflecting on both theories' foci and limits (with reference to Radford's system of principles (*P*), see section 2.3), I propose that personal meaning can be characterized as a boundary object between SDT and social constructivism to connect both theoretical perspectives. Following the present research interest, linking these two theories would help investigate the theoretical connections between learners' personal relevance (individual relevance systems) and their motivational tendency towards the genesis of motivation in the mathematics classroom.

Since both theoretical perspectives emphasize the role of culture, I investigate the hypothesized theoretical connections using data from Germany and Finland to understand how learners' motivation arises in the mathematics classroom with respect to their culture-specific learning contexts. To this effect, I briefly introduce both countries' mathematics curricula and discuss these two educational settings tentatively (see section 2.4). Finally, based on the adopted theoretical perspectives and their ontological basis, I reflect on the remaining challenges and draw implications (see section 2.5) for this study's methodology that will need to be considered in order to examine the research questions of this dissertation (see section 2.6).

[3] In the following, "mathematics" is bracketed when the content of the statement is not restricted to mathematics in an academic context. Furthermore, SDT is not limited to application in academic contexts (Ryan & Deci, 2017).

2.1 Self-Determination Theory (SDT) for Understanding Motivation to Learn

As a grand psychological theory (Assude et al., 2008), *self-determination theory* (SDT) is an *organismic dialectical* approach postulating that all individuals as active and growth-oriented organisms have inherent propensities to develop an ever more coherent sense of self (autonomy) and that they are naturally inclined to interconnect with their social worlds. The dialectic between the organism and its social world represents the nutrient medium for the individual's development (Deci & Ryan, 2000; Ryan & Deci, 2002, p. 5, 2017). Addressing this *organismic dialectical metatheory*, SDT states that every learner is proactively striving towards "psychological growth and integration" in order to experience "learning, mastery and connection with others" (Ryan & Deci, 2020, p. 2). Consequently, education should support this innate tendency to learn and to evolve greater autonomy (Deci & Ryan, 2000, p. 242) in order to facilitate students' natural development, that is, learners' individual processes of assimilation (Piaget, 1971, pp. 172–173), and to support them on their individual journeys towards evolving into "self-directed and lifelong learners" (Liu et al., 2016, p. 2).

Based on this meta-theoretical foundation, SDT represents a robust theoretical framework for understanding students' intrinsic and varied extrinsic quality of behaviours in a mathematics learning context by considering social-contextual factors (e.g., indicators of classroom environment like teacher behaviours) that essentially empower or impede autonomous forms of motivation, internalization (increasing ownership of external sources), and mental well-being (Deci & Ryan, 2000; Ryan & Deci, 2020). To explain learners' quality of motivation, SDT delineates various learning outcomes associated with forms of motivation ranging from self-determined to controlled (Deci & Ryan, 2002; Ryan & Deci, 2017). The most self-determined *regulatory style* is *intrinsic motivation*, followed by autonomous extrinsic regulatory styles (*integrated* and *identified*) and controlled extrinsic forms (*introjected* and *external*). *Amotivation* represents non-regulation. In this connection, SDT conceptualizes three basic psychological needs, the needs for autonomy, competence, and relatedness, as fundamental building blocks for learners' motivational propensity in learning situations. Following the research in the SDT literature, scholars have postulated that the social setting in the (mathematics) classroom can foster autonomously motivated learning outcomes and learners' internalization processes by promoting the satisfaction of students' basic psychological needs (Ryan & Deci, 2017; Vansteenkiste et al., 2020). The degree to which learners experience individual satisfaction of these nutrients is associated with certain dynamics within regulatory styles (autonomous versus controlled orientation in the classroom) and the corresponding content of motivation (or *what*; Deci & Ryan, 2000). Accordingly, to the extent they feel these three psychological basic needs are being fulfilled, learners can attain greater engagement and learning and can identify with external sources: together, these factors explain the differences between individual students' quality of motivation in (the mathematics) classroom (Deci & Ryan, 2000).

How should we address these basic psychological needs and support autonomously motivated learning in the mathematics classroom? To this effect, as indicated earlier (cf. 1), an emphasis on learners' personal relevance in the mathematics classroom represents a promising step to engage students in mathematics-learning activities. To understand the critical role of learners' personal relevance from the perspective of SDT, I first provide the classic definitions and the historical roots of the elements of SDT, *regulatory styles* and *basic psychological needs*, that underlie its metatheory and constitute the foundation for the upcoming discussions on *why* the quality of engagement differs between learners in academic learning contexts (see section 2.1.1).

Secondly, I discuss the SDT-framed research (see section 2.1.2) as follows: The first part of the research review provides a general overview of the regulatory styles and basic-psychological-needs experience in terms of students' quality of behaviour in the classroom. On the one hand, this review intends to point out the importance of autonomous learning to promote students' mathematics-learning process and their personal growth. On the other hand, this elaboration aims to emphasize the existing empirical connections between autonomously motivated learning and the concept of personal relevance. Secondly, based on this general overview, I then highlight the SDT-framed research to date that specifically addresses the role of students' personal relevance and autonomy-supportive teaching behaviours that help learners to experience relevance in the classroom. To conclude this subchapter, I discuss SDT's potential usefulness in examining this study's main purpose (see section 2.1.3). To this effect, besides its strengths, I point out issues that are missing from SDT's ontological principles and that need to be compensated for in order to enhance the epistemological discourse on personal relevance and quality of motivation as intended in this dissertation.

2.1.1 SDT's Elements: Types of Motivation and Basic Psychological Needs

Self-determination theory embraces six mini-theories (retrieved from experimental and field studies), and each of them considers one component of motivation or psychological functioning (Ryan & Deci, 2017). In relation to the present research interest, the relevant elements are empirical knowledge about the consequences of intrinsically and extrinsically motivated behaviours and basic-psychological-needs satisfaction. For this reason, two of these six mini-theories are of crucial importance as they provide the basis for understanding learners' motivational tendencies when learning mathematics in the classroom: namely organismic integration theory (OIT; for understanding intrinsic and extrinsic motivation) and basic-psychological-needs theory (BPNT; for considering experiences of basic psychological needs). These two sub-theories aim to clarify the interconnections between learners' motivated behaviours (the experienced level of self-determination—OIT) in the classroom and their "health and well-being" (BPNT; Ryan & Deci, 2002, p. 10).

Intrinsic and Extrinsic Motivation. Retrospectively, experimental research on intrinsic motivation[4] (Deci, 1971) and the classical differentiation between intrinsically and extrinsically motivated behaviours provided the foundation for Deci and Ryan's research on SDT (Ryan & Deci, 2017, 2020). Early research on intrinsic motivation started in the 1970s when Deci (1971) conducted laboratory and field experiments to examine the effects of external rewards (being rewarded or not) on intrinsic motivation (decrease or enhancement of intrinsic motivation).

Intrinsic motivation. Learners have an internal locus of causality; that is, they feel free to act in a way that advances their interest when involved in intrinsically motivated behaviours

[4] SDT's first mini-theory was cognitive evaluation theory (CET; Deci, 1975; Ryan & Deci, 2017), which particularly addressed social conditions (e.g., rewards, feedback) that facilitate or diminish intrinsic motivation.

(deCharms, 1968; Ryan et al., 1996). They engage in (mathematics) activities *for their own sake* (Deci & Ryan, 2000; Ryan & Deci, 2020); that is, out of interest, enjoyment, and curiosity, as well as inherent satisfaction (Deci & Ryan, 2000; Ryan & Deci, 2020; Vansteenkiste et al., 2018). In line with Ryan and Deci (2020), intrinsic motivation represents the prototype of self-determination (Deci, 1992, p. 44) and "the energizing basis for natural organismic activity" (Deci & Ryan, 1991, p. 244).

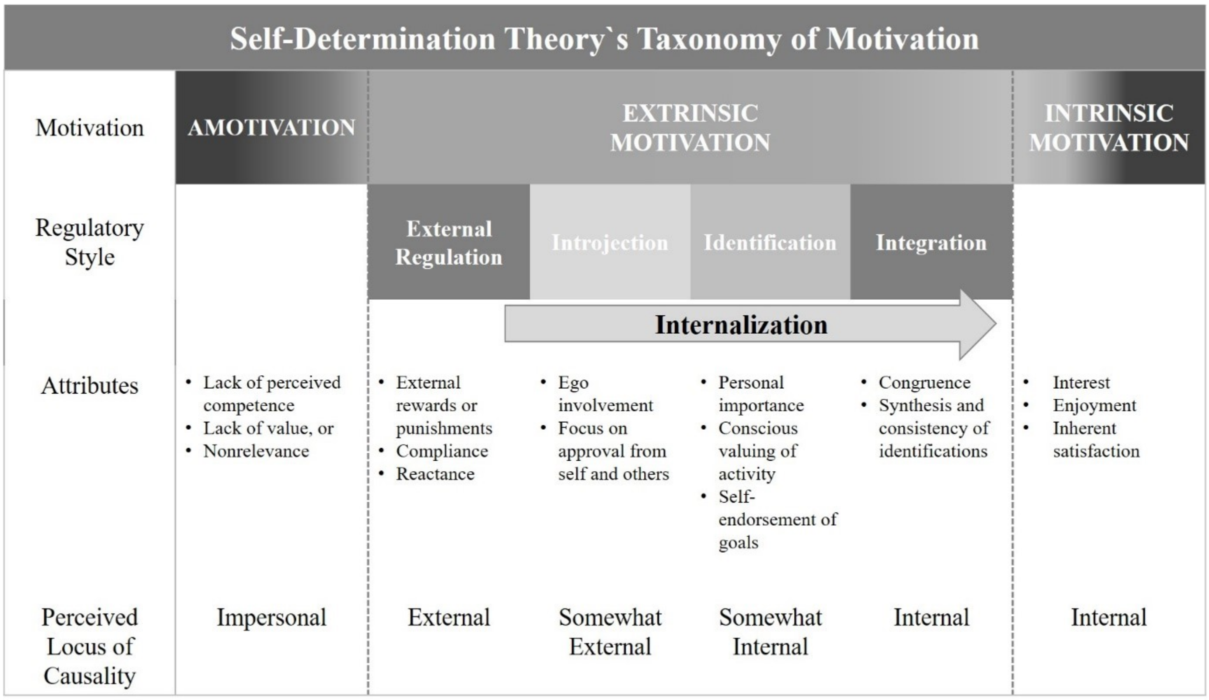
Extrinsic motivation. In contrast to intrinsic motivation, extrinsically motivated behaviours are performed by learners to pursue social-contextual requirements which they personally value (Taylor et al., 2014). As such, those requirements are linked to the learning activity, and learners consider them to be decisive and instrumental in enabling them to be interconnected with the social regulations (Deci & Ryan, 2000). The learner's action is used to achieve a certain outcome, that is, to achieve a particular goal, in which the task is not selected solely on account of the subject matter concerned (e.g., mathematics) (Ryan and Deci, 2017; Vansteenkiste et al., 2018). In the context of extrinsic motivation, learners' intentional actions in the classroom can be distinguished according to their felt locus of causality, which can either be internal or external (deCharms, 1968, pp. 328–329; Deci, 1975; Ryan & Connell, 1989). When learners frame an activity from their socializing environment in the context of the individual values they identify with, an activity which is performed with a feeling of choice and commitment, these autonomous actions refer to an internal perceived locus of causality (similar to intrinsic motivation). In contrast, learners' controlled behaviour (Ryan & Deci, 2017) in class has an external perceived locus of causality – it is regulated by guilt (internal pressure) and threats (external pressure) without learners' having a *true* choice (Deci et al., 1991; Ryan & Deci, 2020; Taylor et al., 2014).

Empirical work in the 1980s focused on extrinsically motivated actions regarding the perceived locus of causality. This research provided the foundation for investigating different levels of extrinsic motivation, which generated the second pillar of the organismic dialectical tendency, *internalization* (Ryan & Connell, 1989; Vansteenkiste et al., 2010). Ryan and Deci (2017) defined internalization as the student's internal psychological mechanism for answering to an "externally observable interpersonal and cultural process of socialization" (p. 180). As a non-uniform conception, extrinsic motivation is specified around internalization, the active process in which learners "attempt to transform socially sanctioned mores or requests into personally endorsed values and self-regulations (…). It is the means through which individuals assimilate and reconstitute formerly external regulations so the individuals can be self-determined while enacting them" (Deci & Ryan, 2000, pp. 235–236). Based on interview data (retrieved from teachers and students from late elementary school), Ryan and Connell (1989) designed the first survey questionnaires to assess students' self-reported reasons for their academic-related behaviour (Ryan & Deci, 2009). Their analyses revealed types of extrinsic motivation associated with different degrees of internalization. That is, unlike other specifications of motivation (dichotomous view: intrinsic versus extrinsic, Harter, 1981; attribution theory, Lepper, 1983), SDT does not conceptualize extrinsically oriented behaviour as a drive from outside the person that forms a contrasting opposite pole to intrinsically motivated behaviour (Ryan & Deci, 2017; Taylor et al., 2014; Vansteenkiste et al., 2018). Instead, in a more sophisticated way, SDT differentiates these regulatory styles so that they fall along a continuum of increasing ownership (internalization). To support learners' internalization in the classroom (from a less to a more fully internalized degree), students must initially experience the valence and importance of (mathematics) learning. A successful internalization process helps them get more integration at the intrapsychic and social level (Deci & Ryan, 2000, p. 236), along the lines of their natural trajectories as proposed by SDT's organismic dialectical metatheory.

OIT illustrates the various behavioural outcomes that are associated with either self-determined or controlled types of motivation to learn. In this connection, learners' instrumental behaviour in terms of different activity engagement in class is associated with four subtypes of extrinsic motivation arranged along the continuum, as a function of internalization (Ryan & Deci, 2017, pp. 191–192). Figure 1 illustrates schematically the varied regulatory styles and their specific sources. As indicated earlier, intrinsic regulation (the far-right end of the motivational continuum) is the hallmark of self-determination. The central section of Figure 1 depicts the different forms of extrinsic regulation ranging from *external* to *integrated regulation*.

Figure 1

Self-Determination Theory's Taxonomy of Regulatory Styles (adapted from Ryan & Deci, 2020).



Within OIT, *external regulation* has an external perceived locus of causality that frames the first type of controlled behaviour (Ryan & Deci, 2020; Vansteenkiste et al., 2018). To avoid negative consequences, students learn to be cooperative in the classroom and work in accordance with the socioculturally required values (Vansteenkiste et al., 2018). They follow the rules they are expected to follow, pay attention in class, and do their schoolwork so as to be rewarded, for example with praise from their teacher (Vansteenkiste et al., 2018). Externally motivated action does not emanate from personal value. Accordingly, no innate attraction to processing the knowledge exists, and thus the learner's behavioural regulation is not internalized (Ryan & Deci, 2020; Vansteenkiste et al., 2018, p. 32).

Rather than responding to pressure from outside (external regulation), *introjected regulated* learners undergo pressure from within themselves, as they force themselves to study (e.g., mathematics) in order to prove their competencies or to feel like a good student (Vansteenkiste et al., 2018). Thereby, their behaviour is a projection of expected characteristics or actions from significant others (Ryan & Deci, 2017). However, the necessity of proving their own value and impressing others is not fully accepted by the learner, and thus it remains external to their sense of self (Deci et al., 1991; Deci & Ryan, 1995, p. 39). This kind of academic motivation is caused by internal coercion that does not reflect the learner's true choice and is thus only partially internalized (Assor et al., 2009). Since this regulation is driven by an internal pressure to increase ego involvement and self-esteem, this behaviour can last longer than external regulation over time but is still a fragile type of motivational behaviour (Deci & Ryan, 2000). Accordingly, this second type of extrinsic motivation constitutes the second form of controlled motivation.[5]

Identified regulation comprises the first autonomous form of extrinsic motivation, in which students recognize the significance of learning activities in the classroom out of personally meaningful reasons that relate to the subject (mathematics) itself. Identified regulation has a more internal perceived locus of causality (Ryan & Deci, 2017), and thus in this stage, the learner has started to internalize the valence of (mathematics) learning (Deci & Ryan, 1995). This type is more volitional than external or introjected regulated behaviour, as learners identify the personal relevance or importance of (mathematics) learning activity that refers to a desired outcome at the present time or in future (Vansteenkiste et al., 2018), for example, doing practice exercises willingly to improve and succeed at the subject (Taylor et al., 2014) or gain good grades for a professional career. Thus, the necessary schoolwork forms a greater part of the integrated cognitions and affects that shape the learner's individuality (Deci & Ryan, 2000). Nonetheless, due to the usefulness of the action, this type remains extrinsic but reflects a full form of internalization (Vansteenkiste et al., 2018) and thus, these learners' actions are highly volitional; that is, similar to those resulting from intrinsic motivation (Ryan & Deci, 2017, 2020).

The fullest form of internalization is *integrated motivation*,[6] which is defined as the most autonomous type of extrinsic motivation (Ryan & Deci, 2020, p. 3). This self-determined extrinsic behaviour appears when learners integrate their personally meaningful values into their coherent sense of self (Deci et al., 1991, p. 330); that is, when learners mutually assimilate the identified values (e.g., of mathematics learning) into harmony with comprehensive deep-rooted aspects of their self, such as their values, (psychological) needs, and identity (e.g., a personal dream of becoming a psychologist, Deci et al., 1991; Deci & Ryan, 2000; Müller & Palekčić, 2005; Ryan & Deci, 2017). Similar to previous empirical studies (to name but a few: Müller & Palekčić, 2005; Ryan & Connell, 1989; Vallerand et al., 1992), I did not include integrated regulation in this present research to avoid issues that appeared regarding distinguishing between integrated regulation and identified type (Vallerand et al., 1992).

[5] In empirical studies, the combination of external and introjected regulation reflects the degree of learners' controlled motivation (Farmer et al., 2020; Koestner et al., 2008).

[6] The sum of self-determined types of motivation, intrinsic, identified regulation (Farmer et al., 2020), and integrated regulation (Koestner et al., 2008) constitutes autonomous motivation in empirical research.

To fully study human behaviours and experiences, Deci and Ryan (1985, p. 150) included a further type of motivation, namely *amotivation* (the far-left category outside of the motivational continuum in Figure 1). A learning situation becomes amotivating when students interpret themselves as unable to master their learning situation because they perceive themselves as incompetent. This category has an impersonal causality (Ryan & Deci, 2017), and thus it is defined neither by personal values nor interest; that is, the learner is neither extrinsically nor intrinsically regulated.

Dozens of studies using survey questionnaires, assessed across numerous cultures, have confirmed the power of these regulatory styles, specifically the knowledge behind the sequential arrangement along the internalization continuum (Deci et al., 1991; Deci & Ryan, 1995; Ryan & Deci, 2020). Thereby, the most researched domain constitutes the academic field (Vallerand & Bissonnette, 1992). More often than not, learners' behaviour in the classroom can become complex, particularly when they are multiply motivated so that different forms co-occur, for example, intrinsic motivation and extrinsic styles; also, different extrinsic regulations can emerge simultaneously for some content (Ryan & Deci, 2017; Ryan & Deci, 2020; Vansteenkiste et al., 2018). That is, the content of schoolwork may simultaneously appear to the learner as interesting (intrinsic) as well as relevant (identified), and sometimes also enjoyable (intrinsic) as it is beneficial for their own self-worth (introjected regulation) in which the personal value to the content is lacking, and so on (Ryan & Deci, 2020; Vansteenkiste et al., 2018). In this connection, the particular content or reason for engaging in a learning activity largely defines which behavioural regulation will most likely become effective (Vansteenkiste et al., 2018, p. 33). For this reason, in addition to examining the specificity of each behavioural regulation, scholars use scores within SDT research that measure, for instance, the composite index, namely the self-determination index (SDI; Levesque et al., 2004, p. 73; Ryan & Deci, 2020). The SDI, computed as SDI = (2 x Intrinsic) + (Identified) – (Introjected) – (2 x External), reflects learners' perceived relative autonomy and is highly predictive of "behavioural persistence" (Ryan & Deci, 2017, p. 195; also see Ryan & Deci, 2020).

In addition, scholars investigated homogenous profiles (Wang et al., 2017) of motivation or of basic psychological needs (Warburton et al., 2020) to examine the different patterns of learners' motivational propensity and establish whether the findings in subgroups result in a theory-consistent explanation of the perceived locus of causality (e.g., the co-occurrence of external and intrinsic regulation towards a goal is theoretically conflicting; Ryan & Deci, 2017, p. 196). As an overall focus, these additional measures help us, in a wide array of ways, to learn more about the predictive power of these regulatory styles (for example, on affective and cognitive aspects, see section 2.1.2; Ryan & Deci, 2020).

This regulation taxonomy is not in itself a developmental continuum, and it is not necessary for learners to proceed through each level of internalization step by step. In fact, learners can optionally internalize a new form of regulation at any level if the social climate in class offers the relevant experiences to satisfy their inter-individual basic psychological needs (Deci & Ryan, 2002).

Basic Psychological Needs. Ryan and Deci (2000b) specified the notion of the basic need concept as follows: "By our definition, a basic need, whether it be a physiological need (…) or a psychological need, is an energizing state that, if satisfied, conduces toward health and well-being but, if not satisfied, contributes to pathology and ill-being" (p. 74). Learners' behaviour and their full functioning during (mathematics) lessons depend accordingly on the fulfilment of both basic physiological (Hull, 1943) and basic psychological needs (Vansteenkiste et al., 2020). Whereas the satisfaction of primary physical needs for, for example, hunger and thirst, leads to a direct reduction of these needs, basic psychological needs for autonomy, competence, and relatedness are permanent and are capable of extending or intensifying under certain favourable cultural and social conditions (Krapp, 2005, p. 631; Ryan & Deci, 2017).

According to basic-psychological-needs theory (BPNT), basic psychological needs are universal; that is, they are valid across time, age, gender, situation, culture, and social context (Ryan & Deci, 2017; Vansteenkiste et al., 2020). Vansteenkiste et al. (2020) explained further: "The effects associated with need-based experiences should be reflected in myriad cognitive, affective, and behavioral outcomes, while also surfacing at different levels, from the psychological to the neurological/biological" (p. 4). Embedded within the growth-oriented nature of the organismic dialectical metatheory of SDT, basic psychological needs serve as the essential nutrients for studying (mathematics) and for energizing the learner's internalization process and integrative functioning, as well as their active and affirmative involvement in (mathematics) learning (Deci & Ryan, 2016, pp. 15–16). In this connection, *relatedness* reflects the learner's strong sense of security, trust, and bonding in the (mathematics) classroom, for instance with their teacher or peers. *Competence* represents feeling efficacious and confident about engaging in (mathematical) tasks to reach the desired outcome as well as feeling mastery. Moreover, *autonomy* refers to the perception of self-initiating one's own (mathematics) activities, accompanied by a sense of psychological freedom (Deci & Ryan, 2000; Ryan & Deci, 2017, 2020).

Historically, White's (1959) criticism of empirical drive theory (Hull, 1943) describes the functional property of an organism to "interact effectively with its environment" (p. 297) as a part of intrinsically motivated behaviour. However, although the need for competence could clarify various motivational forces of intrinsic motivation, it does not entirely explain why people do not engage in activities for which they have the required competence. In this connection, deCharms (1968) emphasized that in addition to the need for competence it is also important for the learner to feel autonomy, arguing that if

> a major factor in the intrinsic dimension is the desire for personal causation, then intrinsically motivating tasks are those in which the person feels that he [i.e., the individual] is in control, that he [the individual] originated the behaviour (as an Origin) with the concomitant feelings of free choice and commitment. (p. 329)

The early studies of Deci (1971) confirmed the importance of these dual basic needs for competence and autonomy to intrinsic motivation and its maintenance over time (Ryan & Deci, 2017, p. 120). Next to the inspiring work of White and deCharms, research with (young) children has stressed that they exhibit intrinsically oriented exploration and childlike curiosity when they are encircled by the feeling of relatedness; that is, warmth, caring, security, and bonding. It is fulfilled through integration and experiencing themselves as significant to others (e.g., their parents or similar supportive carers, Deci & Ryan, 2000; Ryan & Deci, 2017; Vansteenkiste et al., 2020).

Guided by the definitions of the most important elements of SDT in relation to this present research, I now review SDT-framed research on the educational outcomes that correlate with the regulatory styles and the satisfaction/frustration of the basic psychological needs.

2.1.2 SDT's Research to Date on Learners' Classroom Motivation

General Overview: Why Do Learners Experience a Certain Quality of Motivation in the Classroom? Over the last decades, a multitude of studies within SDT have observed very interesting findings about learning outcomes (e.g., behavioural, affective, and cognitive), outcomes that follow the expected pattern regarding the experienced regulatory styles and basic psychological needs in academic contexts. I illustrate a few results to point out why it is important that learning is autonomously motivated and what difference it makes in this connection to perceive a personal relevance to learning.

According to numerous studies of beneficial outcomes, intrinsic motivation can be understood as a desirable form of motivation that fosters students' (mathematics) learning process in the classroom. Emanating from the self, a student's passion for learning (mathematics) is accompanied by spontaneous cognitions and affects about learning that lead to high-quality performance (Ryan & Deci, 2017; Taylor et al., 2014), a high level of concentration (Deci, 1992) and, especially, perseverance when meeting obstacles (Vansteenkiste et al., 2018). The concept of interest (Krapp, 2000, 2002) is closely aligned with intrinsic motivation (Bikner-Ahsbahs, 2005; Deci, 1992; Krapp, 2000, 2002), whereas the flow experience (Csikszentmihalyi, 1975, p. 36) may reflect the purest version of intrinsic motivation, where the learner joyfully gravitates towards an autotelic learning activity and the experience they associate with the object (mathematics) thereby represents both stimulus and reward simultaneously (Deci, 1992). As indicated above, the satisfaction of autonomy and competence nurtures intrinsic motivation (Deci & Ryan, 2000). However, beneficial outcomes also result from autonomous extrinsic forms; namely from integrated and identified regulation. Vansteenkiste et al. (2018) argued in this relation that the notion of personal relevance is critical for both forms, integrated as well as identified regulation.

As indicated above, integrated regulation reflects the advanced form of autonomous extrinsic motivation that leads to a completion of the learner's internalization process. However, integrated motivation requires self-reflection, self-awareness, and great effort on the part of students in terms of expressing who they are and what they value as individuals (Deci et al., 1991; Vansteenkiste et al., 2018). Accordingly, this motivational behaviour requires a certain maturity and occurs mainly at advanced levels of development; that is, the adult stage (Deci et al., 1991). According to Sheldon and Kasser (2001, p. 492), individuals who feel integrated strive willingly to attain a content that satisfies their psychological needs. Younger students, in particular, are not mature enough to associate a sense of integrated value with academic content. Based on these characteristics, students cannot easily attain integrated regulation within the everyday routine (Vansteenkiste et al., 2018) in the (mathematics) classroom and thus it is not easy for teachers to foster that regulation.

In the case of identified regulation, this behaviour co-exists with personal care, better performance (Barkoukis et al., 2014; Ryan & Deci, 2000a), and perseverance, as well as sustained energy (Ryan & Deci, 2017) as the learner links the important values they personally hold (Vansteenkiste et al., 2018) with their motives to study. Here, learners feel ready to overcome various obstacles that the subject (mathematics) or its tasks represent in order to achieve their personally significant outcomes (Vansteenkiste et al., 2018, p. 35). Further, students cope proactively with disappointments in learning situations (Deci & Ryan, 2000). Theoretically, all three basic psychological needs are satisfied in the presence of identified regulation (Deci & Ryan, 2000; Vansteenkiste et al., 2018). The experiences of competence and relatedness are critical to students' starting to own externally offered values (i.e., beginning of internalization; Deci & Ryan, 2000). The need for competence emerges as a need-substitute (Ryan & Deci, 2017) when, for instance, learners want to compensate for their lacking sense of autonomy by displaying power solely because they want to overcome a sense of being confined (Martela et al., 2019, p. 3; Vansteenkiste et al., 2020). The need for competence is also characteristic of ego concerns in particular (Haerens et al., 2015, p. 33). Along similar lines, striving towards a sense of secure relatedness is desired (as a compensatory strategy, Ryan & Deci, 2017) particularly when situations bring feelings of need frustration, a feeling of being controlled, indifference, or having no true choice, as well as being overburdened (Vansteenkiste et al., 2020). Accordingly, to awaken a sense of personal relevance, the social context (e.g., of mathematics) should specifically satisfy a feeling of autonomy, not only of competence and relatedness (Deci & Ryan, 2000; Vansteenkiste et al., 2018).

Altogether, this specific instrumental behaviour—identified regulation—helps learners to become self-determined (Deci & Ryan, 1995), and as such the more internalized the motivation, the more the mathematics activities' personal relevance becomes part of a learner's identity. Thus, referring to this study's context, the properties of this regulatory style, make it *optimal* for motivating students and thus feasible for teachers to use in terms of fostering students' process of internalization in a mathematics-learning environment.

Speaking of the support of internalization, the needs of competence and relatedness suffice to meet introjected regulation. Although the learner has partially accepted the reason for learning (no external prompts are required; Vansteenkiste et al., 2018), the activities remain conflictual to the self. That is, reasons to study, for example being a smart, hard-working student in order to feel competent in mathematics, have not been identified as self-chosen personal values, namely as being a part of the individual learner (Deci et al., 1991, p. 329). In this connection, the reasons for the learners' behaviour, such as anxiety (Ryan & Deci, 2000b) guilt, shame (Bartholomew et al., 2018), or striving towards self-worth to avoid failure, are within the learners themselves (Ryan & Deci, 2017). Ryan and Deci (2017, p. 186) postulated that this kind of behaviour is especially present in achievement-related fields in which competition and inter-individual comparisons exist. This type of motivation leads to inconsistent self-esteem (Deci & Ryan, 1995), as learners are constantly dependent on others' confirmation of their worth, for example in the domain of achievement, where self-worth functions as a guiding strategy for engaging in the (mathematics) classroom (Ryan & Deci, 2017). Accordingly, it is assumed that this kind of introjected self-perfection is detrimental for learners (Assor et al., 2009). In particular, the confusion of self-worth with achievement that frames the search for self-acceptance in a highly competitive environment is counterproductive for students when they feel unable to keep up with the class, but also feel pressure to sustain competency and avoid failure (Covington, 1998, pp. 78–79, 2009). In summary, in relation to this investigation, not only is introjected regulation (Assor et al., 2009) suboptimal for students' mathematics-learning processes and their personal development, but so is external regulation, as will now be discussed.

Initiated by an external source, learners perform a behaviour that is driven by fear, pressure, punishments, threats, and rewards (Deci et al., 1991), for example mediated by their mathematics teacher or a parent. As indicated above, the external pressure drives the learner so that they only perceive a low relevance (e.g., a socially valued one) in schoolwork. The externally regulated student complies with expectations as long as the contingency lasts. That is, without the expectations in the classroom that control learners' behaviour, they would not maintain their behaviour in lessons over the long term and would withdraw from the learning activity, to which little attention and effort was devoted anyway (Deci & Ryan, 2000; Ryan & Deci, 2017). Occurring as the inverse of intrinsic motivation (Deci & Ryan, 2000), this type of motivation is predictive of ill-being and poor academic performance, as a consequence of basic-psychological-need frustration (Warburton et al., 2020).

Complete lack of basic-psychological-needs satisfaction leads to an experience of reactance (Brehm, 1966):

> if a person's behavioral freedom is reduced with reduction or threatened with reduction, he [the individual] will become motivationally aroused. This arousal would presumably be directed against any further loss of freedom and it would also be directed toward the re-establishment of whatever freedom had already been lost or threatened. Since this hypothetical motivational state is in response to the reduction (or threatened reduction) of one's potential for acting, and conceptually may be considered a counterforce, it will be called "psychological reactance". (p. 2)

Further, the absence of a personal intention induces the students to quit and display amotivation (Bartholomew et al., 2018). In most cases, this takes place after the student has experienced a period of consistently negative feedback and poor performance on their part (Deci & Ryan, 1985). This strongly predicts poor learning outcomes and reduced well-being which culminates in personal helplessness (Abramson et al., 1978, p. 52).

Focusing on the pervasive influences, diverse cultures constitute unique pathways for individuals to satisfy their psychological needs that nurture the process of internalization (Ryan & Deci, 2017). What is more, an individual's motives, orientations and identity are contextualised by cultural norms and values (Ryan & Deci, 2017; Vansteenkiste et al., 2020). As a result, the regulations affecting the individual—both social and personal, such as their motives and aspirations— are also a product of the culture in which that individual lives. Thus, each learner follows a distinct path towards well-being and social integration (Ryan & Deci, 2017). Because different types of content differ on a cross-cultural level, they relate to a greater or lesser extent to basic psychological needs and are correspondingly integrated (Chirkov et al., 2003; Ryan & Deci, 2017). However, many SDT-framed studies across cultures (to name but a few: Chen et al., 2015; Chirkov et al., 2003) showed that regardless of cultural communities (e.g., individualism versus collectivism, Chirkov et al., 2003), associations between basic psychological needs and wellness exist. It is especially worth noting that autonomy is necessary for well-being whatever the circumstances, and regardless of the place of autonomy in the cultural context. Specifically, even in cultures where autonomy was not valued, the empirical evidence showed it was essential to well-being (Ryan & Deci, 2017). Therefore, SDT is particularly helpful in terms of understanding how individuals internalize content that they have accumulated through cultural and social channels, as well as the ways in which the cultural context encourages autonomy. In relation to this current investigation, it would be interesting to see how far the cultural elements within mathematics curricula contribute to and provide a frame for studying mathematics autonomously and promote increasing internalization (see section 2.4).

To conclude, from the empirical evidence to date, it appears that there is no doubt about the benefits that come with autonomous motivation for students' learning processes and their personal development. Learners need to recognize the personal value of mathematics learning to assimilate increasingly externally offered contingencies and to display autonomous behaviour. SDT postulates that the robustness of a certain motivational direction and learners' internalization process (that of transferring contingencies from external sources into their own) depends on the involvement of learners' inherent basic psychological needs (Deci et al., 1991). In this regard, the experiences of competence and relatedness are indeed vital for the learner's internalization or to start it. However, to experience deep-rooted personal importance and to put one's heart into learning situations in mathematics, the satisfaction of autonomy is essential. Accordingly, learners effectively experience mathematics learning as personally relevant when the learning environment fulfils their three basic psychological needs (Vansteenkiste et al., 2018) or the experience of autonomy in particular. In a nutshell, within mathematics-learning situations, autonomous types of motivation come with the benefit of cultivating each individual student's natural interest and encouraging it to grow. In comparison to the other types of autonomous motivation (intrinsic or integrated regulation), the teacher's external endeavour can address identified regulation by helping students to understand the value and personal relevance of mathematics learning. Thus, to the extent the learner perceives mathematics learning as important, personally relevant, and self-determined, they will identify with the external mathematical content they are learning and actively internalize it through *identification* (Vallerand et al., 1992, pp. 1006–1007). For this reason, it is a matter of particular interest which mathematics-related personal relevance learners associate with identified regulation.

Importance of Students' Personal Relevance from SDT's Perspective. Scholars within SDT have investigated through varied research settings the process of internalizing by means of the concept personal relevance. Thereby the notion of personal relevance has been addressed using different relevance-related terms, for example rationale, goal, personal meaning, or self-relevance. To give few examples of such a studies, Jang (2008) found that the recognition of rationales provided through an autonomy-supportive climate during uninteresting learning contexts produced positive outcomes for learning, motivation, and engagement. Thereby, teachers explained the value of uninteresting learning activities to their students (college students from educational psychology class) within statistics lessons, so that the participants experienced increasing internalization over time. These rationales were transferred into self-endorsed motives (identified regulation) and induced learners to make an effort for the still-boring learning materials.

The feeling of being involved in classroom processes and having one's opinions taken seriously was also found to be supportive when Steingut et al. (2017), examined within a meta-analysis (including the academic context in that analysis) the unique properties of the rationales provided to the sample (participants were from various developmental stages—elementary, middle, and high school, college, and adult). Their findings were similar to those of previous research carried out within an educational context with early adolescents (Vansteenkiste et al., 2005) in that the specific type of rationale determines the impact of such rationales, and in this relation prosocial and autonomous rationales (e.g., community- and helpfulness-related goals) foster greater engagement and performance than do controlling rationales (e.g., goals relating to money, marks, and social status). Steingut et al. (2017) claimed that prosocial rationales correlate positively with the satisfaction of the basic psychological needs, which were associated with beneficial learning outcomes. Interestingly, their results pointed out that rationale provision may have a negative effect on the competence need. Referring to Hulleman et al. (2010, p. 881), who conducted their study with college students of mathematics and psychology, Steingut et al. (2017) assumed that a decrease in the learner's perceived competence might be associated with low performance. In that case, students might then feel unable to cope and engage when the teacher emphasizes the great importance of the subject mathematics. These results indicated that not all students profit from rationale provision and that, in mathematics, rationales can be experienced at the same time in different ways. To benefit everyone in the class, teachers need to frame the relevance so as to highlight it in light of the social conditions in the classroom and thus begin to understand learners' own rationales first, before providing or highlighting any other rationales in the mathematics classroom.

Referring to classroom conditions, Assor et al. (2002, p. 272) stressed that certain teacher behaviours in elementary schools—"fostering relevance" as well as "suppressing criticism" and making an effort to comprehend their students' subjective values and learning—can support learners' experiences of autonomy in the classroom. In this line, Reeve (2016) underscored the importance of the teacher–student relationship, and Haerens et al. (2016) emphasized the avoidance of controlling teaching styles (e.g., controlling communicative approaches) in order to foster an autonomy-supportive learning environment in the classroom. Controlling teaching (Haerens et al., 2016; Reeve & Cheon, 2021) can be differentiated into *externally controlling* ways of controlling learners' behaviour (e.g., using language that pressures the student, such as "*You have to*!") and *internally controlling* ways, such as psychological control (e.g., expressing disappointment, triggering shame and a sense of guilt). Haerens et al. (2016) also argued that teachers should refrain from using controlling and pressuring behaviour in cases where, for example, the students are not following instructions or advice, as such behaviour frustrates the learners' basic psychological needs. Such frustration in turn associates with controlled motivation and amotivation (Haerens et al., 2015). In particular, a sense of intimidating obligation takes an emotional toll on the student, affecting their well-being and energy, as well as leading to poor performance (Haerens et al., 2016, p. 60).

To conclude, a focus on the students' relevance, or rationales that are student-centred and support the experience of autonomy, as well as autonomy-supportive teaching styles that foster a positive teacher–student relationship contribute to satisfying learners' basic needs experience. These factors in turn foster their processes of internalization and their striving towards autonomy, which is a constant that is independent of the developmental stage (Assor et al., 2002). In a recent article, based on theoretical and empirical considerations, Vansteenkiste et al. (2018) provided a well-elaborated broad overview on the role of personal meaning and self-relevance for promoting students' learning processes. In this connection, Vansteenkiste et al. (2018) not only placed emphasis on highlighting self-relevance in learning contexts, but also discussed the location of personal meaning and self-relevance in terms of the taxonomy of motivation. As an attempt to conceptualize the relations between self-relevance and the internalization continuum, they argued that "although all types of extrinsically motivated activities may be important to people in a very general sense, the notion of self-relevance applies only to the more internalized forms of extrinsic motivation" (p. 33).

2.1.3 Critical Reflection on SDT

In the following, the elaborated theoretical lens of SDT is discussed in terms of its potential to provide answers for this study's main purpose. According to Radford (2008a), a theoretical lens can be viewed "as a way of producing understandings and ways of action based on" *basic principles* (*P*), *methodology* (*M*), and *research questions* (*Q*) (p. 320). Within this tripartite view (P, M, Q) of a theoretical lens, P embraces the system of principles, M is a set of methodologies (technical procedures of data collection and interpretation) facilitating P in producing relevant data, and Q is a set of research questions. These three elements are closely interwoven with each other and help not only in conceptualizing theories (how is understanding produced?), but also in exploring a theory's *boundary*—that is, the limit of the theory's principles. If such limitations emerge in the context of the intended investigation, "Connecting theories can (…) be accomplished at different levels (principles, methodology, research questions), with different levels of intensity" (Radford, 2008b, p. 14). Next, I will consider SDT's system of principles in order to identify the theory's strength and boundary in terms of this study's research interest; that is, by clarifying the conceptual relation between learners' personal relevance (individual relevance systems) and their motivational tendency in the mathematics classroom.

P_1: Briefly, by following an organismic dialectical metatheory, SDT's lens focuses on the understanding of and increasing knowledge about learners' quality of motivation;[7] that is, the kind of motivation, not the amount. In doing so, SDT distinguishes between the general categories of intrinsic and extrinsic motivation in order to emphasize the motivational dynamics underlying the process of internalization, that is, the function of increasing ownership reflected by the extrinsic regulatory styles: external, introjected, identified, and integrated regulation. P_2: Thereby, learners' social context can promote intrinsic motivation and internalization by nurturing the fundamental basic psychological needs of autonomy, competence, and relatedness, which are vital for learners' sense of self, which is analogous to

[7] Again, SDT's theoretical principles are not restricted to the enterprise of schooling (Ryan & Deci, 2017; Vansteenkiste et al., 2020).

SDT's metatheory. These theoretical principles explicitly highlight in this relation the role of social cultural context, which affects the learner's growth and learning in an ongoing dialectic between the learner's basic psychological needs and their social environment (Ryan & Deci, 2000a). Historically, this grand theory emerged over the decades through multiple international research contributions from different contexts. There have been a limited number of qualitative studies that have been important, for instance, to developing the survey instruments (e.g., regulatory styles), but most of the research findings are based on surveys. Thus, SDT's general principles, which have been proven to be robust and predictive, are frequently researched within a quantitative research paradigm (Ryan & Deci, 2020, pp. 17–18). To conclude, the broad framework of SDT seeks to understand the variability in the social cultural conditions that foster individuals' basic-psychological-needs satisfaction in terms of the quality aspect of motivation and differences in personality. In relation to this present study, due to certain limitations, SDT cannot thoroughly explain the interrelations between learners' personal relevance and their motivation.

Based on the well-elaborated knowledge to date, SDT provides an excellent theoretical foundation for understanding learners' classroom behaviour in connection to their individual experiences of basic psychological needs, whereas the central social experiences of classroom motivation are not discussed in the same depth. As indicated above (cf. 2.1), the dialectic between the individual and their social context represents the nutrient medium for an organism's development (i.e., natural growth orientation). Moreover, Vansteenkiste et al. (2018) stated that the notion of personal relevance can only associate with autonomous forms of motivation. Does learners' personal relevance only occur in connection with identified or more internalized motivation? What if learners value the importance of belonging to their mathematics classroom? Would that not be personally meaningful in terms of the organismic integration within SDT? To clarify these questions, it is necessary to find a theoretical lens that explains the development of learners' personal relevance in the mathematics classroom and is compatible with SDT's theoretical conceptualization. To this effect, I include the theoretical approach that is used to understand interpersonal interactions within the classroom, namely social constructivism (see section 2.2.1).

Further, several studies, and in particularly the sophisticated review conducted by Vansteenkiste et al. (2018), have highlighted the importance of personal meaning/importance/self-relevance for the process of internalization and autonomous motivation. They jointly pointed out the significance of understanding exactly what is relevant to learners, instead of providing or clarifying meanings from the teacher's point of view, or the socially valued relevance of learning. Thereby, scholars stated the need for more research (e.g., intervention studies, Vansteenkiste et al., 2018) in future to explore the effect of personal relevance or rationale on motivation (Steingut et al., 2017).

I claim, in this connection, that in order to increase our knowledge of the relationship between the relevance of learning for learners and their motivation, we initially need to clarify the conceptual understanding of learners' personal relevance. To the best of my knowledge, within SDT there exists no theoretical specification of the concept *students' personal relevance* in an academic context. The review of the literature to date (cf. 2.1.2) showed that various scholars have applied this term (relevance) in connection with different notions (e.g., rationale, personal meaning/significance, or self-relevance). Scholars compared their evidence in this research without clarifying and contrasting their fundamental theoretical conceptualizations of these notions (e.g., Vansteenkiste et al., 2018). Recently, Ryan and Deci (2017, pp. 252–254) discussed the general notion of *meaning*, in order to point out that meaning cannot be considered as a basic psychological need. In their view, meaning occurs as a consequence of those needs being satisfied. They postulated that from the perspective of SDT, meaning can be considered as an outcome of the individual's tendency to grow (process of intrinsic motivation) and interconnect with their social worlds (organismic integration). However, with regard to learners' relevance, there is still no clear consensus about the conceptual definition within SDT's research landscape. Although the concept of meaning within SDT seems to theoretically relate to learners' personal relevance, previous scholars have not explained how learners construct personal relevance in learning situations and which factors affect this construction. A clear and well-defined conceptual framework is critical to exploring the relations between personal relevance and motivation. For this reason, a concept is necessary which adopts learners' perspective when learning mathematics, in order to explain their construction of personal relevance with reference to their social context in the classroom and their individual experience of basic psychological needs (to explain motivation). The motivational construct "personal meaning" seems to encompass a theoretical conceptualization that will serve this study's purposes (see section 2.2.2).

In the following, I initially provide a general overview of the theoretical perspective of social constructivism and subsequently introduce the theoretical conceptualization of personal meaning.

2.2 Learners' Construction of Personal Meaning in the Mathematics Classroom

In what follows, the general theoretical understanding of the social constructivist approach is delineated.

2.2.1 The Social Constructivist Perspective

Building on the pioneering research of Vygotsky (1962, 1978, 1987), the perspective of social constructivism helps us to understand and enhance the learning processes that take place in classroom interactions (Hickey, 1997, p. 175). The specific focus of this lens is on "the impact of collaboration, social context, and negotiation on thinking and learning" (Hickey, 1997, p. 175). Referring to concepts from anthropology and sociology, the social constructivist philosophy views each student as indivisibly connected to their surrounding social environment (Ernest, 1998, 2010; Hickey, 1997). This perspective follows the guiding idea of "assisted learning" (Hickey, 1997, p. 175) that emerges within the concept of the *zone of proximal development* (ZPD). Vygotsky (1978) stated: "what children can do with the assistance of others might be in some sense even more indicative of their mental development than what they can do alone" (p. 85). ZPD refers to instances in which a learner, who cannot master a task with their current skills, is "scaffolded" by a more skilled person, who helps them to reach their potential or proximal goal (Ernest, 2010; Hickey, 1997; Vygotsky, 1978). In this spirit, *scaffolding* reflects teachers' supporting learners in the process of internalizing socially accessible knowledge and proficiencies in order to grow and become independent (Bakker et al., 2015; Vygotsky, 1978, p. 130). Without the teacher's guidance the internalization would be too demanding for students to master their proximal goals (Bakker et al., 2015).

Within the Vygotskian theoretical space, the individual's personal development is thereby affected and formed twice, namely through the social and individual internal development processes (Vygotsky, 1978). As a result, we can see the growth and development of the learner's intellect and mind-set as coming from an internalized dialogue with their social world (Ernest, 1998, 2010). Vygotsky's implications (1987) emphasized the vitality of such conversations for the individual's development of knowledge and learning. Guided by Vygotsky's theoretical enterprise, Ernest (2010) proposed a cycle of appropriation, transformation, publication, and conventionalization (see Figure 2) to illustrate the mutual dialogue between the social and the individual that contributes to students' learning; that is, "the development of mind, personal identity, language and knowledge" (Ernest, 2010, p. 44).

Figure 2

Model of Sign Appropriation and Use (adapted from Ernest, 2010).

		Social Location	
	Individual		Collective
Public	Individual`s public utilization of sign to express personal meanings	Conventionalization →	Conventionalized and socially negotiated sign use (via critical response & acceptance)
Ownership	Publication ↑		Appropriation ↓
Private	Individual`s development of personal meanings for sign and its use	Transformation ←	Individual`s own unreflective response to and imitative use of new sign utterance

According to Ernest (2010, pp. 44–45), this scheme illustrates a cycle on the whole where meanings located within the *individual and private* realms as well as utterances within the *collective and public* realms are reciprocally modelled by means of *internalized* conversation (Ernest, 1998). It demonstrates how individuals come to personally appropriate signs by perceiving their prevalent social use. First and foremost, the initial process is that of *appropriation*, in which a learner reacts without reflecting on common usage, and models themselves and their behaviour on well-worn routines within the relevant practical discourse. On the basis of such experiences, a learner passes through the whole cycle of all four processes of appropriation, transformation, publication, and conventionalization several times, so that the individual perceives the routine of applying and expressing that sign in public multiple times. They learn the appropriate link between that sign and its corresponding implicit rules and the references it implies. Based on the experiences gained from this, the learner constitutes their own meaning for that experienced sign as well as its usage, and through the process of *transformation* the sign becomes an item of personal property which is in private hands. At publication stage, the student is ready to apply the sign within "autonomous conversational"

(Ernest, 2010, p. 45) activities. The meaning of the sign that has been thus produced underlies, then, the *processes of conventionalization*. Here, the expressed sign in the social realm is offered on diverse occasions to receive a response from the public context in the form of renegotiation, critique, or compliance, for example. That is, in relation to ZPD (Vygotsky, 1978), the process of conventionalization "takes place in the visible centre of ZPD" (Ernest, 2010, p. 45) where the individual student is taught through their own experiences and instructed on how to use the sign in the community. Looking at all operations within the cycle, appropriation and publication build the boundary actions that enclose learners' sign-internalization process (receiving and producing signs), which takes place collectively through the dialogue between the individual and the social sphere. Within the privately owned sphere, signs come to be transformed from the collective into the individual sphere by means of the production of personal meanings for the sign and its usage. For this reason, a crucial stage of learning occurs in the private sphere.

To conclude, Ernest argues that one can say that the ZPD (Vygotsky, 1978) includes the overall cycle that explains how an individual learns concept development though signs, that is, how learning and knowledge are produced (Ernest, 2010, p. 45). Signs or texts are a kind of cultural representation (although there are others), and they appear in/are relevant to all four quadrants of the model of sign appropriation and use. Since culture encompasses signs and texts, culture affects learning (Ernest, 2010).

Referring to Radford's system of principles (P), the theoretical stance of social constructivism can be discussed in terms of this present research's main purpose. P_1: Generally, the social constructivist philosophical ethos explains the conversation between the social context and the individual that shapes knowledge production in the mathematics classroom. P_2: Thus it places the social sphere in a central position, as individual social phenomena such as conversation are difficult to break down and define individually (Ernest, 1998, pp.135–136). P_3: On the contrary, the social constructivist approach provides the social experiences of "'objective' and 'subjective' knowledge" that are "based on dialectical, socially situated interpersonal 'conversations'" (Ernest, 1998, p. 136) within mathematics education.

In his seminal work, Hickey (1997) postulated that we need to consider classroom motivation through a social constructivist approach so as to increase our understanding of motivation in mathematics learning. The social constructive perspective on social interactions helps us to understand how learners' social experiences of autonomy and self-determination are brought about by interpersonal relations between different people in the mathematics classroom. This lens might become the counterpart of learners' individual classroom experiences in that it could help us to comprehensively understand the development of the learners' individual relevance systems in terms of their motivation in mathematics. The motivational construct personal meaning seems to be grounded on a theoretical conceptualization that synergizes both learners' individual and social experiences into their personal relevance construction in mathematics. Before I discuss these properties in more detail (see section 2.3), I introduce the theoretical conceptualization of the motivational construct *personal meaning* in general, which reflects the personal relevance learners attach to learning mathematics.

2.2.2 The Motivational Construct of Personal Meaning

Generally, all humans *search for meaning* (Frankl, 1978) and desire to have a meaningful life (Bruner, 1990; Frankl, 1978; Ryan & Deci, 2017; Vinner, 2007). The *need for meaning* is fundamental and concerns our longing to understand ideas that were once unknown to us (Combe, 2014). Meaning satisfies this experience of coming to understand something (Combe, 2014; Gebhard, 2014), and Blumenberg expressed this succinctly when he stated that the need for meaning is a demand for a "legibility of the world" (German: *Lesbarkeit der Welt*, Blumenberg, 1981, p. 10, translated by the author). A perceived lack of meaning may have an adverse effect on the individual's well-being, for example by leading to depression (Biller, 1991, p. 105).

The striving towards meaning is not only common within psychology, however. It is also vital in an educational setting, where students need to experience meaning (Vinner, 2007). In this connection, by referring to Postman (1966), Vinner (2007) stressed "that if education does not challenge the students with meaningful goals it will die" (p. 3). Based on the general need for meaning, which must be in place if learning is to be successful, scholars from a Research Training Group (German: *Graduiertenkolleg*), "Bildungsgangforschung", sponsored by the German Research Foundation (DFG), discussed the issues around the meanings attached by students to their personal biographies and to different subject areas, as well as their feelings about lessons at school. They intended to study which conditions affect the construction of meaning from the learner's perspective, and how they do so. Thereby, they specifically considered students' biographies (that is, life stories and longer-term considerations) and the debate about subjective and objective requirements learners need to face when learning (Koller, 2008). Through this discussion, several researchers from different disciplines (e.g., physical education, physics, linguistics, and mathematics education) discussed the topic of learners' personal meaning in an educational context (see Koller, 2008 for more). Vollstedt and Vorhölter (2008) elaborated the notion of meaning within mathematics education. Next, I will present the motivational construct of personal meaning with regard to mathematics education.

Learning mathematics within a classroom is in its nature a collective and cultural phenomenon (Cobb et al., 1992). However, the teacher's impulses and the mathematics activities engaged in in class are not collectively experienced in exactly the same manner. Within one learning environment, the teacher's instructions are perceived and interpreted differently, as students ask themselves: "*What is the personal relevance of learning mathematics for me*?" (Kilpatrick et al., 2005; Vollstedt, 2011b). Dependent on their biographical lens, each learner actively constructs a subjective personal relevance for the school subject mathematics that fits their own personal life story and identity (Blumer, 1969; Cobb et al., 1992; Mason & Johnston-Wilder, 2004). The individual world that is constructed in this fashion functions as a *mental compass* providing students with their own meaningful learning orientation in the mathematics classroom. Hence, it does not matter how well prepared mathematics educators and their concepts of teaching styles are as long as they fail to connect to those individual "worlds" that direct students in the classroom (Blumer, 1969). Accordingly, teachers should not just understand how they personally perceive the subject of mathematics and then assign their own perspectives to students; rather, they should understand how mathematics learning appears to those students, so that the stimuli they provide can successfully match the individual mental compass that each student uses to study mathematics. To stress this notion, Blumer (1969) stated:

> The contention that people act on the basis of the meaning of their objects has profound methodological implications. It signifies immediately that if the scholar wishes to understand the action of people it is necessary for him [them] to see them. Failure to see their objects as they see them, or a substitution of his [their] meanings of the objects for their meanings, is the gravest kind of error that the social scientist can commit. It leads to the setting up of a fictitious world. Simply put, people act toward things on the basis of the meaning that these things have for them, not on the basis of the meaning that these things have for the outside scholar. (pp. 50–51)

However, it is not an easy task to capture those different student voices in one classroom when the teacher does not have access to the individual learners' worlds (Blumer, 1969). Referring to this issue in a mathematics-learning context, Vollstedt and Vorhölter (2008) jointly developed the theoretical conceptualization of *personal meaning*, a motivational construct reflecting the personal relevance that learners associated with studying mathematics, and its relation to the activities, content, and people involved (e.g., teacher or peers) in an educational context. Initially, Vollstedt and Vorhölter used existing theoretical as well as empirical evidence to formulate the theoretical concept behind personal meaning (Vorhölter, 2009). Subsequently, two qualitative studies, focusing on the relations between German students' personal meanings and modelling tasks in mathematics (Vorhölter, 2009) and on the different kinds of personal meaning in Germany and Hong Kong (Vollstedt, 2011b), contributed profound and solid knowledge which enriched and refined the conceptualization of personal meaning (Vorhölter, 2008). Both studies were based on Grounded Theory (Strauss & Corbin, 1996).

Briefly, three aspects are essential within the methodology of Grounded Theory (Strauss & Corbin, 1996). Firstly, the type of coding is theoretical. That is, theoretical concepts are constituted that have an explanatory value for the examined phenomena. Secondly, sampling sources for the source data are identified, from which the theory is developed, and further samples are carried out successively and iteratively throughout the entire research process (*theoretical sampling*). Thirdly, as a result of the constant comparisons between the phenomena and research contexts, the theoretical concepts emerge; that is, these sensitizing concepts are empirically based results.

Vorhölter's study considered students from grade 10, whereas Vollstedt studied ninth and tenth graders' kinds of personal meaning. Since this present investigation focuses on the interrelations between the patterns of personal meaning (individual relevance system) students assign to mathematics learning and quality of motivation, I mainly concentrate on the evidence of Vollstedt's study, as elaborated on below.

Figure 3

Theoretical Concept of Personal Meaning (adapted from Vorhölter, 2009; see also Vollstedt, 2011b).

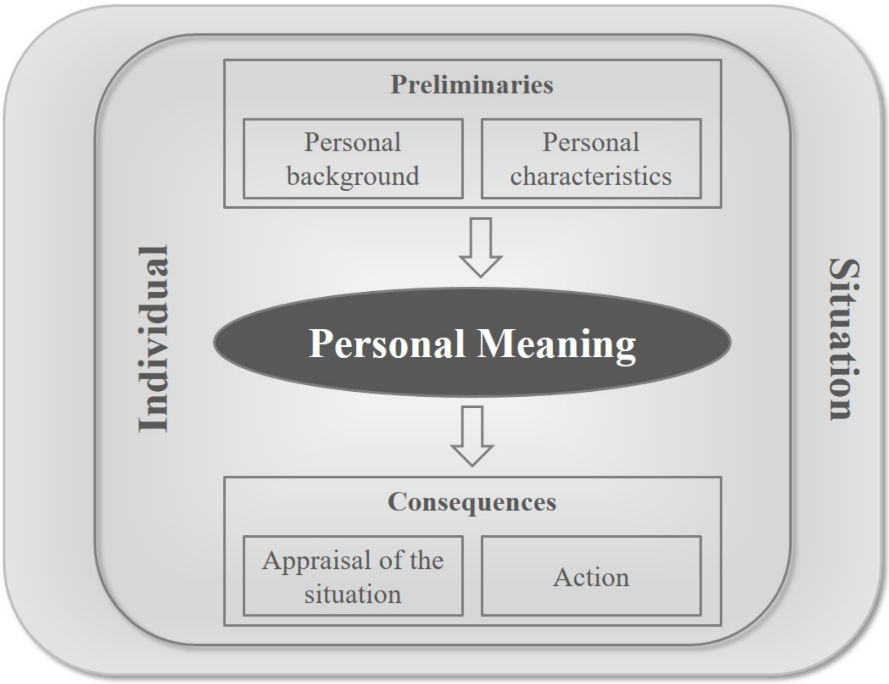


Figure 3 depicts the theoretical conceptualization of personal meaning. The subjectively perceived personal meaning can assume the shape of a personal aim, purpose, goal, value, or a benefit that the individual associates with an object or an action (Vollstedt, 2011b; Vollstedt & Duchhardt, 2019; Vollstedt & Vorhölter, 2008; Vorhölter, 2009). By adopting the perspective of the *individual* learner, this conceptualization describes the construction of personal meaning in the context of the *mathematics-learning situation*. Within this learning context, students' construction of personal meaning is individually affected by preliminaries, classified as their personal background (stable properties) and personal characteristics (influenceable properties). Learners' personal background refers to attributes (e.g., gender, family, cultural and social contexts, and age) that belong to the individual and as such cannot be influenced by themselves or the outer environment. On the contrary, learners' personal characteristics are variable by nature, referring to concepts from different scientific disciplines, such as educational psychology (interest, Krapp, 2002; basic psychological needs for autonomy, competence, and relatedness, Ryan & Deci, 2017; Vansteenkiste et al., 2020), educational science (developmental tasks; Havighurst, 1972), and particularly with regard to mathematics education (beliefs; Törner, 2002). Based on these prerequisites as well as the learning settings in the classroom, students individually construct a personal meaning that is accordingly associated with the evaluation of the experienced situation and the consequences of their action, which may be either supportive or obstructive to mathematics learning. Learners' need for meaning is permanent, and as such for each (new) item of mathematics-learning content the personal relevance has to be continually interpreted and subjectively constructed anew (Fischer & Malle, 1985). A specific construction cannot persist unaltered when the student's learning situation and the influences on their personal characteristics change. These changes can make their own demands on subsequent situations; for instance, the factors that affect the construction in a certain way can become personal requirements for the learners and influence their personal development (Vorhölter, 2009). Accordingly, the teacher cannot impose a personal meaning from outside (Meyer, 2008), but can only assist their students to recognize their own personal meaning so that they can find the mental compass that supports them in their personal development.

To conclude, the construction of personal meaning involves an interaction between individual (endogenous personal characteristics) and social (exogenous to the mathematics-learning situation in the classroom) factors (cf. Figure 3). Moreover, personal meaning in its nature can become conscious, albeit not always (Vollstedt, 2011b; Vorhölter, 2009). Vollstedt (2011b) and Vorhölter (2009) postulated that not only may different learners simultaneously assign different meanings to the same mathematics context, but also that one learner can associate various personal meanings to a specific learning situation. However, what kinds of personal meaning do learners construct in mathematics-learning situations?

With regard to this issue, Vollstedt (2011b) identified different kinds of personal meaning students related to the subject mathematics. As mentioned earlier, in a qualitative study, 34 students, ninth and tenth graders (aged 15–17), from Germany and Hong Kong (17 from each country) were interviewed to investigate the influence of culture on the construction of personal meaning, in order to further validate and refine its theoretical conceptualization. The interviews conducted by Vollstedt started with a sequence of stimulated recall (Gass & Mackey, 2000). Thereby the students watched a short video sequence (5–10 min) of their last mathematics lessons. The sequence was intended to provide an example of a situation in which the learners dealt with something new for them. This situation both reaffirm existing kinds of personal meaning and help construct new ones. Then, the scholar asked the students to reflect on the thoughts they had when they were attending the lesson and while watching the sequence. Within subsequent interviews (approx. 35–45 min) several topics inspired by the relational framework of personal meaning were elaborated (cf. Figure 3). Thereby, the scholar discussed questions like: How did you like this mathematics lesson? What was especially interesting? What feelings do you relate to mathematics lessons? Why do you learn mathematics? What can mathematics be used for? (see Vollstedt, 2011a for more). Accordingly, learners' social experiences in the classroom were individually reflected upon within the interviews. As a central result, Vollstedt reconstructed 17 kinds of personal meaning with relation to mathematics learning in an educational context. The 17 kinds of personal meaning varied between different notions ranging from *duty* (e.g., I mainly deal with mathematics because I have to) to *purism of mathematics* (e.g., mathematics is beautiful to me as it is unique in its formalism).

In the following, I provide a brief description (see Vollstedt, 2011b, pp. 139–227 for more) of each kind of personal meaning that has been reconstructed on the basis of qualitative empirical data to date (apart from "reference to reality", as explained below).

Active practice of mathematics: When learning/dealing with mathematics, it is personally relevant for the learner to actively deal with the mathematical content themselves. The basic focus lies on their own activity, which gives pleasure to them. Active pursuit can promote the learning process, for instance by being useful in helping the student to prepare for exams or by providing them with a source of enjoyment. The task variety stimulates to deal with them.

Balance and even-temperedness[8]: Mathematics learning is personally relevant for the learner when they feel balance. Engagement with mathematical content is freely chosen during free time or in the classroom, in which the learner feels relaxed while they deal with mathematical tasks.

Classroom management[9]: This type of personal meaning refers to the classroom structure when learning mathematics. The student learns mathematics within a well-organized learning atmosphere in which the teacher is not only someone they trust but also has authority, cares about the learning atmosphere, and can ensure positive classroom management.

Cognitive challenge: This personal meaning has a strong relation to mathematical content and the learner strives towards solving complex and difficult tasks in order to be and feel challenged for its own sake. When the learner experiences mathematics learning as cognitively challenging, they want to improve their performance or to compete with themselves or their peers.

Duty: The learner deals with mathematics as it is a compulsory school subject and to fulfil external (e.g., from family, people who have promised rewards) and their own expectations. They pile pressure on themselves by competing and are well prepared to meet the requirements in examinations.

Emotional-affective relation to the teacher: The student learns mathematics in the classroom due to the affective bond between them and their mathematics teacher. This bond is strengthened through a positive and friendly learning atmosphere, for example by joking. This respect or appreciation for the teacher might also become the basic centre of the learner's attention.

Examination: Mathematics learning is personally relevant as it is necessary to pass unavoidable exams at school. This personal meaning is about both the effects that passing or failing might have (e.g., for present and future education) and the type of exam preparation (e.g., constant/short-term learning).

Experience of autonomy: Mathematics learning becomes relevant when the student can independently deal with mathematics-related tasks/content and decide which activities to work on as well as complete the learning activities on their own in the classroom. A personal characteristic of the individual is here the focus, namely, the desire for self-determination of one's own actions.

Experience of competence: Learning mathematics is personally relevant when the learner can feel competent and successful. The experience of competence is a consequence of certain prerequisites being fulfilled, either through one's own activities (e.g., presenting one's own solutions in front of the others, achieving high marks) or through interaction with others in the classroom (e.g., supporting other students successfully and independently).

Experience of social relatedness: The learner studies mathematics in the classroom to feel integrated in the group. In this connection, friendship can be intensified through

[8] Within a previous study (Suriakumaran et al., 2019) that considered the same data base as this present study, balance and even-temperedness did not fit the data; consequently this personal meaning was excluded from this present research.

[9] As explained below, within subsequent studies a slight change was made concerning the personal meaning "efficiency" from Vollstedt (2011b); it was changed to "classroom management".

mathematics-related or non-mathematics-related activities as well as by solving tasks on the blackboard. Specifically here, the focus is on collective learning.

General education: The student learns mathematics in the classroom to improve their general education in order to participate in society as well as to act as a responsible citizen.

Marks: Learning mathematics is personally relevant to getting good grades that are linked to various goals, such as meeting the learner's own performance standards or feeling competent, evaluating their own qualifications/knowledge, or because further education depends on mathematics performance.

Positive image: Mathematics learning is perceived to be personally relevant in order to gain recognition from significant others (e.g., parents, peers, teachers, or people from the social environment).

Purism of mathematics: The individual has an emotional relation to mathematics itself. Its formal language and logical structure is valued by the learner as it helps students to understand mathematics easily. The applied approach of mathematics in life is perceived as annoying and uninteresting. The student is fascinated by the characteristics of mathematics, such as the following: its answers can be either right or wrong, it is easy to memorize as only formulae are necessary, and it is easy to understand as it is explainable inner-mathematically; i.e., it can be explained using its own terms and frames of reference (and not by taking an applied approach to life).

Reference to reality[10]: Mathematics learning is personally meaningful as it provides and embraces knowledge that can be found in the student's reality. Its connection to everyday life shows the relevance of studying mathematics.

Self-perfection: Mathematics activities are relevant as they help to improve the learner's personal skills and performance-related factors that bring them to perfection. Lifelong learning is appraised as important. So, the learner strives to learn mathematics as they associate thinking logically and thorough understanding with fun and they enjoy mastering cognitive skills. Mathematics learning helps the learner to improve themselves (including their performance) and their self-confidence.

Support by the teacher: Mathematics learning is perceived to be personally relevant when the learner experiences support from their teacher. The support can be realized through interpersonal activities where the learner experiences the teacher's fair and empathetic personal characteristics or through classroom teaching that is affected by a friendly atmosphere, appreciation, and respect.

Vocational precondition: Learning mathematics is personally relevant for the student when mathematical skills are required or perceived as important prerequisites for their desired profession or further education.

Although various concepts were considered as influencing factors during the coding process (see Vollstedt, 2011b), the three basic psychological needs, beliefs, and interest became relevant concepts (in the sense of Grounded Theory) for the construction of the learner's personal meaning (Vollstedt, 2011b; also in the study by Vorhölter, 2009). Whereas in both empirical studies, those of Vorhölter (2009) and Vollstedt (2011b), the concept of basic

[10] This personal meaning did not emerge in Vollstedt's (2011b) qualitative research. As explained below, it was included on the basis of subsequent studies.

psychological needs proved to be relevant, within Vollstedt's study it was possible to reconstruct three personal meanings that reflected learners' personal meanings in relation to EXPERIENCE(ING) AUTONOMY, COMPETENCE, and SOCIAL RELATEDNESS[11] when learning mathematics. Moreover, based on theoretical considerations, Vollstedt (2011b) suggested structuring the 17 personal meanings respectively in terms of the dimensions *intensity of relatedness to mathematics* and *intensity of relatedness to individual* (Vollstedt & Duchhardt, 2019, p. 147). This theoretical procedure led to distinguishing the personal meanings into seven superordinate types of personal meaning: The type *fulfilment of societal demands* encompassed duty, examination, positive image, and vocational precondition. *Interaction with mathematical contents* consists of active practice of mathematics. *Efficient and supportive lesson design* encompassed classroom management and support by the teacher. The type *cognitive self-development* embraced cognitive challenge, EXPERIENCE OF AUTONOMY, purism of mathematics, and self-perfection. *Relevance of application* consisted of application in life, and *well-being due to own achievement* compromised EXPERIENCE OF COMPETENCE and marks. The type *emotionally effected evolvement* contained balance and even-temperedness, emotional-affective relationship to the teacher, and EXPERIENCE OF RELATEDNESS (see Vollstedt, 2011b for more).

In subsequent studies, a research team at the University of Bremen investigated the assessment and the structure of the various personal meanings by means of a survey questionnaire (Büssing, 2016; Schröder, 2016; cf. Vollstedt & Duchhardt, 2019; Wieferich, 2016). Based on these studies, a reliable questionnaire was developed by following a complex validation procedure (see Vollstedt & Duchhardt, 2019 for more). The results revealed further knowledge about the conceptualization and kinds of personal meaning: Büssing's (2016) study found out that an additional personal meaning needed to be considered in order to depict relevance of application (one of the seven types of personal meaning), so "reference to reality" was included as an additional kind of personal meaning. Moreover, within the personal meaning of efficiency, one of its two facets (learners' efficient ways of working) could not be assessed; consequently, this personal meaning was termed "classroom management" to address the remaining scale, namely efficient classroom management (Vollstedt, 2011b).

However, further results showed that a scale revision was necessary as the scale of classroom management could not fit the data due to poorly fitting items (Vollstedt & Duchhardt, 2019). In addition, the model fit values indicated that the two dimensions suggested by Vollstedt (2011b) to structure her typology, intensity of relatedness to mathematics and intensity of relatedness to individual, were not exact counterparts to the 17 personal meanings (reference to reality was not included). Therefore, Vollstedt and Duchhardt (2019, p. 161) recommended that future research investigate appropriate dimensions or meta-factors by reviewing the relations to relevant concepts, for example motivation, or interest, which theoretically affect the learner's construction of personal meaning.

[11] In what follows, the three (specific) personal meanings, EXPERIENCE OF AUTONOMY, COMPETENCE, and SOCIAL RELATEDNESS appear in small capitals in order to differentiate them from the three basic psychological needs.

2.2.3 Critical Reflection on the Concept of Personal Meaning

Referring to Radford's system of principles,[12] the theoretical conceptualization behind this motivational construct addresses the personal relevance learners associate with studying mathematics, which is affected by learners' personal preconditions (personal background and personal characteristics) and their mathematics-learning situation (P_1). The reconstructed personal meaning(s) (which include the later-added reference to reality) cannot be assigned; on the contrary it is constructed by synergizing individual and social experiences within learning situations (P_2). Vorhölter (2009) and Vollstedt (2011b) argued that one learner constructs various personal meanings towards mathematics learning (P_4). Although the theoretical concept of personal meaning is quite new, it has proven to be valid within several investigations in qualitative (Vollstedt, 2011b; Vorhölter, 2009) and quantitative paradigms (Suriakumaran et al., 2019; Vollstedt & Duchhardt, 2019). The theoretical conceptualization of personal meaning can be used to study the effects of mathematics-subject-related features (e.g., modelling tasks, Vorhölter, 2009) and the impact of learners' cultural context on the construction of personal meaning (Vollstedt, 2011b).

Even though Vollstedt's personal meanings have been empirically found[13] in data from Germany (Suriakumaran et al., 2019; Vollstedt, 2011b; Vollstedt & Duchhardt, 2019), Hong Kong (Vollstedt, 2011b), and Finland (Suriakumaran et al., 2019), they cannot (yet) claim a universal existence, as the social cultural context has a huge impact on what relevance learners see in mathematics learning. That is so because the theoretical concept behind personal meaning considers not only the learners' social cultural environment that offers the learning situations, but also the learner as a subject who has a background and long-term perspective shaped by cultural values and goals. Accordingly, within the construction of personal meaning, as explained within the social constructivist approach, a conversation takes place between the individual learner and their social context in the classroom. For this reason, these personal meanings might not exist in all cultural contexts, particularly when cultures follow distinct values, motives, and so on, and likewise further personal meanings may exist within these different cultural contexts that are not detected yet.

In relation to this current research, the concept of personal meaning has several supportive functions. This motivational construct encompasses a well-elaborated theoretical conceptualization and provides various kinds of personal meaning that learners construct when studying mathematics. Moreover, Vollstedt (2011b) and Vorhölter (2009) stated that learners construct various patterns of personal meaning in relation to learning situations. An insight into those patterns, or (here termed as) learners' *individual relevance systems*, might help us to understand what individual relevance systems students construct as their own mental compasses, and which are preferred most or least.

Nevertheless, certain limitations and questions remain that need to be clarified. If patterns of personal meaning exist, it nonetheless cannot be assumed that this insight into learners'

[12] In general, Radford's triplets (basic principles, methodology, and research questions) are intended to explain the conceptualization of a theory. It should be clarified here that those triplets are used in this context to describe the conceptualization of the motivational construct of personal meaning (see also Suriakumaran et al., 2020).

[13] A master's student from the University of Bremen investigated Thai secondary students' personal meanings. However, the measurement accuracy was limited as the scales' psychometric properties were not analysed in detail within her research (Mayer, 2017).

individual relevance systems automatically says anything about the development of those systems. Are there certain conditions within the social context or the individual that affect this construction? Furthermore, which motivational propensity (autonomous or controlled) is associated with each constructed individual relevance system in class? If empirical evidence exists for those individual relevance systems, these patterns of personal meaning can be used to examine the theoretical interplay between individual relevance systems and quality of motivation.

Furthermore, Vollstedt (2011b) stated that in both cultural contexts (Germany and Hong Kong), three personal meanings occurred that indicated learners' personal relevance to EXPERIENCING AUTONOMY, COMPETENCE, and SOCIAL RELATEDNESS in mathematics-learning situations. It is notable that not all the affective psychological concepts that are the ingredients of personal meaning (such as interest or mathematical beliefs) were personally relevant for the learner (Vollstedt, 2011b). Although Vollstedt claimed that these three personal meanings turned out to be particularly relevant for students studying mathematics, it is not clear yet how these three personal meanings are either related to the basic psychological needs or differ from the concept of needs. Basic psychological needs are unique, and are based on nine key criteria, as stressed by Vansteenkiste et al. (2020): "psychological, essential, inherent, distinct, universal, pervasive, content-specific, directional, and explanatory" (p. 4). An overall frustration of these three basic psychological needs leads to ill-being. At this stage, it is important to state that basic psychological needs are not the same as goals, motives (Krapp; 2005; Ryan & Deci, 2017) or, as in this case, personal meanings. Since the basic psychological needs are relevant nutrients for learners' construction of personal meaning, it should be clarified how the concept of basic psychological needs is connected with the constructed personal meanings, particularly with EXPERIENCE(ING) AUTONOMY, COMPETENCE, and SOCIAL RELATEDNESS.

A further clarification refers to the two dimensions of personal meaning proposed by Vollstedt (2011b). The dimensions "intensity of relatedness to mathematics" and "intensity of relatedness to individual" were not exact counterparts to the 17 personal meanings. Since this present investigation refers to motivation in mathematics education, the kinds of personal meaning could be structured along the meta-factors "orientation towards the relatedness of mathematics" and "orientation towards the regulatory styles". This approach might be a possible strategy of relating the kinds of personal meaning to motivation in order to increase our understanding of the relationship between personal relevance and motivation.

The following section 2.3 discusses these specific issues by relating the construct of personal meaning to the theoretical lenses of SDT and social constructivism.

2.3 Contrasting Juxtaposition of SDT and Social Constructivism

"There is nothing so practical as a good theory" (Lewin, 1951, p. 169). Lewin, a seminal scholar whose work led to substantial progress in the field of motivation, stressed that a good theory encompasses the potential to accurately and meaningfully verify a prediction in practice. Due to its exactness and practicability on broad contexts, theory focuses on specific questions (and any relevant factors associated with them) about a subject (Schoenfeld, 2002). Accordingly, one theory cannot provide reliable and predictive responses for all kinds of questions about the researched object of discourse.

On the basis of the theoretical considerations presented above, SDT (2.1) mainly provides a broad framework for understanding the factors behind *why* learners display certain motivational propensities (intrinsic and types of extrinsic motivation) and why these propensities bring benefits but also an emotional cost while learning. In this relation, the concept of basic psychological needs is outlined as an essential key factor within learners' social context (i.e., of the classroom) that has a robust effect on students' individual well-being and quality of behaviour within educational settings. SDT emphasizes the importance of learners' personal relevance (which is close to the individual's sense of self) within the social context to promote students' internalization, satisfy their basic psychological needs, and foster their learning processes. In a nutshell, SDT is a powerful lens with which to discuss the basic needs mechanisms within the individual that belong to the learner's individual biological nature. Although SDT considers the important role of social context in the classroom, it does not provide a well-founded conceptual base to investigate how the social context affects the construction of personal relevance within the classroom context, and it is not clear what specific personal relevance learners construct within their social context when learning (in this case mathematics).

This can probably be clarified through the lens of social constructivism (2.2.1), which explains students' learning as well as knowledge production as a conversation between the individual and social mechanisms in the mathematics classroom. Thus, the cycle proposed by Ernest (2010) can be applied to understand the development of learners' personal relevance (or individual relevance system) through their experiences in the mathematics classroom. That is, in contrast to SDT, this perspective primarily uses a cyclical model of four processes to describe how learners internalize externally offered sources by reflecting their own experiences. Hence, this lens also concerns the fundamental intercommunication between the social context and the individual learner. But in contrast to SDT, it remains unresolved which key factors guide learners' individual "meaning of sign" development processes in private; that is, the processes of appropriation and transformation (cf. Figure 2).

Accordingly, an isolated view of each of the lenses cannot help us to fulfil the main purpose of this dissertation; that is, to clarify the interplay between learners' individual relevance systems and motivation, as both theories have limitations. Therefore, the use of SDT or social constructivism alone would fail to address the subject of this thesis. However, both lenses can be used to view motivation from different angles by dissecting focus-relevant questions with respect to the role of cultural imprinting. That is, building a bridge between both theoretical pathways, the social and individual context, seems to be a promising step to getting an enriched and more meaningful insight into the subject of mathematics-related motivation. Nevertheless, both views are rooted within different research domains and should be treated accordingly. In this connection, the motivational construct of personal meaning that was introduced earlier (2.2.2) seems to have a specific conceptual nature that is "plastic and solid enough" (Star & Griesemer, 1989, p. 393) to function as a bridge between both theoretical worlds (Akkerman & Bakker, 2011) as well as to be understood from both perspectives.

The connection between theories can take place at the level of principles, methodologies, or of questions, or by combining these elements (Radford, 2008b). In this study, I consider the connections at the level of principles. In the following, I provide an argument explaining why personal meaning has the potential to be considered as a so-called "boundary object" (Akkerman & Bakker, 2011) between SDT and social constructivism. In this connection, I provide my theoretical assumptions about how personal meaning can be viewed through both theories' viewpoints. Moreover, I discuss how the nature of personal meaning applies to both *ontological* and *epistemological worlds* (Radford, 2008b) when it is considered through the two perspectives of SDT and social constructivism.

2.3.1 SDT's Perspective on Personal Meaning – Theoretical Assumptions

To investigate how the concept of personal meaning can be understood from the perspective of SDT, it is necessary to clarify the conceptual links between SDT's central elements as they relate to learners' quality of motivation (basic psychological needs and regulatory styles) and (kinds of) personal meaning. A theoretical clarification, at the level of theoretical conceptualization, could explain how the nature of personal meaning appears through the lens of SDT.

Basic Psychological Needs and Personal Meaning. As indicated above (2.2.2) the basic needs concept of SDT was found to be a relevant influencing factor among the personal characteristics that made up learners' construction of personal meaning (Vollstedt, 2011b; Vorhölter, 2009). This knowledge was then reinforced further in both of the relevant empirical studies, namely Vorhölter (2009) and Vollstedt (2011b). Whereas both studies confirmed the involvement of the basic psychological needs within the theoretical conceptualization of personal meaning, Vollstedt (2011b) reconstructed three personal meanings that reflect learners' experience of personal relevance when studying mathematics: EXPERIENCE AUTONOMY, COMPETENCE, and SOCIAL RELATEDNESS. As pointed out earlier, the concept of basic psychological needs and the conceptualization of personal meaning are not equal and thus should not be treated as such. To understand their conceptual relationships, theoretical assumptions are made.

According to the conceptualization of personal meaning, for the construction of a personal meaning—no matter what kind—the basic psychological needs are always involved as relevant personal characteristics. It seems that depending on the constructed personal meaning(s), one can follow that the basic psychological needs are experienced/fulfilled to a certain individual level. For instance, the basic psychological needs seem to be experienced differently when a learner constructs purism of mathematics or duty as the personal relevance of studying mathematics that applies to them. Theoretically, I would expect that (at least) the need for autonomy is satisfied when the learner perceives purism of mathematics as their own personal relevance, while a personal relevance of duty indicates frustration of the need for autonomy. Accordingly, I assume that the individual perception of basic psychological needs essentially contributes to learners' construction of personal meaning in the classroom. By referring to the different notions behind the kinds of personal meaning (cf. 2.2.2), among all the various personal meanings, I presume that the three basic psychological needs (autonomy, competence, and relatedness) are closer to the three which we shall call *specific personal meanings* (to EXPERIENCE AUTONOMY, COMPETENCE, and SOCIAL RELATEDNESS). That is, I assume that through the construction of the three specific personal meanings, the three basic psychological needs are also addressed, as the specific meanings address within their core the notions of autonomy, competence, and relatedness. Within the specific personal meanings, a fundamental connection can be assumed between the personal meaning EXPERIENCE OF AUTONOMY and the need for autonomy, the EXPERIENCE OF COMPETENCE and the need for competence, and the EXPERIENCE FOR SOCIAL RELATEDNESS and the need for relatedness.

If so, due to the construction of a specific personal meaning in the mathematical context, the corresponding basic psychological need in mathematics lessons can be satisfied. For instance, if the learner constructs the personal meaning EXPERIENCE OF AUTONOMY, they might satisfy the need for autonomy, and so on. And if the three specific meanings are not constructed, the respective basic psychological needs cannot be satisfied in the classroom. That is, if the learner does not construct the personal meaning EXPERIENCE OF COMPETENCE, their need for competence in the mathematics classroom might not be satisfied, for example.

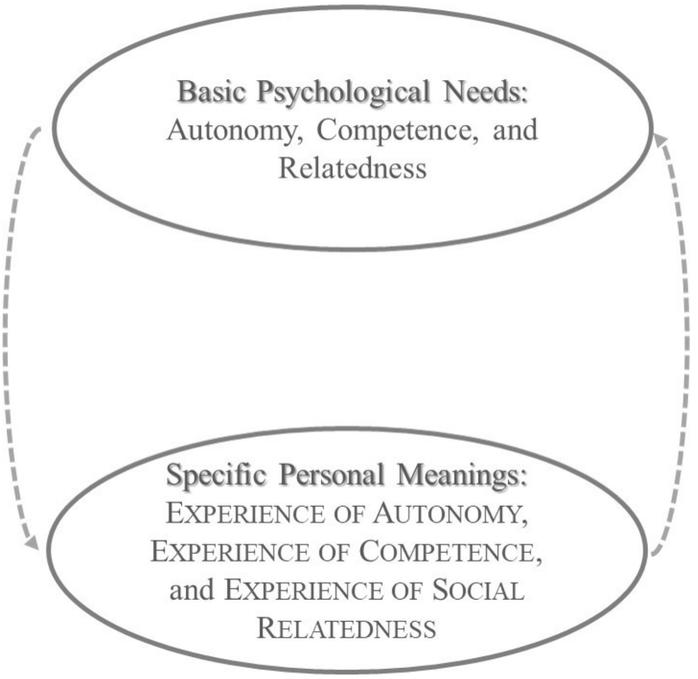
In turn, when their basic psychological needs are satisfied, the learner constructs the specific personal meanings in the classroom. That is, when a student experiences fulfilment of the need for autonomy during mathematics-learning situations, they construct EXPERIENCE OF AUTONOMY, which strengthens their well-being in the classroom. Likewise, if the basic psychological needs are not satisfied, then learners' well-being decreases, as something is missing. To that effect, I assume that the specific personal meanings in the mathematical context correspond to the basic psychological needs. There seems to exist an *inner feedback mechanism* (see Figure 4) between the basic psychological needs and the specific personal meanings in the classroom.

If so, I expect that these three specific personal meanings determine what specific pattern of personal meaning the learner constructs in the classroom (see section 2.3.2). Furthermore, based on this inner feedback mechanism, I would expect to find the same associations between the basic psychological needs and the learners' well-being and internalization continuum as exist between the three specific personal meanings and the learners' well-being and internalization continuum. In a nutshell, if an inner feedback mechanism between the three specific personal meanings and the basic psychological needs exists, then

1. the specific personal meanings determine the patterns of personal meaning (individual relevance system).
2. the specific personal meanings exert an influence on the learners' self-determination in the classroom (autonomous or controlled motivation).
3. the same regulatory mechanisms should also be found between the specific personal meanings and the regulatory styles as are found between the basic psychological needs and the regulatory styles (cf. 2.1.2).

Figure 4

Hypothesized Link Between Basic Psychological Needs and the Three Specific Personal Meanings.



As indicated earlier (cf. 2.1.2), support of autonomy occurs to a varying extent between cultures. Nevertheless, the association between basic psychological needs and well-being and an understanding of the essential role of autonomy for wellness is universal. For this reason, the predicted inner feedback mechanism should exist irrespective of social cultural contexts; that is, both in the data from Germany and the data from Finland.

Regulatory Styles and Personal Meaning. Based on the assumptions above, we shall explore further hypothetical connections between the regulatory styles and kinds of personal meaning. The taxonomic overview of the subtypes of motivation reflects the levels of autonomy leading towards learners' increasing sense of ownership (cf. Figure 1). In this connection, Vansteenkiste et al. (2018, p. 33) stressed that self-relevance is solely associated with autonomous extrinsic regulatory styles (identified, integrated), in which the regulation of society is less involved compared to controlled extrinsic motivation (introjected, external). I completely agree that in this case, autonomous-oriented activities are closer to the individual learner and more connected to their values and personal aspirations, which leads to multiple beneficial outcomes in their learning and promotes their internalization of external content due to the satisfaction of basic psychological needs. However, it might also be personally relevant for the learner to feel that they are a member of their social group, as this aspect, striving towards integration, belongs to their organismic dialectical nature (Deci & Ryan, 2000; Ryan & Deci, 2017). Hence, although controlled forms are alien to the individual's sense of self and thus not as close to the individual as well-internalized (autonomous) extrinsic actions, students occasionally also identify with those actions in order to experience integration within the mathematics classroom. For this reason, I claim, by referring to the theoretical concept of personal meaning, that learning activities in the mathematics classroom can still be personally relevant when those processes are closer to external sources and less a part of the student's identity (Ryan & Deci, 2020). Those less-internalized extrinsic forms may emerge in connection with substitute personal meanings when learners cannot construct a personal relevance that closely relates to their sense of self and the subject of mathematics (Vorhölter, 2009; Vollstedt, 2011b). For this reason, to fully understand the conceptual relations between motivation and personal meaning and to have a full picture of individual learners' personal relevance in mathematics, it is necessary to consider all the personal meanings that can be constructed within learning situations; that is, regardless of their relation to more-or-less-internalized forms of motivation.

As indicated above (1), I am interested in the patterns of personal meaning (individual relevance systems) that learners construct. Before I empirically relate the individual relevance systems to their motivational propensity, it would be interesting in this regard to investigate the theoretical correlations between the regulatory styles and the 17 personal meanings; that is, to identify which kind of personal meaning refers to which regulatory style.

In the following, hypothetical connections are explored[14]on the basis of the existing theoretical knowledge of regulatory styles and personal meanings, and the links between these are subsequently examined empirically. Firstly, this procedure should help us to understand the theoretical connections between the regulatory styles and the various personal meanings. Secondly, if meaningful connections exist, they will be applied in order to understand the constitution of individual relevance systems and their respective types of motivational behaviour in the mathematics classroom. Both subsequent analyses will show to what extent there is evidence that highlights the conceptual relations between regulatory styles and personal meaning, and how personal meaning can be understood through the lens of SDT.

To explore the theoretical links between both concepts, I initially used two theoretically reasonable meta-factors as orientations along which we can assume connections consistent with theory (between the regulatory styles and the kinds of personal meaning). These theoretical correlations apply to an alternative procedure of structuring the personal meanings by examining the relations between personal meaning and motivation (cf. Vollstedt & Duchhardt, 2019). Hence, I used the connectivity to the subject mathematics and the orientation guided by the taxonomy of motivation to understand how the different personal meanings associate with the regulatory styles. As such, the 17 personal meanings can be systematically located in terms of the two meta-factors, *orientation towards the regulatory styles* and *orientation towards the relatedness of mathematics* (see Figure 5).

When locating the 17 personal meanings into the matrix, each personal meaning is placed at one specific level of relatedness to mathematics (ranging from very low to complete harmony). Thereby, each personal meaning is associated with several regulatory styles but has a strong relation with one specific regulatory style. As indicated earlier (cf. 2.1.1), theoretically, learners' motives can be multiply motivated (intrinsic and forms of extrinsic or several extrinsic regulated forms). In this connection, Ryan and Deci (2017) stated "the contents of the regulations and values that are culturally transmitted and internalized make an important

[14] In general, scientific instructions (e.g., provided by Kelle & Kluge, 2010, pp. 91–107) help to group empirical cases into multidimensional types in order to investigate theoretical systems. In the present case, I first assume theoretical connections in order to test them subsequently in empirical research. For this reason, no specific instructions are followed, but the procedure for formulating the assumptions follows a similar pathway as the guidelines proposed by Kelle and Kluge (2010).

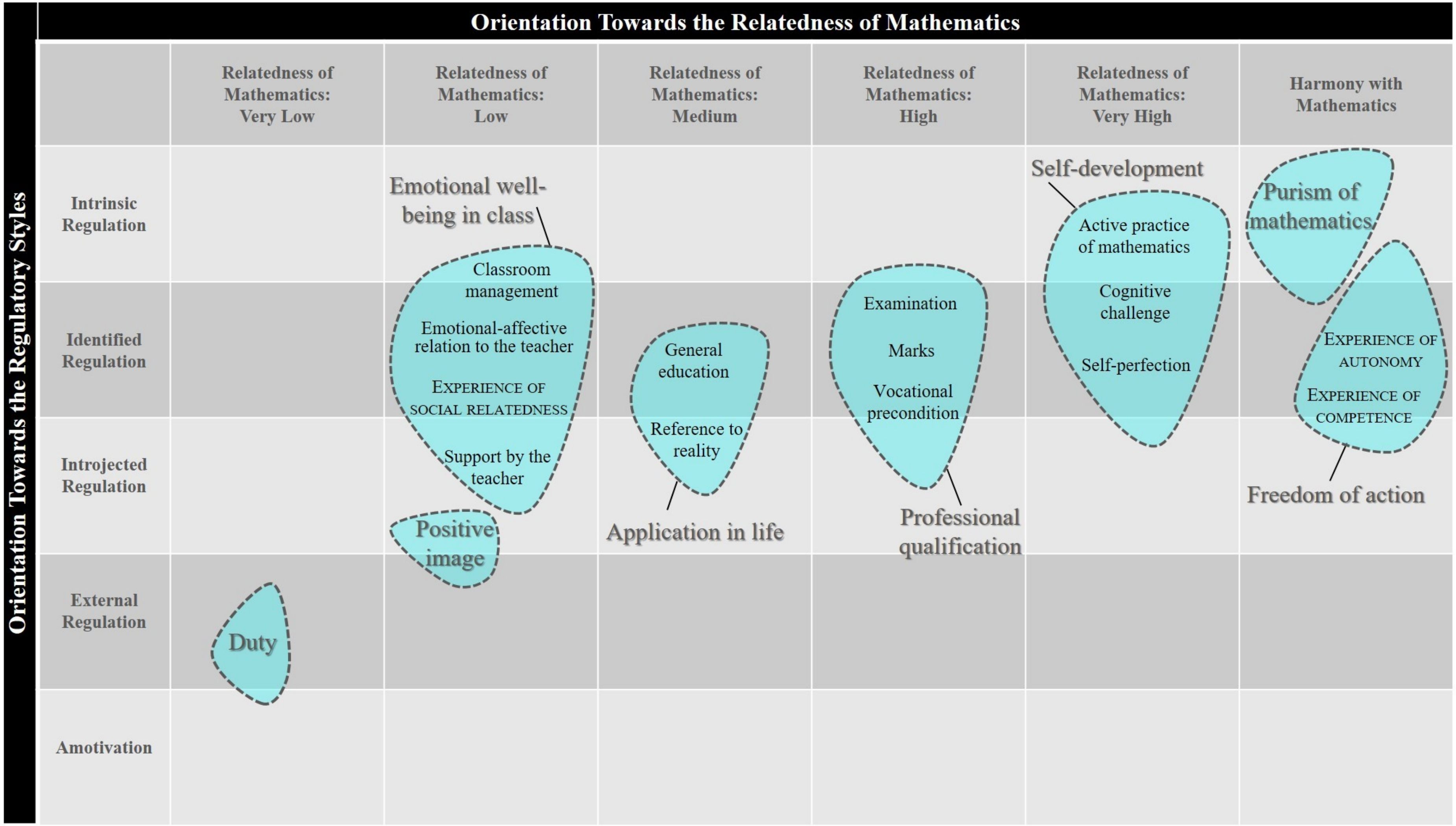
difference, with some contents more or less fitting with basic psychological needs and therefore more or less easily integrated" (p. 181). In this relation, referring to Ryan and Connell (1989), Vansteenkiste et al. (2018) stated that the correlates of the regulatory styles follow a *simplex pattern*; that is, with correlates "becoming increasingly less negative and more positive as one moves along the continuum from external to intrinsic motivation" (p. 34). Our existing knowledge about regulatory styles' correlational styles (multiply motivated and simplex pattern) were used to assume the connections. Additional knowledge that was used to locate the personal meanings came from the perspective of culture. This present investigation considers data from two different cultural contexts, namely Germany and Finland. Although these countries' educational systems are different (see section 2.4), their cultural norms and values and ways that these influences are transmitted to the educational system, particularly in mathematics curricula, are more autonomy-supportive than authoritarian. Therefore, based on the cultural contexts that are dealt with within this present study (students from Germany and Finland), I assumed the existence of the illustrated links between both personal meanings and regulatory styles (see Figure 5).

In so doing, the 17 personal meanings could be theoretically divided into the following groups: eight are content-specific dimensions[15]of personal meaning; five of them are multifaceted (*application in life, emotional well-being in class, freedom of action, professional qualification,* and *self-development*); and three of them are self-sufficient (duty, positive image, and purism of mathematics). The irregular forms that surround the dimensions reflect the fact that the kinds of/dimensions of personal meaning do not relate only to one specific regulatory style. In the following, I briefly describe the contexts and the characterization of each dimension by referring to the properties of each meta-factor, namely its type of orientations towards the regulatory styles (external–internal locus of causality) and towards the relatedness of mathematics (less strong–strong relation to mathematics).

[15] The kinds within the multifaceted dimensions of personal meaning are sorted in alphabetical order.

Figure 5

Hypothesized Links Between Regulatory Styles and Personal Meanings.



Application in life: This dimension emerged when I combined the personal meanings of general education and reference to reality. Here I assumed a relatedness of mathematics that is based on a balanced mixture of the social and the individual contexts, as the learner not only constructs these personal meanings in order to participate in the social realm but also values mathematics knowledge as a part of their identity. Hence, I assume that this dimension and its personal meanings, in which learners have a somewhat internal locus of causality, could be *strongly associated with identified regulation*.

Duty: Here the relatedness of mathematics learning is very low. In cases where this meaning is constructed, I expect the learner not to be interested in mathematics, thus the learner participates in class as they intend to avoid sanctions, which in turn are motivated by an intention to regulate society. Accordingly, the learner's actions have an external locus of causality and due to their lack of interest in mathematics and its low relevance for them, they feel controlled in the classroom. Here, I expect a *strong relation to external regulation*. Since this personal meaning focuses uniquely on the external obligations associated with mathematics and the social regulations that preponderate, I did not group this.

Emotional well-being in class: This dimension encompasses personal meanings that relate to emotional-affective factors within the classroom. Specifically, all these personal meanings are largely related to the role of the teacher. Here, I assume a *strong relation to introjected regulation*, as the focus gaining their teacher's approval or making a positive impression on others in the classroom may lead the learner to internally pressure themselves to study mathematics. Accordingly, the learner's actions may have a somewhat external locus of causality and the relatedness of mathematics is perceived as low, as an orientation towards social regulations preponderates.

Freedom of action: This dimension encompasses two out of the three[16] specific personal meanings. Based on the hypothesized links between the three specific personal meanings and basic psychological needs, I expect EXPERIENCE OF AUTONOMY and COMPETENCE to be associated with activities that are in harmony with mathematics as an object. Accordingly, I assume that when both personal meanings merge together as freedom of action, the learner has an internal locus of causality and there might be a *strong experience of intrinsic motivation*.

Positive image: In this case, I expect the learner not to be interested in mathematics, but to value mathematics learning in order to avoid feelings of guilt and shame. Accordingly, the learner's actions have a somewhat external locus of causality, and they see a low relatedness in mathematics as well as feeling controlled internally. Here, I expect a *strong relation to introjected regulation*. Since this personal meaning uniquely focuses on the internal obligations associated with mathematics, I did not group this with other personal meanings that show a similar direction in mathematics learning.

Professional qualification: This dimension encompasses personal meanings that matter for the learner's vocational planning. Since the desired planning for their profession—or for their further education—is personally meaningful, learning mathematics has a high relevance; here, the relatedness of mathematics is high. The perceived locus of causality is somewhat internal, and I expect a *strong association with identified regulation*.

[16] The three specific personal meanings are EXPERIENCE OF AUTONOMY, COMPETENCE, and SOCIAL RELATEDNESS.

Purism of mathematics: This personal meaning represents the hallmark of the subject mathematics. In cases where this meaning is constructed, I expect the learner to be interested in mathematics, thus in mathematics as an object itself. Accordingly, the learner's actions have an internal locus of causality and due to their interest (Krapp, 2002) in mathematics, these actions harmonize with their experiences in the classroom. Since this personal meaning is unique in that it focuses on mathematics as an object, I did not group this with other personal meanings that show a similar direction in mathematics learning. I expect a *strong association with intrinsic regulation*.

Self-development: This dimension encompasses personal meanings that matter for the learner's personal progress. Since the individual values mathematical skills as they guide them towards perfection and improvement, mathematics has a very high relevance in which an orientation towards the relatedness of mathematics preponderates. The perceived locus of causality is internal, and I expect a *strong association with intrinsic regulation*.

Investigating these assumptions—the relation between basic psychological needs and personal meaning as well as the associations between regulatory styles and personal meanings—will show whether personal meaning can be understood from the perspective of SDT. Since both concepts, personal meaning and motivation, emphasize the role of culture, it would be interesting to see when such theoretical correlations exist; whether, for example, they vary across different cultures and whether they follow similar associations (e.g., purism of mathematics relates to intrinsic motivation) when the patterns of personal meaning are related to their corresponding motivational tendency (in this example, autonomous motivation).

2.3.2 Social Constructivist Perspective on Personal Meaning – Theoretical Assumptions

The current investigation is interested in the core subject of motivation in the mathematics classroom. In this connection, as indicated above (cf. 2.2.1), the social constructivist approach helps us in that we can incorporate its view on social interactions in order to explore how students' experiences of autonomy and self-determination are brought about by interpersonal relations with different people in the mathematics classroom. On this specific topic, within his excellent article, Hickey (1997) precisely reviewed and discussed the notion of motivation in the sense of social constructivism. As an important impetus towards classroom motivation, he considered to view classroom motivational issues by means of a social constructivist approach in order to increase awareness of the contradictions in both conceptual understandings. A holistic perspective that surveys each viewpoint using the tools of the others is important as it is an approach that avoids bringing the social constructivist perspective and motivation theory into conflict with each other and that also assists our comprehension and optimization of learning in the classroom context (Hickey, 1997).

In its appreciation of the central role of the social context within its conceptual understanding, SDT seems to be theoretically commensurate with the social constructivist approach. However, it is necessary to look more closely at the way SDT and social constructivism communicate and to identify whether their roots conflict. To achieve this, I propose that personal meaning should also be viewed through the social constructivist perspective. According to the theoretical conceptualization of personal meaning, learners construct their individual relevance systems within the mathematics-learning situation (cf.

Figure 3). As a result, they are integrated into the social world of the classroom and take part in interpersonal conversations. Such individual relevance systems, which I want to identify, have to appear in the classroom and the question is how such relevance systems can be created in the classroom at all; I wish to draw attention not only to their existence, but also to how they manifest themselves in a teaching context and to address the issue of how they are created in the first place. It will only be possible to describe these individual relevance systems theoretically after these points have been addressed. I used Ernest's model (2010) create a social regulatory mechanism as a counterpart that applied to everyday lessons through the lens of social constructivism. Effectively, in order to examine how interpersonal dialogue affects learners' constructions, I adapted Ernest's cycle and applied it to the ways in which learners produce patterns of personal meaning in the mathematics classroom; that is, the *social regulatory mechanism of the individual relevance system* (see Figure 6). The application of this adapted cycle may help us to increase our understanding of the development of learners' individual relevance systems in mathematics.

By respecting the core notions behind each of the processes—appropriation, transformation, publication, and the processes of conventionalization—I theoretically explain the production of the learner's individual relevance system through the following adjustments (see Figure 6): the adapted model schematically explains how the individual relevance system becomes *appropriate* to the learner through their experience of the personal relevance that mathematics learning can have.

Figure 6

Social Regulatory Mechanism of the Individual Relevance System (cycle adapted from Ernest, 2010).

		Social Location: Individual		*Social Location*: Collective
Ownership	Public	Individual uses the individual relevance system as mental compass to orientate meaningfully in classroom	**Conventionalization** →	Production of the individual relevance system is promoted by conventionalization experiences
		Publication ↑		**Appropriation ↓**
	Private	Individual`s construction of own individual relevance system (via three specific personal meanings)	**Transformation** ←	Individual receives own impression of what personal relevance mathematics learning can have

As a first instance, the operation of *appropriation* (creating one's own impression) sees the learner receive their experiences within their social context in the mathematics classroom, which provides them with their own impression of what mathematics learning is about. Then, the learner experiences this a number of times, each time completing a cycle in which they process the personal relevance of mathematics learning for them. This cycle consists of four processes: appropriation, transformation, publication, and conventionalization. The learner examines each experience in terms of its relevance to their own meaning construction and aspirations and *transfers* that perception into their own personal meanings for mathematics as a subject.

Within the individual development process, which takes place in private (and which consists of two of the above-mentioned operations, appropriation and transformation), I specifically assume that the three specific personal meanings as key factors guide the learner's construction on the basis of their experience (non-/construction). That is, here the effects of the hypothesized *inner feedback mechanism* (cf. Figure 4) between basic psychological needs and the three specific personal meanings impacts the learner's reflection process in terms of the pattern of personal meaning. For this reason, this process of reflection, which is based on appropriation and transformation, plays a major part in the construction of the individual's relevance system. This is because this reflection process helps prepare the learning orientation for mathematics for the *publication* stage (orientation), by defining the pattern of personal meaning that provides the learner with the mental compass they will use to enter/attend mathematics-classroom interactions. In connection with the others in classroom, through *processes of conventionalization* (reflection), individual relevance systems are individually differentiated as local patterns for the individuals, just as they fit the individual.

It is notable that the model initiated by Ernest (2010) followed a semiotic perspective (development of a concept through signs), whereas the adapted cycle here focuses on the importance of social interactions within classroom situations that affect the learner's development of their individual relevance system through social interactions in the classroom. This theoretical construction is by its nature hypothetical, and will be further examined within the context of this study.

2.3.3 Consideration of Personal Meaning as a Boundary Object

According to the theoretical assumptions presented above, there seem to exist conceptual links between SDT and personal meaning as well as between social constructivism and personal meaning.

On the one hand, SDT's ontological principles (cf. 2.1.3) provide a lens with which to focus on learners' individual experiences. That is, SDT is a powerful lens to discuss the basic psychological needs mechanisms within the individual that belong to learners' individual biological natures. As such, these basic need mechanisms generate motivation. When considering personal meaning from the perspective of SDT's ontological principles, personal meaning appears as an individual psychological phenomenon. That is so because personal meaning encompasses conceptual characteristics: the biological aspect that considers learners as individuals, and which captures, understands, and translates SDT's ontological principles. Based on this structural complexion of personal meaning, conceptual links between SDT's central elements (basic psychological needs and regulatory styles) and personal meaning could be assumed on the level of theory.

When SDT sheds light on personal meaning, within the individual level of personal meaning there seems to exist a *biological regulatory mechanism* that addresses learners' individual experiences of meaning construction. As such, this biological regulatory mechanism may become epistemologically observable in learners' motivation.

On the other hand, the ontological principles of social constructivism (cf. 2.2.1) highlight students' social contexts in the classroom. In general, this lens helps to explore individuals' social experiences, in which they subjectively assimilate the social regulations within the classroom. As such, the conversation—which belongs to learners' social nature—between the social realm and the individual is used to explain the production of learning and knowledge. However, when considering personal meaning by means of the ontological principles of social constructivism, personal meaning appears as a social phenomenon. That is so because personal meaning (also) includes characteristics—or the social aspect that considers learners as members of the social world—that captures, understands, and translates the ontological principles of social constructivism. Based on this nature of personal meaning, conceptual links between the cycle proposed by Ernest (model of sign appropriation and use) and personal meaning could be combined in the development of the learner's individual relevance system. The social nature of personal meaning helps to transform learners' interpersonal experiences which have a major impact on learners' production of individual relevance systems.

When social constructivism sheds light on personal meaning, within the social level of personal meaning there seems to exist a *social regulatory mechanism*. As such, this social regulatory mechanism may become epistemologically observable in learners' social experiences in the classroom; that is, be reflected within learners' individual relevance systems.

Accordingly, personal meaning seems to have the potential to fulfil "a bridging function" (Akkerman & Bakker, 2011, p. 133) as a boundary object at the boundary of social constructivism and SDT. According to Star and Griesemer (1989), *boundary objects* are analytic concepts that

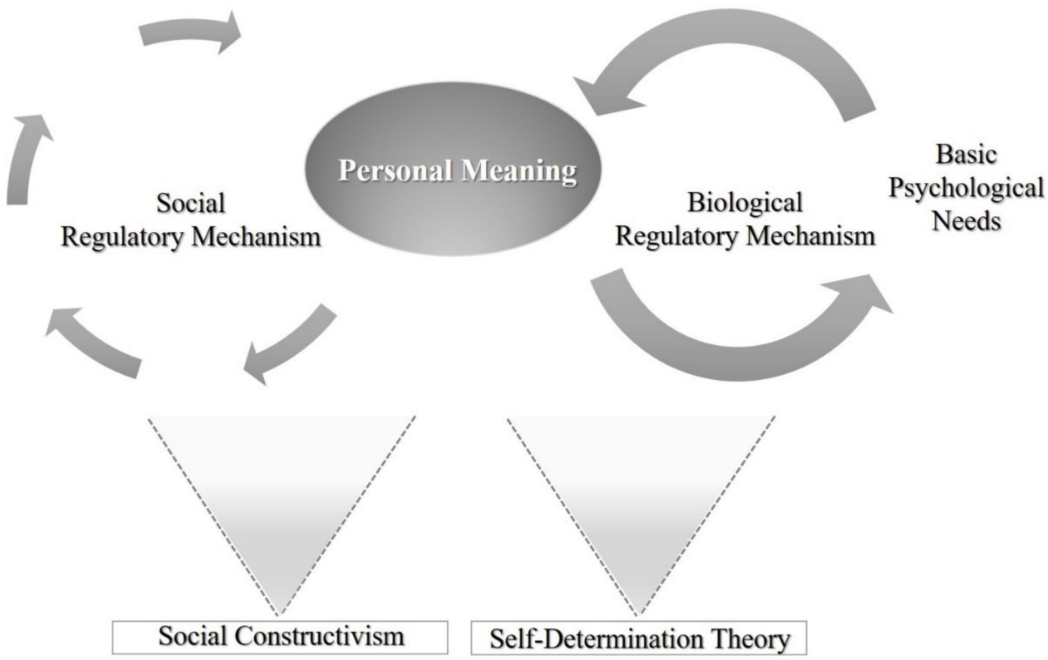
> both inhabit several intersecting social worlds … and satisfy the informational requirements of each of them …. [These concepts are] both plastic enough to adapt to local needs and the constraints of the several parties employing them, yet robust enough to maintain a common identity across sites. They are weakly structured in common use, and become strongly structured in individual site use. These objects may be abstract or concrete. They have different meanings in different social worlds but their structure is common enough to more than one world to make them recognizable, a means of translation. (p. 393)

To conclude, if personal meaning can be viewed and understood from two different theoretical angles, SDT and social constructivism, then it seems to encompass two different ontological and epistemological natures. The hypothesized interconnection between both, biological and social regulatory mechanisms, is illustrated in Figure 7. The lens of SDT reflects its biological or individual ontological nature. Epistemologically, this aspect comes to the fore through the learner's motivational behaviour in the classroom. The perspective of social constructivism focuses on the social nature (on the level of ontology) that epistemologically exists in the learner's constructed individual relevance system. Accordingly, personal meaning seems to have a double ontological nature. In this case, it would be interesting to explore how these regulatory mechanisms, biological and social, interact. That is, these different ontological natures of personal meaning can be used to study the interplay between learners' individual relevance systems and their motivation in mathematics education.

The exploration of the interrelation between learners' individual relevance systems (social regulatory mechanism) and their quality of motivation (biological regulatory mechanism) may clarify how learners' motivation emerges in the mathematics classroom. To elaborate these theoretical links, both ontological natures need to come into a dialogue by means of networking, so that we can study their interplay and how it relates to the genesis of motivation in mathematics.

Figure 7

Personal Meaning Characterized as Located at the Boundary of Social Constructivism and Self-Determination Theory.



2.4 Cross-Cultural Settings for Validation and Refinement

As indicated above, both theoretical viewpoints highlight the central role of teaching in sociocultural classroom contexts. Ryan and Deci (2020) stated that a country's educational reforms and instructed curricula have an impact on learners' motivation (and also on the teacher's behaviour and performance). In particular, an "excessive emphasis on grades, performance goals, and pressures from high-stakes tests" (p. 10) have a negative effect on learners (and also on teachers). In the face of the mandates that the teacher needs to follow in the classroom, certain constraints within the curricula of a subject hamper and challenge autonomy-supportive teaching (Ryan & Deci, 2020). Referring to Hickey's major leap ahead in terms of discussing motivational issues within social constructivism, how are motivational issues (e.g., support of autonomy) targeted within mathematics curricula approaches? Do the mathematics curricula formulate instructions that pay the same attention to learners' intellects as they do to the individuals' affective states? This is important because "motivation cannot be distinguished from the larger realm of activity, and the individual's activity cannot be distinguished from the larger sociocultural context" (Hickey, 1997, p. 178). Hickey (1997) emphasized that motivation cannot be separated from cognitive activities; therefore, to study one without the other would serve no purpose.

To understand the hypothesized connections between learners' individual relevance systems and their motivation in the mathematics classroom, data from Germany and Finland are considered here. This decision was made for two reasons: Firstly, both theoretical views, SDT and social constructivism, emphasize the pervasive impacts of cultural norms, behaviour, and practices on students' individual relevance systems and motivation. This present study investigates German and Finnish ninth graders.[17] Referring to scholastic performance in mathematics, Finland has earned international renown for its high performance in international comparative studies. In fact, Finnish and German students were both above average in the context of their performance in PISA 2012–2018. Finnish students outperformed their German counterparts by a narrow margin (OECD, 2014a, 2016, 2019). However, a few differences between the two groups were notable in relation to learners' affect in mathematics learning. For instance, in terms of the association between mathematics anxiety and performance, Finnish students felt less anxiety than German students (OECD, 2015, p. 2). Moreover, German students felt more anxious than Finnish students did when their schoolmates performed better than they did (OECD, 2015, p. 3). Further, *between 2003 and 2012 the mean index of intrinsic motivation to study mathematics* increased in Finland, whereas in Germany the level decreased (OECD, 2013, p. 74). Accordingly, it would be interesting to see which patterns of personal meaning German and Finnish learners construct on the basis of their social regulations and autonomy support in mathematics classroom.

Secondly, analysing results from two cultural contexts may help to contrast and refine the interconnections between learners' individual relevance system (social regulatory mechanism) and motivational behaviour (biological regulatory mechanism) as they relate to the students' development of motivation in the mathematics classroom. To this effect, I provide a brief insight into both countries' mathematics curricula (intended curriculum) in order to explore which aspects of their educational policies and practices support the learning of mathematics in the classroom.

2.4.1 German and Finnish Mathematics Curricula

A country's educational system and curriculum represent an image of its culture and its society's major elements. In terms of the mathematics curriculum, an exploration of its institutional mandates can take place within different arenas. According to Valverde et al. (2002) a country's overall mathematics curriculum can be viewed as a tripartite model, namely the *intended, implemented,* and the *attained curriculum*. As the name suggests, the intended curriculum formulates the official framework of the encompassing guidelines and key approaches like values, goals, competencies, and behaviour that the country's educational authorities usually communicate within official documents, like textbooks, exams, and syllabi

[17] Again, the theoretical conceptualization of personal meaning is based on students' data from grades nine and ten.

for all learners in that society. It also sets out the way in which those general skills are addressed and transmitted in mathematics-classroom practice/interactions and how they should be discussed within the implemented curriculum. This arena particularly focuses on the interpersonal learning activities in the classroom; that is, those that take place between teacher and students and, of course, between students. The curriculum that specifies the attitudes, knowledge, or competencies that teachers should mainly follow in order to guide these social interactions in practice is known as the attained curriculum. The instructions at this level specify the knowledge and skills to be taught in mathematics-learning situations. A detailed comparison of both countries' educational systems at each level would go beyond the scope of this thesis. For this reason, I limit these (tentative) explorations to the German and Finnish intended curricula of the subject mathematics. Notably, the following analyses do not conform to the criteria-guided empirical methods that are generally applied to educational system comparisons. Nevertheless, since the aim here is to acquire an understanding of the philosophy behind the mandated instructions for mathematics learning in the classroom, a tentative analysis might also be helpful in obtaining an impression of the virtues of those instructions, which may affect the construction of ninth graders' patterns of personal meaning and their experiences of autonomy support in the mathematics classroom.

German Mathematics Curriculum. In response to the major educational initiative "Getting ahead through education" (German: *Aufstieg durch Bildung*), German educational policies emphasized an emerging need for skilled and well-educated workers. A list of aims and measures were decided upon by the federal government and all 16 federal states (KMK, 2019, p. 307):

- Education is to have top priority in Germany
- Every child should have the best possible starting conditions
- Everyone should be able to gain school-leaving and vocational qualifications
- Everyone should have the opportunity to get ahead through education
- More young people should take a degree course
- More people should be filled with enthusiasm for scientific and technical vocations
- More people should take advantage of the opportunity for continuing education

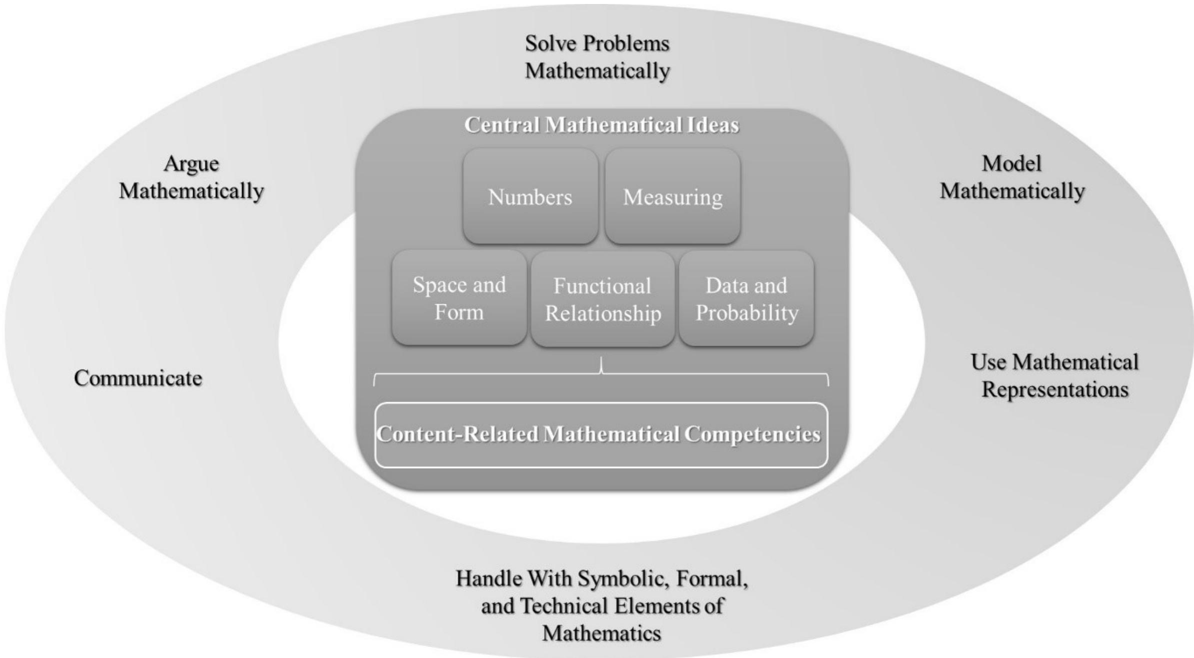
What these objectives had in common was their key aims: to make education a higher national priority and to reduce the number of adolescents leaving school without formal qualifications while raising the number of those obtaining them. Compared to the former version (KMK, 2016, p. 289) the updated (in 2019) key objectives particularly emphasized the effort required to address the causes of the problems the educational system faced; it stated that due to "the challenges posed by digitisation and the ongoing migration of refugees, great efforts must be made to develop the German education system in the years ahead" (KMK, 2019, p. 307).

In Germany, the school systems in each state are dependent on the federal state. In fact, the variation among the types of schools (mostly focused on different performance levels or vocational orientations) and the time spans over which students visit them represent a "mishmash". The variation within the 16 federal states aims to accommodate the heterogeneous diversity in the country by providing the best education possible for each student, which reflects their academic level and requirements (KMK, 2019). To provide adequate care and support for each learner, all federal states provide an early selection process (typically after four or six years of school attendance, depending on the federal state in which students live) in which students are streamed according to their academic performances.

In relation to the German intended curriculum in mathematics, inspired by the PISA results in 2003, the standing conference of the ministers of education and cultural affairs (German: Kultusministerkonferenz—KMK) introduced so-called general educational standards (German: *Allgemeine mathematische Kompetenzen*; Blum, 2006; KMK, 2005, p. 7) for major subjects in school that should be reached after grade four (primary level) and after grade nine (intermediate degree). To facilitate the acquisition of these educational standards, various content-related mathematical competencies[18] (German: *Inhaltsbezogene Kompetenzen*) are formulated that are in turn assigned to central mathematical ideas (German: *Mathematische Leitideen*; Biehler & Hartung, 2006; KMK, 2005, p. 8). Each central idea is a theme that is woven like DNA (i.e., spirally) into the mathematical curriculum and helps learners to achieve an understanding of basic mathematical terms, interdisciplinary thinking, and joined-up thinking (Biehler & Hartung, 2006). Figure 8 provides a brief overview of these major components and the relationships between them.

Figure 8

Overview of General Educational Standards in Mathematics to be Reached After Grade Nine Which are Acquired Through Content-Related Mathematical Competencies and Central Mathematical Ideas (adapted from KMK, 2005, translated by the author).



When looking at these mandated standards, what immediately catches the eye is that all of them (including the mathematical competencies and the central ideas) only discuss those

[18] A detailed description of the content-related mathematical competencies can be found in KMK, 2005, pp. 9–11.

competencies that refer to the subject mathematics itself. The review of the core curriculum (KMK, 2005) pointed out that the promotion of learners' individual development is also an important aspect of mathematics learning: "The mission of school education goes beyond the acquisition of subject-specific skills. Together with other subjects, mathematics lessons also aim at personal development and value orientation" (KMK, 2005, p. 6, translated by the author). However, later amendments to the curriculum did not pick up on this fundamental topic at all. This issue was mentioned by Blum (2006) and Heymann (2005), whose criticism was that these standards should be redefined as *performance standards*, as the only detailed definition they provide concerns what students should have learned in mathematics by the end of grade nine (Blum, 2006, p.15). They neither consider students' everyday learning situations in class (Blum, 2006, p. 15), nor value individuals' social, ethical, and personal development (Heymann, 2005, p. 40).

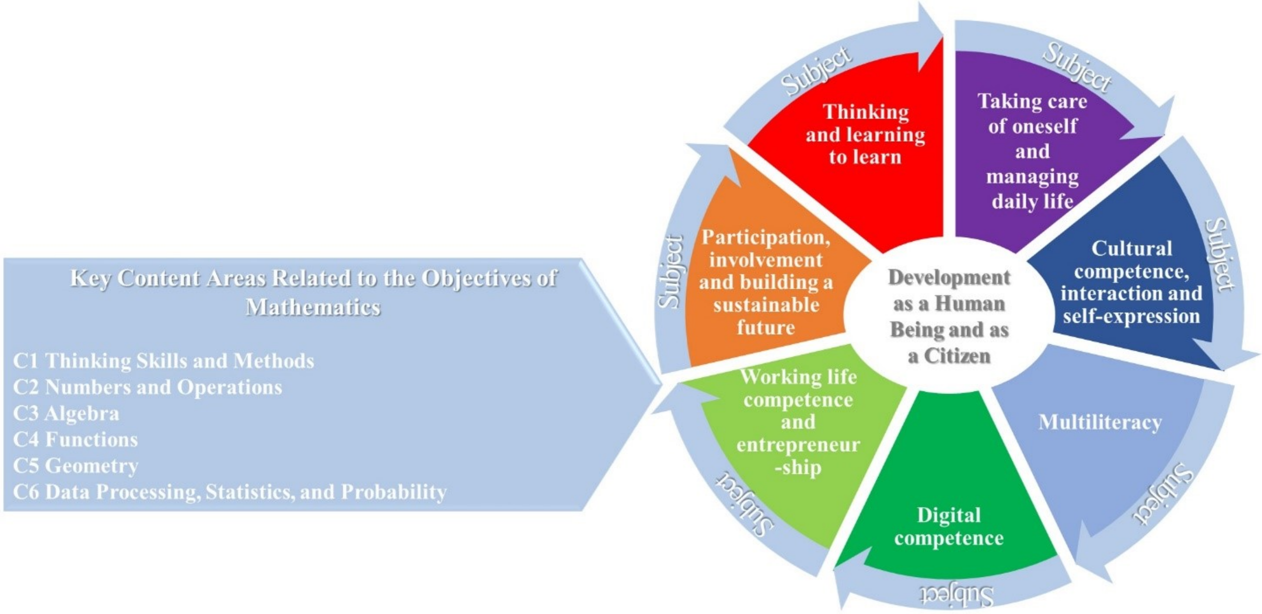
Finnish Mathematics Curriculum. The educational philosophy of Finland places emphasis on the guideline "Every pupil is unique and has the right to high-quality education and training" (Finnish National Agency for Education, 2017, p. 6). This ethos highlights in particular the importance of equality to students' growth and learning processes, and emphasizes the need for a high quality of learning if students are to be prepared for lifelong learning (Finnish National Agency for Education, 2017, p. 7). Following a single structure, Finland has a comprehensive system for the basic education covering grades one to nine. Accordingly, students of all academic abilities attend comprehensive school from primary to lower secondary level. They are taught in mixed-ability classrooms, and receive individual support according to their academic needs (Finnish National Agency for Education, 2017).

Guided by the education authorities' quality assurance plan, "Steering instead of controlling" (Finnish National Agency for Education, 2017, p. 13), the national core curriculum for basic education defines seven transversal competencies for all subjects. Besides these, objectives of instructions and content areas related to those objectives are central elements of the syllabus. The objectives of instructions[19] can be differentiated into "significance, values, and attitudes", "working skills", and "conceptual objectives and objectives specific to the field of knowledge" (Finnish National Board of Education, 2016, p. 403). Figure 7 briefly illustrates the content areas in mathematics and the transversal competencies (Finnish National Board of Education, 2016).

[19] A detailed description of the objectives of instructions can be found in Finnish National Board of Education (2016, pp. 402–408).

Figure 9

Key Content Areas Related to the Objectives of Mathematics in Grades 7–9 and Transversal Competencies in Basic Education (adapted from Finnish National Board of Education, 2016; see also Halinen, 2018).



Accordingly, the task of mathematics education encompasses not only developing students' "logical, precise, and creative mathematical thinking" (Finnish National Board of Education, 2016, p. 402) in mathematical modelling and problem solving; it also covers transversal competence areas that are cross-curricular and that largely concern students' personal development, such as self-actualization and developing confidence both as a learner and as an individual in society (Finnish National Board of Education, 2016). Therefore, in mathematics lessons students' learning process is assisted through an approach that supports their development of a healthy self-concept and self-esteem. They are encouraged to share responsibility with others and set their own goals, while the teacher provides them with strategies and methods for self-improvement. What is more, the teacher enables the students to develop their proximal skills through self-assessment and to support each other in terms of cooperative learning based on constructive feedback, an experience during which it is ensured that they are not ignored but heard. To conclude, the Finnish curriculum places emphasis on cooperation rather than competition. Teachers provide the "scaffolding" that supports their students and helps them to become active, self-confident lifelong learners.

2.4.2 Answerable Questions

Based on this tentative examination of the German and Finnish mathematics curricula, it appears that both countries aim to improve learners' mathematics performance but focus on them differently. In general, both countries, both of which are located in Western Europe, tend to teach mathematics in a way that is more supportive of autonomy than it is authoritarian. What is more, Finland is one of those countries that has adopted the German concept of *Bildung* as a key pillar in its approach to education (Autio, 2021). Despite this German influence, the

Finnish system is distinct from the German system in that it is equally concerned with learners' mathematical skills and with their individual development, whereas the German system mainly emphasizes learners' performance skills in mathematics. In a nutshell, the tentative examination of the mathematics curricula showed that in Finland, autonomy support seems to be more strongly represented than in Germany. The empirical evidence will show which similarities and differences result from these issues in terms of learners' individual relevance systems and motivation, and the interplay of both affective concepts. Since this study does not include any empirical classroom observations, its conclusions on the mathematics curricula/learning environment can only be based on theoretical knowledge and should be drawn with caution.

2.5 Résumé

This study's main purpose is to explore the interplay between learners' individual relevance systems and motivation in mathematics. To this effect, this chapter set out to clarify their interrelation on the level of theory. In so doing, it pointed out that we need to consider the learners' individual (through the lens of SDT) and social experiences (through the lens of social constructivism) in the classroom to reach a more comprehensive research result.

The theoretical conceptualization of personal meaning seems to encompass a biological and a social ontological nature, so that this construct can be viewed as lying at the boundary of SDT and social constructivism. If personal meaning is considered as a boundary object, from the perspective of SDT there seems to exist a biological regulatory mechanism explaining learners' motivational behaviour. Likewise, from the perspective of social constructivism there seems to be a social regulatory mechanism explaining the construction of the individual relevance system. Accordingly, personal meaning seems to link both regulatory mechanisms if we consider it as being at the boundary of both theoretical lenses. Studying personal meaning's dual ontological nature might help to understand how both regulatory mechanisms are interconnected in terms of learners' motivation in mathematics. Besides capturing both ontological natures, a robust and systematic procedure is necessary to network the theoretical lenses while respecting both theoretical roots. This examination will show how these lenses are connected and complement each other at the level of theoretical principles (*P*) so that both theories' articulation within the context of personal meaning leads to a common research result (*R* as fourth element, Radford, 2012).

2.6 Research Questions

As indicated earlier, within SDT's research landscape, for instance, Wang et al. (2017) have considered the patterns of regulatory styles (motivational profiles) in mathematics education in order to identify configurations of behavioural regulations. Focusing solely on these behavioural regulations is not enough to promote motivation in the mathematics classroom. In this connection, research that takes an SDT approach has pointed out that highlighting learners' personal relevance seems to be a promising pathway towards fostering their autonomously oriented motivation. In comparison to the other kinds of autonomous motivation (intrinsic or integrated regulation), the teacher's external efforts can address identified regulation by helping students to understand (e.g.,) the personal relevance of mathematics learning. For this reason, it is a matter of particular interest which mathematics-related personal relevance learners associate with identified regulation. However, to the best of my knowledge, there is no clear theoretical concept behind learners' personal relevance or its conceptual relation to learners' motivational behaviour.

Studies within mathematics education have addressed this issue and proposed a relational framework of the motivational construct personal meaning based on Grounded Theory (Vollstedt, 2011b; Vorhölter, 2009). However, it has not been clarified how individual and social factors affect the development of learners' individual relevance systems and which kinds of individual relevance systems learners subjectively construct in order to engage in mathematics learning. Although the theoretical concept of personal meaning encompasses the basic psychological needs as relevant concepts, the conceptual relation between personal meaning and motivation in mathematics has not been clarified yet.

For this *problematique*, this dissertation now moves to the next level and focuses on the interconnection between the learners' individual relevance system and their behavioural regulations. To understand their relation in mathematics, two different theoretical lenses are adopted, SDT and social constructivism, in order to obtain a comprehensive research result that involves both learners' individual and social experiences when learning mathematics. Concerning the main objectives of this study, understanding the interplay between the learners' individual relevance systems (or profiles of personal meaning) and motivational behaviour from both theoretical perspectives may help to increase our theoretical knowledge and conceptual understanding of both affective constructs. Furthermore, the comparison of German and Finnish data may allow us to contrast the interconnections and to refine the interplay between personal relevance and motivation. Accordingly, this exploration may contribute to clarifying the genesis of motivation by focusing on the role of the personal relevance experienced by the learner in the mathematics classroom.

The following research questions (RQs) are considered within this investigation as a means of tackling the main objectives of this dissertation. Firstly, in order to investigate the interconnection between individual relevance systems and motivation, empirical evidence is required that allows a fair comparison of German and Finnish ninth graders' data. Based on this clarification, the data of German and Finnish learners can be compared with regard to (dimensions/kinds of) personal meaning and motivation. Secondly, the evidence provided by empirical patterns of personal relevance is necessary to theoretically describe the development of these patterns within the context of social interaction and their association with motivation. To this effect, RQ 1–RQ 3 are examined in the first part of the study.

(RQ 1) What are the psychometric properties of the affective constructs personal meaning and regulatory styles across Germany and Finland?
(RQ 2) What cross-cultural differences exist on the level of personal meanings and regulatory styles in Germany and Finland?
(RQ 3) How many distinct profiles of personal meaning can be detected in Germany and Finland?

Thirdly, to understand the interplay between individual relevance systems and motivation, the interconnections found in German and Finnish samples are separately analysed on a theoretical level. Based on that, the results are compared and related to the respective mathematics curricula. Drawing on these comparisons, the genesis of motivation in mathematics is theoretically explained. Therefore, RQ 4–RQ 6 are studied in the second part of this investigation.

(RQ 4) How do the individual relevance systems and motivation interact in Germany and Finland, when viewed from a perspective coordinating SDT and social constructivism?
(RQ 5) How does the interaction found in German and Finnish data differ in view of learners' mathematics curricula?
(RQ 6) How does the conceptual framework built by coordinating SDT and social constructivism by means of the boundary object of personal meaning explain the development of motivation in mathematics classroom?

Next, I will present the research design used and the methodological considerations that should be taken into account before exploring the research questions of this dissertation (see chapters 3 and 4).

3. Empirical Work

In general, when scientifically investigating an empirical phenomenon, the questions always arise of whether the respective theoretical object is measurable and how it can be assessed in an appropriate way that respects its theoretical nature. To this effect, researchers' assumptions about the ontological and the epistemological natures of the phenomenon under investigation strongly determine the picture of the scientific world that they build up. This in turn affects the methodological approaches that the researcher selects (Buchholtz, 2021; Slevitch, 2011). Within the mathematics-related affect research, both qualitative and quantitative approaches are adequate (Goldin, 2014). These two research paradigms facilitate distinct insights into empirical phenomena. However, the suitable research direction should ultimately depend on two decisive factors, namely the theoretical roots (Radford, 2008a) of the research object and the problem statement that one aims to study. Therefore, it is necessary to identify the methodology that is useful for the research question, concepts, and theories to be used, and also the larger goal of relating theories or, as in this case, viewing an object from the perspective of two theory cultures at the same time. Accordingly, before deciding to apply a certain research approach, the researcher should clarify whether they have explicitly considered the theoretical conceptualization of the phenomenon and whether their research methods are appropriate for the intended targets of the research questions (Prediger et al., 2008; Slevitch, 2011).

Based on the theoretical framework, this study's substantive concept is clear: As elaborated earlier (cf. 2.3.3), the motivational construct of personal meaning (Vollstedt, 2011b; Vorhölter, 2009) can be characterized as located at the boundary of two theories; namely, self-determination theory (SDT; Deci & Ryan, 2000; Ryan & Deci, 2017) and social constructivism (Ernest, 2010; Hickey, 1997; Vygotsky, 1978). Accordingly, personal meaning seems to present as having *multimodal ontological natures* (Radford, 2014, p. 359); that is, both as an individual psychological phenomenon (viewed through the lens of SDT) and as a social one (viewed through the lens of social constructivism). Corresponding to these two ontological worlds, personal meaning can be viewed from the perspective of both theories and also comes epistemologically to the fore in two different ways. The connection between personal meaning and SDT—that is, the biological regulatory mechanism—becomes observable in learners' motivational behaviour. As the counterpart to these individual experiences, the connection between personal meaning and social constructivism, or the social regulatory mechanism, is observable in those social interactions on the part of learners that account for the construction of that learners' individual relevance systems. Although the biological and the social worlds are ontologically different concepts, they seem to be theoretically aligned by means of the theoretical construct of personal meaning. The clarification of the interconnections between social and biological regulatory mechanisms might help to increase our knowledge about learners' quality of motivation in the mathematics classroom.

For this reason, this study's entire methodology is tailored to studying the double ontological nature of personal meaning. Since learners' social regulation is also determined by culture, I investigated the connections between both theories not only in the German, but also in the Finnish mathematics-learning context. Based on the similarities and the differences between both countries' mathematics curricula (cf. 2.4), the results may provide important evidence about the interplay between learners' social and biological regulatory mechanisms. The question that arises now is that of how best to study the boundary object of personal meaning, which, it seems, can be viewed from the perspectives of two different ontological worlds.

3.1 Adopting a Sequential-Dependent Approach to Studying Personal Meaning's Multimodal Nature

This study integrates two different methodological approaches to studying the interconnection between the biological and social regulatory mechanisms within personal meaning in terms of motivation to learn mathematics. In the following, I briefly outline the necessity of and the justification for my choice to take this approach. Subsequently, I set out the exact nature of this study's methodological combination to anchor that approach within the methodological discourse.

First and foremost, it was necessary to gather empirical data. Personal meaning is produced by individuals. For this reason, the initial empirical work, in which I applied a quantitative research approach, drew on the different kinds of personal meaning as well as the regulatory styles from SDT, as the motivational elements are a central component of the hypothesized biological regulatory mechanism of learners. Subsequently, the empirical profiles that were generated in this way were used to clarify the other social regulatory mechanism in the classroom. To illustrate, if we were to look at the empirical profiles (personal meaning as an individual psychological phenomenon) from the perspective of social constructivism (personal meaning as a social phenomenon), then we would be able to explain how these individual relevance systems arise and unfold in the classroom. This is because, if we can finally use this social regulatory mechanism to explain how these empirical profiles have developed, then we would have good reason to accept this specific link, between biological and social regulatory mechanisms, as meaningful. For this reason, it was necessary to develop these individual relevance systems and the motivational elements empirically so that they were independent of the hypothesized adapted model – the social regulatory mechanism of the individual relevance system (cf. Figure 6). This was done in order that the gathered empirical evidence could then challenge/support the adapted model of social regulatory mechanism by providing the explanation/interpretation of how these profiles had developed, on an evidence-based basis. Within the quantitative research approach, there is no research activity which helps to link theories, i.e., of biological and social regulatory mechanisms. For this reason, in order to capture the social regulatory mechanism by linking both theoretical perspectives, I also used the networking of theories approach.

In a nutshell, the initial empirical work, which focused on a classical survey (self-report data), only captured the biological ontological nature of personal meaning by means of quantitative statistical analyses. As a response to the weakness of the quantitative research approach, I subsequently considered networking theories approach and applied the coordination analysis method to capture the social ontological nature of personal meaning. This research activity, that of coordinating theories, was helpful not only to studying the second ontological nature of the empirical phenomenon, but also in investigating the articulation of the biological (motivational behaviour) and social regulatory mechanisms (construction of the individual relevance system) as regards learners' motivation in mathematics education.

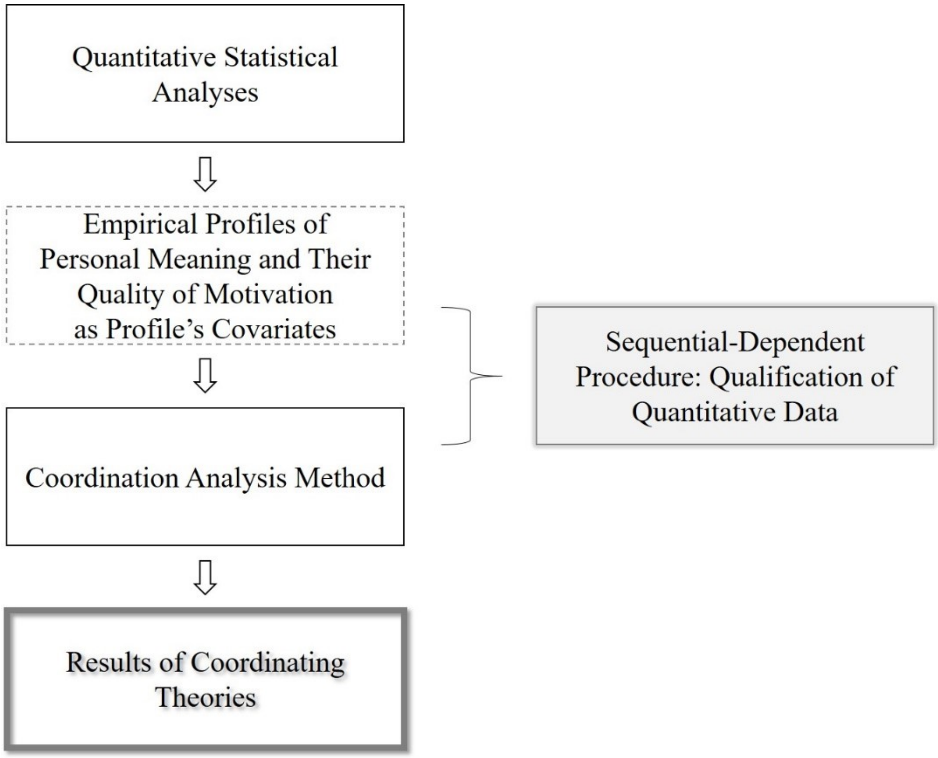
This study's whole methodology thus falls into a research design that follows a sequential procedure (Schoonenboom & Johnson, 2017). That is, by

> using the outcomes of the first research component, the researcher decides what to do in the second component. Depending on the outcomes of the first research component, the researcher will do something else in the second component. If this is so, the research activities involved are said to be sequential-dependent, and any component preceded by another component should appropriately build on the previous component. (p. 115)

According to this working definition, the following figure (Figure 10) provides a condensed overview of this study's overall methodical procedure, which frames the conducted empirical and the theory-driven analyses of the empirical data within a sequential-dependent research design. This present study will also discuss to what extent this procedure helped to counterbalance the methodological issues that emerged.

Figure 10

The Present Investigation's Sequential-Dependent Research Design.



To reflect this study's dual methodological approaches for the data analyses, I present these in two different chapters. In this chapter, Chapter 3, I first describe the empirical work that solely focuses on quantitative research methods. I discuss in detail the strengths of this empirical approach, and then explain why a mono-methodological quantitative research approach (Kelle & Buchholtz, 2015) is insufficient to study personal meaning's double ontological nature. As a response to that weakness, I describe in Chapter 4 why building the networking strategy of coordinating on the basis of the existing empirical work has potential as a means of reaching the research aim (cf. 2.6) of the present study.

In the following section (see section 3.2), I discuss the philosophical background of the conducted statistical analyses, before presenting the quantitative methodical procedure (see section 3.3) I used to study personal meaning as an individual psychological phenomenon.

3.2 Methodological Considerations for the Quantitative Analyses

As indicated above, the existing empirical work aims to capture learners' individual experiences. Therefore, the empirical data used in this study should reflect learners' mathematics-classroom experiences. As an initial step to achieving this end, empirical evidence needs to be developed in two different cultural contexts. I first clarify which concepts I assessed, and subsequently, why the quantitative research approach was chosen in particular as a means of studying the empirical evidence at the individual level.

Vorhölter (2009) and Vollstedt (2011b) theoretically stated that learners construct individually preferred patterns of personal meaning (individual relevance systems) within their mathematics-learning environment (cf. 2.2.2). For this empirical evidence, the assessment of the 17 personal meanings is essential to see whether those patterns of personal meaning are also empirically measurable. The regulatory styles of SDT (amotivation, extrinsic regulation, introjected regulation, identified regulation, and intrinsic regulation;[20] Ryan & Deci, 2017) can be used to investigate the subjective motivation associated with each individual relevance system (cf. 2.3.1). As indicated above (cf. 2.1.1), learners' motivational behaviour can be multiply motivated. To understand which behavioural regulation will most likely be manifested (Vansteenkiste et al., 2018, p. 33), besides examining the co-occurrence of the specificity regulatory styles, I used a composite index, or the SDI (Levesque et al., 2004, p. 73). Consequently, the 17 personal meanings and the five regulatory styles need to be assessed.

As we have seen, in a previous qualitative study Vollstedt (2011b) reconstructed 17 kinds of personal meaning students relate to learning mathematics in an educational context (cf. 2.2.2). The identification of the patterns of personal meaning and the investigation of their relationship to motivation within learners' natural learning environments by means of qualitative research methods is theoretically also possible. However, this procedure presents various hurdles, since in this context an increased effort generates only a low number of cases that are not selected as random samples (Lamnek & Krell, 2016, p. 16; Neuman, 2011). More often than not, self-report questionnaires are used for profiling individual response patterns (Geiser, 2011; Lazarsfeld & Henry, 1968; Rost, 2006). The application of a survey instrument helps us to straightforwardly gather plenty of heterogeneous empirical cases, which in turn allows us to consider numerous random samples (Kelle & Buchholtz, 2015, p. 333; Loewenthal, 2001; Neuman, 2011). This quantitative procedure of pattern (or profile) identification at the inter-individual level (e.g., latent class analysis, Geiser, 2011; Lazarsfeld & Henry, 1968; Rost, 2006) is one good way to identify learners' constructed individual relevance systems in connection with their quality of motivation. The self-report data thus collected can be used to study how personal meaning's biological regulatory mechanism becomes observable in learners' motivational behaviour. Based on the need to consider a large amount of random cases, in this present investigation the conceptualization and operationalization of personal meaning and the regulatory styles are carried out using a deductive linear pathway ("top-down" procedure; Neuman, 2011, pp. 69–70), by means of quantitative research methods. The decision to use a quantitative approach was also based on the need to apply highly structured and standardized statistical procedures, which are elaborated on below (see section 3.2.2).

After deciding to apply a self-report questionnaire, the criteria for this study's survey (or *self-report*; see Borkenau, 2006) were chosen that would be used to measure the 17 personal meanings and five regulatory styles in Germany and Finland, as I describe in the following section.

[20] As indicated earlier (cf. 2.1.1), integrated regulation is excluded as this style cannot be empirically distinguished from identified regulation (Vallerand et al., 1992).

3.2.1 Selection of the Classical Survey Instruments

In order to select the most suitable survey instrument(s), it is important to identify the properties of this study's affective constructs. Both affective constructs, personal meaning (Vollstedt, 2011b; Vorhölter, 2009) and regulatory styles (Deci & Ryan, 2000), are so-called latent psychological constructs (Pospeschill, 2010; p. 177), defined as "mental structures and processes that are not directly observable; rather, they are postulated to explain observable behaviour" (Macho, 2015, p. 1, translated by the author). On the basis of this particular nature of personal meaning and regulatory styles, these theoretical constructs can be measured by a range of specific observable (manifest) variables (Backhaus et al., 2015; Brown, 2006; Moosbrugger & Schermelleh-Engel, 2006; Rost, 2006). In order to measure these constructs, the immanent theoretical concepts need to be specified (Bühner, 2011; Pospeschill, 2010). In this connection, the latent theoretical construct is operationalized through several standardized items that systematically cover the whole range of the latent construct's conceptual knowledge (Rost, 2004; Steyer & Eid, 2001).

Generally, the assessment, evaluation, and subsequent selection of adequate survey instrument(s) are critical when exploring scholars' individual research objectives. More often than not, the empirical phenomenon appears too specific, so that established surveys cannot be applied (Loewenthal, 2001). As a solution, some scholars develop their own survey instrument. In many cases, it is possible to use already existing validated scales with good psychometric properties that can either be applied directly (if the research phenomenon is the same) or after modification as an adapted version of the instrument (Döring & Bortz, 2016, p. 410; Moosbrugger & Höfling, 2006). If scholars need to develop an instrument for a new theoretical concept (such as for the assessment of the construct personal meaning) or a very specific target—for instance if there are no established instruments for the phenomenon of interest—then a robust and systematic procedure is necessary for scale construction (Döring & Bortz, 2016; Pospeschill, 2010). If that is the case, self-developed survey questionnaires should be validated in terms of their psychometric properties before they can be applied in a study (Moosbrugger & Höfling, 2006; Pospeschill, 2010; Schermelleh-Engel et al., 2006). The methodical process used to validate the test standards for a new instrument is made up of a large number of intricate statistical procedures, which are used to assess and evaluate the relevant quality benchmarks. Particularly in cross-cultural studies, the survey instrument should also be available in the corresponding national language (Kelle & Buchholtz, 2015). A literal translation alone is not enough. In order to ensure that the survey items are understood in the same way in all countries, the survey instrument should be subjected to a thorough check by a native speaker based on local knowledge (e.g., population's age group and their social desirability; Chen, 2008) of the population (Kelle & Buchholtz, 2015) as well as through robust statistical estimation measures (e.g., test of measurement invariance). Due to the intricate and laborious nature of the instrument-validation process, academics have recommended using established survey instruments where possible (Döring & Bortz, 2016; Loewenthal, 2001).

In this study, the literature review has drawn the following conclusions on the application of survey instruments: Motivational regulatory styles can be measured, for instance, by a survey questionnaire (Ryan & Connell, 1989; Ryan & Deci, 2017; Thomas & Müller, 2011). Various empirical studies have confirmed their successful application in different contexts (including mathematics education; Ryan & Deci, 2017). In this present study, a standardized survey instrument (Thomas & Müller, 2011) is used to assess each learners' quality of motivation in mathematics lessons across countries. Since this instrument has been developed for the German-speaking countries, translation work and further statistical examinations are necessary to integrate Finnish students' data into this present investigation (see section 3.3.3).

Regarding personal meaning, as already mentioned (cf. 2.2.2), a research team at the University of Bremen (Büssing, 2016; Schröder, 2016; Vollstedt & Duchhardt, 2019; Wieferich, 2016) developed a survey instrument to assess the 17 personal meanings students assign to mathematics learning. Concerning the validation process of this young instrument, the following insights from previous investigations are of particular importance for this present study: Almost all scales of the 17 personal meanings showed good psychometric properties, apart from the scale for "classroom management" (Vollstedt & Duchhardt, 2019). A revision was thus necessary in order to consider classroom management in this present investigation (see section 3.3.2). I used an adapted version of the PISA 2012 items (for disciplinary climate in the classroom; OECD, 2014b, p. 331) to estimate classroom management that exhibits efficient organization and structure of teaching, which is one facet of the personal meaning "efficiency" (Vollstedt, 2011b). Furthermore, Büssing's (2016) study found that an additional personal meaning needed to be considered in order to depict "relevance of application" (one of the seven types of personal meaning, Vollstedt, 2011b), so "reference to reality" was included within this survey as an additional kind of personal meaning. Moreover, a previous study (Suriakumaran et al., 2019) had considered the same database as this present dissertation, in which the scale for the personal meaning "balance and even-temperedness" did not fit the data. Consequently, this personal meaning was excluded from this research.

A further result concerned Vollstedt's typology (2011b). As indicated earlier, from a theoretical perspective, Vollstedt summarized the 17 reconstructed kinds of personal meaning from her study that were related to two dimensions, intensity of relatedness to mathematics and intensity of relatedness to individual, into seven types of personal meaning (cf. 2.2.2). Model-fit values indicated that these two meta-factors could not hold the data (Vollstedt & Duchhardt, 2019). To seek an adequate measurement model, Vollstedt and Duchhardt (2019, p. 161) recommended that future research investigate appropriate dimensions or meta-factors by reviewing the relations to relevant concepts, for example motivation, or interest, which theoretically affect learners' constructions of personal meaning.

Highlighting the theoretical links between personal meaning and SDT, I hypothesized an alternative segmentation (cf. 2.3.1) for this present study that structured the 17 personal meanings into eight dimensions with respect to two meta-factors: the orientation towards the regulatory styles and the orientation towards the relatedness of mathematics. In this investigation, two crucial aspects motivated my exploration of the eight dimensions in separate measurement models.

Firstly, the categorizations into separate dimensions were made by specifying the theoretical links between the 17 personal meanings and five regulatory styles (cf. overview of predicted dimensionality in Figure 5). Both meta-factors thus reflected the theoretical conceptualizations behind personal meaning (Vollstedt, 2011b; Vorhölter, 2009) and regulatory styles (Ryan & Deci, 2017). This suggestion focused in particular on the theoretical correlations between them. Hence, through this investigation, I was able not only to examine how far these eight dimensions or eight measurement models relate to the German and Finnish data, but also how they cross-culturally relate to the regulatory styles and appear in students' profiles of personal meaning (as profile specifications with respect to their particular quality of motivation). In this spirit, testing the empirical factor validity of the predicted dimensions would empirically support the hypothesized links between personal meaning and regulatory styles (SDT).

Secondly, the specification of too many factors in one overall measurement model is challenging, especially if a matured conceptual rationale is not provided (Brown, 2006). That is to say, considering a huge amount of variables (in this case, 17 personal meanings) from a relatively new theoretical construct in one complex measurement model is risky. Therefore, I aimed to differentiate the 17 personal meanings in a meaningful fashion (with respect to this research interest), where I expected high correlations between the factors (kinds of personal meaning within one dimension) to occur. These were the two main factors that motivated the splitting of the 17 kinds of personal meaning into eight dimensions. Regarding the suggestion of Vollstedt and Duchhardt (2019), this specification may provide a meaningful way not only of conducting the statistical analyses, but also of increasing the empirical knowledge about the theoretical concept of personal meaning in relation to quality of motivation.

Considering 17 personal meanings and five regulatory styles within one survey instrument is demanding, particularly in a cross-cultural setting. This complex composition (the combination of a new with an advanced instrument) thus required a highly structured and standardized procedure to be adopted that suits the focus of the current research project. The standardization applied in quantitative research provided a secure framework on which to carry out the appropriate statistical procedures. These standards help us to determine the psychometric properties of these affective constructs. In particular, they provide us with methods for dealing with the methodological challenges and biasing effects associated with studying established properties across cultures (Chen, 2008).

However, when quantitative statistical procedures were conducted, profiles of personal meaning and their corresponding quality of motivation are available in the form of numerical data. The examination of the identified empirical profiles provided an insight into German and Finnish students' individual experiences in the classroom. However, when it comes to interpreting the double ontological natures of personal meaning not only in the biological, but also in the social world, the quantitative mono-methodological research design showed technical limitations. To compensate for the limitations of quantitative methods, methodological strategies are required that support the consideration of personal meaning as a boundary object in both worlds. If we are to counterbalance this weakness by means of an additional strategy, firstly the strengths and limitations of the quantitative tradition should be clarified with regard to this study's research interest.

3.2.2 The Quantitative Research Approach to Studying Personal Meaning's Individual Nature

The quantitative research paradigm refers to basic assumptions that formulate a purely rational structure to plan the required methodical procedure in terms of measuring latent constructs in a cross-cultural context. Which specific limitations of quantitative research need to be compensated for to reach this study's research objectives?

Strengths of the Quantitative Research Approach as Regards the Goal of This Study. To start with the strengths, the *hypothetico-deductive method* (Lawson, 2015; Popper, 1968) associated with the philosophical perspective of the quantitative paradigm, that is, *critical rationalism* (Popper, 1989) formulated maxims of how to conclude theoretical knowledge by emphasizing the philosophical ethos of Popper's idea of falsification (Popper, 1989, 2000; Przyborski & Wohlrab-Sahr, 2014). Following a deductive direction, this type of research predicts hypotheses that are based on existing theoretical knowledge. Next, consequences are deduced and empirically measured in order to test the theory in practice and gather evidence (Lawson, 2015; Neuman, 2011; Popper, 1968). Therefore, precise steps for theory-driven problems are formulated in such a way that they are empirically testable and falsifiable (Döring & Bortz, 2016; Lamnek & Krell, 2016; Przyborski & Wohlrab-Sahr, 2014). For instance, theoretical antecedents (preferably connected to related qualitative work) depict assumptions about the reality. Then, progress in findings is gained in a stepwise fashion through structured processes of statistical operations that examine the empirical observations that have been obtained in order to address unsustainable theories (Döring & Bortz, 2016; Neuman, 2011; Popper, 2000). In the quantitative approach, the use of standardized (numerical) data, gathered from a large number of cases (in contrast to qualitative research; Döring & Bortz, 2016), is very typical (Kelle & Buchholtz, 2015; Neuman, 2011). The purpose of this is to develop object-related theories further (Przyborski & Wohlrab-Sahr, 2014) and to tentatively approve those theoretical findings that have repeatedly been tested but have not been falsified (Döring & Bortz, 2016, p. 37).

An important line of critical rationalism is the critical review of the research quality, as well as the disclosure of the methodical procedures and survey instruments of an investigation (Döring & Bortz, 2016). The criteria used to standardize these quantitative methods include objectivity (observer independency), reliability (consistency of results) and, most importantly, validity (validity of resulted empirical evidence; see section 3.2.3; Döring & Bortz, 2016; Kelle & Buchholtz, 2015). The applied object-related theories are established at the beginning and the end of the quantitative research frame, whereas the development of new theories is not the focus in this research paradigm (Neuman, 2011; Przyborski & Wohlrab-Sahr, 2014).

At the same time, this tradition contains several methodological issues that may arise in particular for this study's research context: To help ensure that this study adds to our store of knowledge of this subject, this research approach adheres to a falsification process by confronting empirical evidence with potential inaccuracy in theory, as well as taking into account methodical effects that may result from any issues and difficulties associated with the survey instruments (Döring & Bortz, 2016, p. 37). This was necessary because, more often than not, contradictory empirical results sometimes occur as a result of measurement or operationalization errors. An example of this is the *problem of correspondence* (Döring & Bortz, 2016), which refers to the problem of verifying whether the manifest variables are a precise measure of the correct theoretical concepts. This type of inconsistency, between the theoretical knowledge and the standardized data, should not only be viewed as a contradiction of the hypothesized theory to be tested, but also as a cause conditioned by the measurement theory to assess the empirical phenomenon, as both are always connected to one another. That is so because each data set has an individual methodological and methodical formation and its biography ultimately become a crucial element of the study's evidence and in turn the implications derived from that evidence (Radford, 2008a; Schoenfeld, 2002).

Additionally, to ensure a balanced scientific approach, it is recommended that when researchers study and interpret the empirical evidence produced, they should critically analyse alternative theoretical explanations as well as their own theoretical ideas (Döring & Bortz, 2016; Fiedler et al., 2012). Therefore, *meta-theoretical knowledge* that confronts the empirical evidence with related or competing theories is essential (here referring to the affective constructs of personal meaning and motivation; Döring & Bortz, 2016; Fiedler et al., 2012; Przyborski & Wohlrab-Sahr, 2014). However, most quantitative studies ignore the meta-theoretical consideration of the object-related theory (Przyborski & Wohlrab-Sahr, 2014).

Moreover, there are some further critical aspects that need to be taken into account in order to compensate for any possible misconstruction of evidence from different cultural contexts. Especially in an educational context, culture-specific knowledge about learners' particular living environments plays a central role (Chen, 2008). The relations between the student, teacher, school, subject curricula, and educational system are shaped by the respective cultural factors, such as social desirability (Chen, 2008). In cross-cultural studies, typically the sociocultural background of the scholars who conduct the study and the subjects who participate in the study vary as a result of missing *heuristics of common-sense knowledge* (Kelle & Lüdemann, 2019, p. 121). Furthermore, limited knowledge concerning the sample's living environment may lead to *problems of operationalization and measurement* (Kelle & Buchholtz, 2015). This issue could arise if scholars have not considered the respondents' linguistic usage or the topics that are relevant to them (in this case, German and Finnish ninth graders) so that the respondents do not understand the scholars' intended message in the survey items (Kelle & Buchholtz, 2015).

It is therefore clear that acting by rote (mechanically implementing statistical methods like following a formula) is not sufficient to meet the requirements of scientific research (Döring & Bortz, 2016). This discussion has illustrated that even a purely quantitative research study cannot function without reference to a qualitative interpretation work, that is, theoretical conclusion (Kleemann et al., 2013).

Given the above concerns, the current study aims to address all of the types of knowledge that emerged during the quantitative data evaluation. Furthermore, experts from Germany and Finland evaluated the survey instrument to ensure it took account of respondents' language and sociocultural issues. This helped to counter heuristics of common sense and problems of operationalization and measurement (see section 3.3.2).

The first part of the empirical data analyses studied the psychometric properties of the latent constructs of personal meaning and regulatory styles using multivariate analysis methods (confirmatory factor analysis). These statistical procedures (see section 3.3.3) aimed primarily to transform the theoretical knowledge into numerical data. The examination included studying how these properties are operationalized in different cultural contexts. As several cross-cultural studies have demonstrated, the use of survey instruments in one culture that originated in another may present significant challenges (Byrne, 2000; Bofah & Hannula, 2015), for example, poor construct validity (Chen, 2008). Therefore, robust methods were used that counter biasing effects in survey instrument (test of measurement invariance; Marsh & Köller, 2004). Further, for the identification of profiles (individual relevance systems of personal meaning), I used explorative research methods; that is, latent class analysis with distal outcomes. To compare similar profile outputs across countries, I conducted Mann–Whitney U tests. For a condensed overview of this investigation's empirical work, see Figure 11 in section 3.3.3.

In contrast to the regulatory styles, a new survey instrument measured the 17 personal meanings in two different cultural settings. Both instruments are at different stages of development. To compensate for problems of correspondence, a rigorous investigation of theoretical concepts' psychometric properties is necessary. If indicators of theory inconsistency exist, this false-positive theoretical and statistical empirical evidence (Döring & Bortz, 2016) may reflect the existence of hidden knowledge in terms of the study's theoretical concepts. A critical discussion of these paradoxical indicators may establish opportunities to enhance concepts' descriptive/explanatory power (Schoenfeld, 2002) or to improve the survey instrument of personal meaning for future research.

Weakness of the Quantitative Research Approach as Regards This Study's Goal. The methodological considerations elaborated from the conducted empirical work provided the philosophical background for the application of self-report questionnaires in the present investigation. With regard to this study's aims, these quantitative statistical analyses support the study of personal meaning as an individual psychological phenomenon when considering the concept through the lens of SDT (biological regulatory mechanism). Therefore, the self-reported data can be explored in terms of the predicted biological regulatory mechanism (connection between SDT and personal meaning) that becomes observable in learners' motivational behaviour in the classroom.

However, to examine whether personal meaning also exists in the social world, in which it becomes observable in learners' social interaction, it is necessary that the empirical evidence, on an individual level, can be interpreted through the lens of social constructivism; that is, in relation to the adapted model predicting the *social regulatory mechanism of* the *individual relevance system* (cf. Figure 6). Therefore, the empirical evidence (the detected profiles of personal meaning and their motivation) and the adapted model should come into a theory-driven dialogue.

Within the quantitative research paradigm, there is no clear description of how to interpret empirical evidence in view of networking theoretical perspectives. Without this interpretation work (Bernard & Ryan, 2010), the multimodal nature of personal meaning cannot be thoroughly discussed as intended in this research. As a response to this weakness of the quantitative approach, it is not only necessary but also justified to incorporate an additional research practice in order to consider the social nature of personal meaning; namely, networking of theories (cf. Prediger & Bikner-Ahsbahs, 2014; Prediger et al., 2008). In other words, a theory-driven interpretation of quantitative data, in terms of systematically coordinating the two theoretical perspectives, follows the quantitative statistical analyses in order to qualify the quantitative data (Schoonenboom & Johnson, 2017). As indicated above, the methodology of the empirical work and the methodological considerations of the theory-driven analyses differ. For this reason, I describe the methodological considerations and the applied coordinating analyses method in more detail in the next chapter (see Chapter 4).

After presenting the strengths and limitations of this study's quantitative research approach, I highlight the methodological properties of this empirical work.

3.2.3 Quality Criterion and Quality Assurance

Science should by definition always refer to consistent, common standards that adhere to the relevant quality benchmarks (Döring & Bortz, 2016; Neuman, 2011; Rost, 2004). The level of scientific quality produced by these methods is complex to assess and should always be examined individually, taking into account the strengths and weaknesses of the study in question (Döring & Bortz, 2016). Therefore, it is important to communicate any necessary methodological limitations transparently to the scientific community (Döring & Bortz, 2016; Neuman, 2011). In the following section, I briefly refer to the quality criteria of quantitative research that relate specifically to this study's properties.

Meta-Theoretical Reflection. Before conducting an empirical study, the clarification of meta-theoretical preconditions plays a fundamental role. As theoretical precursors of the substantive framework, meta-theoretical matters encompass comprehensive ideas concerning the research phenomenon, the concept on which it is based, its prerequisites, and its significance (Przyborski & Wohlrab-Sahr, 2014). This philosophical foundation is usually omitted in quantitative studies or is referred to only obliquely (Przyborski & Wohlrab-Sahr, 2014). The reflection of a profound research logic behind empiricism is not only essential for the progress of knowledge in a discipline, but also for the coherent and consistent understanding of the results in scientific discourse (Döring & Bortz, 2016; Przyborski & Wohlrab-Sahr, 2014). Thereby, the argumentation of meta-theory as a concrete scientific justification also forms part of these research results in order that this knowledge can be applied or addressed in future research (Döring & Bortz, 2016; Przyborski & Wohlrab-Sahr, 2014).

In this study, the conceptualization of motivation is briefly elaborated while pointing out the differences between the quality and quantity aspects of motivation (cf. 1). As mentioned earlier, both kinds of motivation are relevant within the academic context. However, in contrast to achievement-related behaviour, the quality of motivation explains the differences in learners' types of behaviour and their quality of functioning (Ryan et al., 1996) in the mathematics classroom.

Furthermore, in previous research (Suriakumaran et al., 2020), so as to study learners' motivation systematically, affective constructs were distinguished from theoretically comparable concepts by highlighting their conceptualizations. In this spirit, certain theoretical aspects of quality of motivation (basic psychological needs and regulatory styles) were targeted as they play a crucial role in the educational context (Krapp, 2005; Nuttin, 1984; Ryan & Deci, 2020). On this basis, scholars identified which motivational construct would increase our knowledge of the quality aspect of motivation by considering the fundamental theoretical elements of motivation. A tentative comparison between values (Wigfield & Eccles, 2000) and personal meaning (Vollstedt, 2011b; Vorhölter, 2009) helped to select an appropriate affective construct, namely personal meaning, which represents learners' personal relevance in a mathematics-learning context (see Suriakumaran et al., 2020 for more). The concept specification (Döring & Bortz, 2016) of personal meaning explicitly considers critical components of quality of motivation (basic psychological needs).

Within the frame of this quantitative study, the theoretical elaborations referred to are those meta-theoretical knowledge aspects that could be ascertained (Przyborski & Wohlrab-Sahr, 2014). If relevant indicators emerged in the evidence, the relevant meta-theoretical knowledge was retrieved and is discussed here in terms of this present study's research interest.

Construct Validity. Construct validity exists if the empirical results represent statements that reflect the estimated theoretical constructs; that is, if the validity of actual measures can be shown (Döring & Bortz, 2016; Schermelleh-Engel et al., 2006). Generally, latent constructs are measured by means of directly observable indicator variables (Backhaus et al., 2015). When examining latent constructs, the respective theoretical concept should be formulated precisely so that the construct can be empirically reflected (Rost, 2004). Thereby, the manifest variables should represent the various content-related facets of the construct (Moosbrugger & Schermelleh-Engel, 2006; Rost, 2004). The reflective measurement model built in this way (Backhaus et al., 2015) reflects by means of the indicator variables the direction of change that belongs to the associated construct (Rost, 2006). At this point, the roles of reliability and objectivity comes into play; these are briefly explained in what follows. Moreover, with respect to construct validity, the problem of correspondence according to critical rationalism should be taken into account. Therefore, critical reflection regarding the agreement between the intended theoretical concepts and the actual empirically measured data is essential (Döring & Bortz, 2016). To assess the construct validity, I considered certain scientific criteria (see confirmatory factor analysis in section 3.3.3) to guarantee methodical rigour (Backhaus et al., 2015; Döring & Bortz, 2016; Moosbrugger & Schermelleh-Engel, 2006).

Since this study examined data from two different countries, it was essential to check constructs' measurement invariance. In particular, for further investigation of data (e.g., comparison of affective constructs' latent means), it is important to estimate the actually existing measurement invariance in order to guarantee the required level of measurement invariance for such comparisons. At a minimum, (partial) scalar invariance is required to conduct latent mean comparisons over groups (see the test of measurement invariance in section 3.3.3). In addition, data cleansing plays a crucial role, as unstable data quality can reduce construct validity and impair the interpretation of results (Döring & Bortz, 2016). To ensure adequate handling of the missing values, I used the FIML (Full Information Maximum Likelihood) algorithm in Mplus (see missing data in section 3.3.3).

Objectivity. The existence of test objectivity guarantees that another test leader, independent of the scholar, could achieve exactly the same results by following their procedures in research. The criterion of test objectivity can be judged as fulfilled if the implementation, evaluation, and subsequent interpretation of methodical procedures are explicitly stated in the study (Rost, 2006). According to this, the procedures followed in this work, from data collection through statistical analyses to interpretation of the results, are presented in detail in the following chapters.

Regarding the theory-driven analyses (described in Chapter 4), to guarantee that the interpretation work that had been conducted was *intersubjectively* comprehensible and verifiable (Bohnsack, 2014) in terms of the coordinating theories, these analyses were also discussed with German and Finnish experts who were familiar with their respective educational systems in order to avoid the author's making a one-sided interpretation. The intersubjective decisions that were made with a group of experts give an objectifying character to the theory-driven analyses, especially when tentatively discussing the results in relation to mathematics curricula and the educational systems behind them.

Reliability. In the sense of classical test theory (Brown, 2006), the existence of test objectivity is a prerequisite for measurement accuracy; namely, reliability (Döring & Bortz, 2016). In this study, consistency is estimated using composite scale reliabilities ρ (Bentler, 2009; Raykov, 2009; Sijtsma, 2009). The German survey instrument of personal meaning is not based on established valid scales. The use of these new scales in a different country (other than Germany) may lead to weaker scale reliability. The possibility of reduced reliability (in this case in Finland) can negatively impact on the validity of the results as well as decreasing the statistical significance and effect sizes (Raykov, 2012). These aspects should be taken into account, before proceeding to the comparative analyses.

3.2.4 Generalizability and Representativeness

Since the investigated constructs, personal meaning and regulatory styles, are affected by the cultural context, I collected the sample in two different educational cultures in order to enrich the validity of the evidence as well as to contrast and refine the research results. As already mentioned above, the educational system in Germany is diverse, while the Finnish system is largely uniform (cf. 2.4). In Finland, data from schools from one province were considered, while the German data came from four federal states (see section 3.3.1).

Regarding profiling classes, by using quantitative methods *average types* were constituted; that is, profiles based on learners' mean values of the 17 personal meanings. These types are strongly bound in time and space (Weber, 1972). In accordance with the quantitative tradition, this study thus aims for representative data (although it is very limited here due to the relatively small sample size; see section 3.3.1) and not the representation of the social meaning (in case of *ideal types*; Weber, 1972). However, the empirical profiles in Germany and Finland cannot easily claim validity for the whole country, as they are simply "this dissertation's results", which I have collected from a few locations in these countries. Therefore, I consistently refer to "Results or profiles in Germany/Finland" instead of "German/Finnish results or profiles".

Based on the moderate sample size used in this research, a sound generalizability of research results is limited (Kelle & Buchholtz, 2015). However, if the empirical evidence found in Germany and Finland leads to theory-consistent explanations when testing the applied background theories that have been applied, this should increase our confidence in the validity of the evidence collected, at least on a preliminary basis.

3.3 Empirical Methods

This section describes the particular quantitative research methods that I used in the present investigation. At the end of this chapter, I reflect on the planned empirical work with respect to the remaining challenges.

3.3.1 Data Collection

Data was collected in Germany and Finland. Since the theoretical conceptualization of personal meaning (Vollstedt, 2011b; Vorhölter, 2009) was retrieved from ninth and tenth graders' interview data, I considered learners from a similar age cohort, namely ninth graders. The 276 German participants (♀: 46%) from 17 classes were from four different federal states (Bavaria, Baden-Württemberg, Bremen, and North Rhine-Westphalia) and attended different schools according to their academic performance (three *Gymnasien* (for high achievers), one *Realschule* (for middle achievers), three *Oberschulen* (comprehensive schools with streaming of classes according to academic performance), and two *Hauptschulen* (for low achievers). In this study, the focus is on the personal meanings used by all students when dealing with mathematics in an educational context. Therefore, to guarantee a heterogeneous group of participants, there was no separation between high and low achievers. The data in Finland is from 256 ninth graders (♀: 48%) from four comprehensive schools and 13 classes from the region of Uusimaa.

The permission for the survey was granted according to the respective German data protection act and by the German *Landesdatenschutzgesetze* (Verbund FDB, 2020), whereby I obtained separate authorizations for all four of the German federal states. In Finland, I followed the local regulations, according to which an ethics review was not required to conduct the survey. As a result, permission was requested in an information letter for students and parents, while a detailed description of the research plan was submitted to the teachers. For one school (in Vantaa), the author had to first ask the director of basic education for permission; once this was obtained, the director delegated responsibility for overseeing the study to the school's head teacher. In both countries, the author's colleagues and private contacts helped to find schools to participate in the study. The whole survey (assessment of both instruments: Personal meaning and regulatory styles) of the Germany and Finnish ninth graders by means of a 45-min (one lesson across countries) paper-and-pencil test took place from October 2016 to March 2018 during the school year (not at the beginning or at the end of the school year). In Germany, the author carried out the surveys in all classes on her own, whereas in Finland Dr Eeva Haataja, a Finnish-speaking PhD student from the Department of Education at the University of Helsinki, supported the data-collection processes in some schools.

3.3.2 Survey Instrument

As indicated earlier (cf. 2.2.2), a research team at the University of Bremen (Büssing, 2016; Schröder, 2016; Vollstedt & Duchhardt, 2019; Wieferich, 2016) developed the German instrument to assess students' different personal meanings. As we have seen, besides the personal meanings reconstructed by Vollstedt (2011), according to Büssing's (2016) suggestion, "reference to reality" was also included within this survey as an additional kind of personal meaning. Within a previous study (Suriakumaran et al., 2019) that considered the same data base as this present study, "balance and even-temperedness" did not fit the data; for this reason, this personal meaning was excluded from this research. Furthermore, as a consequence of the validation process (Vollstedt & Duchhardt, 2019), in this study a few items are adapted from established scales (OECD, 2014b; see Table 1) to measure "classroom management". I used an adapted version of PISA 2012 items (*for disciplinary climate in classroom*, OECD, 2014b, p. 331) to estimate classroom management representing efficient organization and structure of teaching; that is, one facet of the personal meaning "efficiency" (Vollstedt, 2011b). To conclude, 17 kinds of personal meaning were considered in this research. The scale for the study contained items that were formulated as self-oriented statements (e.g., "*I engage with mathematics in order to …*").

As mentioned earlier, regulatory styles were assessed using established scales (Thomas & Müller, 2011). This survey questionnaire considers the regulatory styles in an educational context, adapted for the subject mathematics. For both instruments, a 4-point Likert scale (0 = strongly disagree to 3 = strongly agree) is used to rate the survey. The following Table 1 illustrates the scales of 17 different kinds of personal meaning and the five regulatory styles that were considered in this dissertation.

To investigate Finnish students, the German instruments of personal meaning and regulatory styles were translated[21] into Finnish by Jessica Salminen-Saari and Dr Eeva Haataja. At the time that the data was being collected in Finland, both were PhD students at the University of Helsinki. To avoid translation mistakes or misconceptions (heuristics of common sense and problems of operationalization and measurement), the instrument was validated in a cognitive lab (Zucker et al., 2004). In collaboration with Dr Eeva Haataja, we conducted the cognitive lab with two Finnish ninth graders. Based on their feedback, retrieved from the cognitive lab, some items were correspondingly revised in the final version. Additionally, from the original total[22] of 131 items of personal meaning and 21 items of regulatory style, those satisfying expectations regarding psychometric properties and factor loadings across both German and Finnish data were retained. Consequently, the statistical analyses conducted in this study considered a total of 68 items of personal meaning in addition to 17 items of regulatory style. Moreover, mathematics performance (in Germany marks range from 1 to 6 (highest to lowest), whereas in Finland they rank from 10 to 4

[21] Initially, the author translated the German instruments into English.
[22] For the original survey instrument, see the additional file in Appendix A.

(highest to lowest)) and gender were additionally integrated as individual-related data. The focus of this study is neither on students' mathematics performance nor on their gender. Nevertheless, the inclusion of this background information may provide an implicit contribution to the meta-theoretical knowledge relating to this study's research interest.

3.3.3 Quantitative Statistical Analyses

In the following section, I describe the quantitative statistical methods used within this empirical work in more detail. The following overview (Figure 11) summarizes the planned empirical work in a condensed format.

Figure 11

Overview of the Present Investigation's Empirical Work.

Rationale	Data	Analysis Method
Investigation of affective constructs' psychometric properties and their relations in Germany and Finland	Personal meanings and regulatory styles	Reliability CFA Correlation analyses Measurement invariance
Identification of profiles of personal meaning in Germany and Finland	Personal meanings and regulatory styles	Latent class analysis with distal outcomes
Differences in profile outputs in Germany and Finland	Personal meanings and regulatory styles	Mann–Whitney U test

Multivariate Analysis Methods. Software. Statistical analyses were carried out using the Mplus statistical package (version 8; Muthén & Muthén, 1998–2017).

Missing Data. In this present investigation the estimated percentage of missing data was low (at a maximum of 1.9% in Finland and a maximum of 1.1% in Germany). Table 1 provides the exact amount of missing data for each item. I used the Mplus FIML function to treat missing data. The FIML process assumes that values are not randomly omitted and takes into account all the information provided in order to estimate the parameters of the model (Enders, 2010). Complex models require a large sample size and thus listwise deletion was ignored (Hoyle & Gottfredson, 2014).

Table 1.
Descriptive Statistics of Sample, Means, and Standard Deviations for all Items of Personal Meaning and Regulatory Style in German and Finnish Students.

Item	Germany			Finland		
	N	*M*	*SD*	*N*	*M*	*SD*
Active practice						
I enjoy doing calculations.	274	1.52	0.96	255	1.73	0.96
I am happy when I can do mathematical tasks.	272	1.07	0.90	253	1.68	0.98
I like to deal actively with mathematics.	273	1.33	0.91	253	1.75	0.91
Classroom management						
It annoys me if no one listens to what the teacher says.	274	1.88	0.95	255	1.84	0.96
It bothers me if the maths lesson is noisy and chaotic.	276	2.12	0.90	256	2.08	0.93
I don't like it if my teacher has to wait a long time until it becomes quiet in the lesson.	275	1.98	0.90	254	2.17	0.82
It is important to me that we can work well in maths lessons.	276	2.34	0.62	255	2.36	0.65
It annoys me if the students only start working a long time after the lesson has already begun.	276	1.44	0.88	253	1.45	0.93
Cognitive challenge						
If I cannot solve tricky tasks immediately, I think about it until I get it.	275	1.74	0.91	253	1.97	0.89
It is important to me that I feel challenged from mathematics.	272	1.47	0.83	254	1.63	0.91
I am unmotivated when the tasks in the maths lessons are too simple.	273	1.15	0.90	256	1.24	1.00
Duty						
I mainly deal with mathematics because I have to.	276	1.58	0.94	252	1.37	0.99
I deal with mathematics mainly because it is a compulsory subject.	272	1.76	0.94	249	1.57	1.00
Dealing with mathematics is nothing more than an obligation.	274	1.45	0.90	254	1.30	0.92
To deal with mathematics is an obligation to me.	275	1.55	0.97	252	1.54	0.98
Emotional-affective relation to the teacher						
It is important to me that I have a good relationship to my teacher in maths lessons.	273	2.20	0.80	248	1.98	0.91
I pay more attention in maths lessons if I like the teacher.	273	1.99	0.92	255	2.22	0.83

I prefer to go to maths lessons when I realize that the teacher is interested in me.	276	1.64	0.93	252	1.51	0.98
It is important to me that I feel valued by my teacher.	275	1.98	0.82	247	1.91	0.86
Examination						
I run through a large number of practice exercises in order to have a good feeling before exams.	273	1.55	0.94	256	1.69	0.94
Before exams I work on more tasks than those given to us so that I know that I understand the topic.	273	1.52	0.99	248	1.76	1.04
I prepare intensively before a maths examination in order to do well.	274	1.97	0.90	256	1.75	0.96
EXPERIENCE OF AUTONOMY						
It is important to me to independently find the solution for new types of tasks.	275	1.78	0.83	253	1.74	0.88
Working autonomously is important to me when it comes to mathematics.	275	1.70	0.77	251	1.69	0.83
It is important to me to explore new mathematical contents independently.	271	1.34	0.79	251	1.36	0.88
When dealing with mathematics it is important to me to recognize that I can manage tasks without any help.	275	1.91	0.81	254	1.98	0.91
EXPERIENCE OF COMPETENCE						
I have a positive feeling when I succeed in mathematics.	272	2.47	0.66	255	2.51	0.73
I am proud of myself when I solve a mathematical task which was difficult for me.	276	2.33	0.75	253	2.37	0.83
I feel good when I can help others in maths lessons.	270	2.19	0.76	249	2.00	0.87
I am proud of myself when I can present my solution in front of my class.	273	1.89	0.90	254	1.61	0.96
EXPERIENCE OF RELATEDNESS						
It is important to me that there is a good sense of community in maths lessons.	275	1.89	0.75	254	1.87	0.79
It is important to me that preferably everybody should understand the content in maths lessons.	275	1.46	0.92	246	1.70	0.94
It is important to me that there is a positive classroom atmosphere during maths lessons.	274	2.10	0.72	253	2.13	0.80
If I can, I am happy to help the others with their problems in mathematics.	273	2.24	0.78	249	2.08	0.94
General knowledge						
Learning mathematics is meaningful/sensible/reasonable because responsible	271	1.79	0.77	251	1.96	0.85

citizens should have mathematical skills.						
I think that learning mathematics is important because it is of major importance to society.	273	1.66	0.85	251	1.83	0.89
Mathematics is important to me because thinking mathematically has many advantages.	275	1.81	0.83	252	1.86	0.93
I deal with mathematics because logical thinking pertains to general education.	272	2.11	0.77	251	2.06	0.88
Marks						
I deal with mathematics in order to be proud of my marks.	274	1.81	0.91	254	2.11	0.82
I make an effort in mathematics in order to be pleased with my good marks.	274	2.08	0.84	255	2.30	0.76
I deal with mathematics as good marks strengthen my self-confidence.	274	1.73	0.93	254	2.01	0.88
Positive image						
It is important to me that I am given credit for my achievements in mathematics from people who are important for me.	270	1.43	0.98	249	1.74	0.88
It is important to me that others get a positive impression of me through my mathematics achievements.	273	1.34	0.90	249	1.51	0.94
I deal with mathematics in order to get appreciation for my achievements.	273	1.22	0.87	249	1.45	0.86
It is important to me that my family can be proud of my mathematics achievements.	272	1.92	0.96	247	1.90	0.87
Purism of mathematics						
In its purity, mathematics is uniquely beautiful to me.	270	1.36	0.95	255	1.38	0.97
The structure of mathematics fascinates me.	273	1.27	0.91	253	1.44	0.97
The beauty of mathematics appears to me in simple results for difficult problems.	270	1.40	0.87	253	1.45	0.94
I am overjoyed by the uniqueness of the mathematical technical language.	269	1.07	0.81	254	1.14	0.84
Mathematics is beautiful to me as it is unique in its formalism.	270	1.10	0.85	250	1.12	0.91
Due to the logical structure of mathematics it is easy for me to understand the content.	273	1.75	0.81	256	1.70	0.92
I am excited about mathematics because mathematical statements are either right or wrong.	274	1.37	0.81	254	1.30	0.90
Reference to reality						
It is important to me that the content of maths lessons refers to everyday life.	273	1.45	0.88	251	1.35	0.86

I understand mathematics more easily if tasks refer to examples from my life.	272	1.68	0.89	250	1.48	0.90
I prefer mathematical tasks which apply to situations from real life.	273	1.77	0.87	251	1.81	0.82
Tasks referring to everyday life make me feel more involved with the mathematical content.	271	1.43	0.86	251	1.30	0.86
Self-perfection						
I work on many mathematical tasks in order to become better and faster.	273	1.26	0.82	255	1.68	0.87
I deal with mathematics in order to improve my logical thinking.	273	1.60	0.81	255	1.99	0.87
I deal with mathematics in order to increase my mathematical self-confidence.	271	1.20	0.80	253	1.64	0.91
Mathematics is important to me because I can develop myself in a number of ways.	273	1.41	0.84	254	1.51	0.89
Support by the teacher						
My teacher's support is important for my learning process in mathematics.	272	2.04	0.84	255	1.98	0.89
It is important to me that my teacher communicates my progress in mathematics to me.	271	2.10	0.81	250	1.88	0.87
It is important to me that my teacher is interested in my progress in maths lessons.	270	1.96	0.81	247	1.85	0.81
Vocational precondition						
I deal with mathematics as I need it for my desired profession.	270	1.30	1.06	251	1.85	0.99
I deal with mathematics because I will have a wider selection of professions later on.	273	1.75	0.92	251	2.08	0.88
I try to be good at mathematics because thereby I will have better opportunities in a profession later on.	273	2.02	0.87	250	2.32	0.80
I learn mathematical content because I have to apply it during my studies/vocational training.	271	1.79	0.88	248	1.97	0.83
I will not need school mathematics for my professional life.	274	1.87	0.95	251	2.26	0.86
Amotivation:						
In maths lessons, I am just physically present but I don’t think.	273	0.69	0.78	252	0.83	0.89
If the teacher does not notice, I do other things.	274	1.12	0.84	249	.1.29	0.90
I do not care about maths lessons.	274	0.64	0.84	254	0.79	0.83

External regulation:

I work and learn in the subject mathematics…						
… because otherwise I will get into trouble at home.	273	0.98	0.97	247	1.00	0.97
… because otherwise I will get in trouble with my teacher.	273	0.82	0.72	247	0.90	0.82
… because my parents require that from me.	273	1.26	1.03	247	1.55	0.92
Introjected regulation:						
I work and learn in the subject mathematics…						
… because I like to be better than my classmates.	272	1.09	0.95	247	1.13	0.98
… because I want the other students to think I am smart.	273	1.16	0.97	247	1.22	0.94
… because I like to receive praise.	270	1.43	0.97	245	1.19	0.89
Identified regulation:						
I work and learn in the subject mathematics…						
… because thereby I will have more opportunities to choose a profession.	272	1.92	0.91	246	2.30	0.79
… in order to be able to do further education.	273	1.98	0.95	247	2.40	0.73
… because thereby I can get a better job.	273	2.04	0.89	247	2.26	0.85
Intrinsic regulation:						
I work and learn in the subject mathematics…						
… because it is fun.	271	1.45	0.95	247	1.49	0.95
… because I like to deal with mathematics.	270	1.42	0.94	246	1.42	0.93
… because I like to solve mathematical tasks.	273	1.37	0.96	247	1.57	0.98
… because I like to think about the topics in the subject mathematics.	273	1.25	0.87	247	1.43	0.91
… because I enjoy dealing with mathematics.	272	1.16	0.88	246	1.48	0.95

Reliability of Scales.[23] Using latent variable modelling, I assessed the scale reliability ρ (Raykov, 2012). The estimator ρ reflects the composite reliability, an alternative to the more widely used Cronbach's alpha. The decision to choose ρ instead of Cronbach's alpha was underpinned by the need to obtain a precise estimate of the respective latent constructs and avoid inconsistency, such as overestimation or underestimation of the reliability (Bollen, 1989; Geldhof et al., 2014; Raykov, 2009). For all 17 personal meanings and five regulatory styles, I estimated the scale reliability ρ (see Table 2). Regarding Germany, the composite reliabilities for personal meanings and regulatory styles were acceptable to good. The values for personal meanings varied between .61 (EXPERIENCE OF SOCIAL RELATEDNESS) and .87 (purism of mathematics) and for types of motivation from .76 (amotivation) to .90 (intrinsic regulation). Furthermore, for Finland, the values of the measured composite reliabilities were also acceptable to good. They ranged from .66 (EXPERIENCE OF SOCIAL RELATEDNESS) to .90 (purism of mathematics) for personal meanings, and between .71 (external regulation) and .89 (intrinsic regulation) for regulatory styles. As imported affective constructs, the scales did not result in lower reliability in Finland than in Germany, despite the translation of material from the original language.

[23] Reliabilities are measured in nine separate measurement models reflecting the eight dimensions of personal meaning and one model encompassing the regulatory styles (see section 5.1 for more).

Table 2.
Scale Reliabilities for Regulatory Style and Personal Meaning Scales in German and Finnish Students.

Scale	Amount of items	ρ Germany	ρ Finland
Regulatory Style			
Amotivation	3	.76	.76
External regulation	3	.81	.71
Introjected regulation	3	.82	.84
Identified regulation	3	.89	.89
Intrinsic regulation	5	.90	.89
Personal meaning			
Active practice	3	.82	.77
Classroom management	5	.78	.80
Cognitive challenge	3	.67	.67
Duty	4	.81	.87
Emotional-affective relation to the teacher	4	.70	.75
Examination	3	.78	.83
EXPERIENCE OF AUTONOMY	4	.71	.72
EXPERIENCE OF COMPETENCE	4	.70	.72
EXPERIENCE OF RELATEDNESS	4	.61	.66
General knowledge	4	.78	.84
Marks	3	.85	.84
Positive image	4	.72	.79
Purism of mathematics	7	.87	.90
Reference to reality	4	.82	.80
Self-perfection	4	.71	.81
Support by the teacher	3	.70	.78
Vocational precondition	5	.81	.80

Confirmatory Factor Analysis. I recorded the reliability and the validity of the hypothetical constructs by means of confirmatory factor analysis (CFA). As an advanced method of multivariate analysis, CFA is used to operationalize theoretically based latent variables using directly observed indicators on an empirical level (Backhaus et al., 2015). Following a reflective measurement model (Backhaus et al., 2015; Moosbrugger & Schermelleh-Engel, 2006), the indicator variables entirely represent the latent variable. For this reason, the assignment of the observed variables, the determination of the amount of factors, and their meaning in terms of the theoretical consideration should be explicitly considered (Backhaus et al., 2015; Brown, 2006).

Data were analysed with the Mplus robust maximum likelihood estimator (MLR). The estimator MLR is robust to non-normality of the manifest variables. Because of the sample's nested design (interest in modelling the differences at the individual level, not across classes), design-based correction of standard errors, and model-fit statistics, MLR is used with the analysis function "complex" (Muthén & Muthén, 1998–2017; Muthén & Satorra, 1995). In both countries, I considered "student's grade of school" as the clustering level. For the model evaluation, several model-fit indices should be considered. In this study, statistical goodness-of-fit of CFA was measured using the Comparative Fit Index (CFI), the Tucker Lewis Index (TLI), the root mean square error of approximation (RMSEA), and the standardized root mean square residual (SRMR). For chi-square difference testing, the Satorra-Bentler (Satorra & Bentler, 2001) scaled χ^2 is included to estimate chi-square under non-normality. The CFI and TLI fluctuate from 0 to 1; indices > 0.90 show an acceptable model fit to the data, while those ≥ 0.95 are an excellent model fit (Hu & Bentler, 1999). The RMSEA values $\leq .06$ show good and $\leq .08$ an acceptable approximate model fit (Hu & Bentler, 1999). The SRMR are indicated as reasonable if the values are $\leq .08$ (Hu & Bentler, 1999).

Test of Measurement Invariance. The concept of measurement invariance is an essential requirement for the comparison of relationships and latent means of constructs in at least two different grouping variables, as here in Germany and Finland. Hence, students should respond in the same way across groups if they have the same score on the underlying construct. This statistical method tests whether the cross-cultural comparisons are equivalent and the survey instrument is operating as theoretically intended between both countries, and allows the cross-cultural generalizability of the estimated factor models (Steenkamp & Baumgartner, 1998). When testing measurement invariance, the context conditions of an investigation comparing German and Finnish students can be defined. Non-invariance indicates invalid or biased group comparison due to a different contextual understanding of the items (DIF items; French & Holmes Finch, 2016). If non-invariance is ignored, the results provide an unfair, faulty, and meaningless comparison (i.e., comparing apples and oranges; Cheung & Rensvold, 2002). In this study, students may understand DIF items differently due to the influence of their educational systems, inspired by the corresponding cultural context.

The existence of a measurement invariance can be gradually examined to demonstrate the validity of the hypothesized model in different groups. This investigation is carried out on the basis of a sequence of hierarchically nested linear multiple-group CFAs. These nested CFAs become increasingly restricted. While the least restrictive baseline model has no invariance constraints, the most constrained model requires the parameters in question to be equal across the countries (Bollen, 1989; Brown, 2006; Chen, 2007; Meredith, 1993). Within these nested models, the initial three models of the series have a certain role when comparing the mean structure in Germany and Finland, namely configural, metric, and scalar invariance. The baseline model, so-called *configural invariance*, indicates with acceptable model fit, identical factor amount, and factor loading patterns between indicators and latent variables. In the subsequent model, the factor loadings are required to be equal across groups by testing *metric invariance*. This level constitutes weak measurement invariance. Metric invariance is a precondition for all other upcoming logically constituted levels of increasingly stringent restrictiveness. *Scalar invariance* requires identical factor loadings (metric invariance) and item intercepts of indicator variables and implies strong measurement invariance. Latent mean comparisons across groups are only possible if models show scalar invariance. When scalar invariance is reached, the latent constructs respond to the certain peculiarity of indicator variables. Hence, the presentation of configural, metric, and scalar invariance supports the comparison of latent mean differences for a meaningful interpretation (Cheung & Rensvold, 2002). Lastly, the most restrictive level is *strict measurement invariance*, which confirms equal item uniqueness. As this level is not compulsory to a comparison of the construct's latent means, this model was not estimated in this case.

However, achieving scalar invariance is demanding, especially when relatively new instruments are implemented in the survey (as in the present study). In this case, as an alternative approach, *partial (scalar) invariance* facilitates the comparison of differences in latent means (Byrne et al., 1989; Steenkamp & Baumgartner, 1998). In other words, if additional model restrictions (from metric to scalar invariance) lead to a significant drop in goodness of model fit, individual indicators can be identified and freely estimated in the groups. The identification of indicators should be based on the theoretical considerations that could follow an explorative approach (Vandenberg & Lance, 2000), for instance by considering modification indices (MIs) and expected parameter changes. Meaningful and valid group comparisons for the measured constructs are possible, if at least two indicators of each factor supports metric and scalar invariance (Byrne et al., 1989; Steenkamp & Baumgartner, 1998).

The chi-square difference test was not conducted because of its sensitivity to sample size (Chen, 2007). Instead, Chen's (2007) and Cheung and Rensvold's (2002) guiding criteria were used when comparing the set of nested parsimonious-to-constraint model change. With regard to Chen (2007), a decrease of the model fit CFI at (ΔCFI) $\leq$.010 in the more parsimonious model or a drop in RMSEA (ΔRMSEA) $< .015$ indicates empirical support for the less constrained model and the respective level of invariance.

Profile Identification - Latent Class Analysis. The statistical procedure of latent class analysis (LCA) was conducted to classify German and Finnish students into homogenous profiles based on the 17 personal meanings in mathematics. This exploratory classification was based on students' response patterns regarding the observed items (Lazarsfeld & Henry, 1968). The inter-individual differences in the patterns that were observed are attributable to class affiliation (Geiser, 2011; Rost, 2006). These latent classes correspond to class-specific response profiles (Geiser, 2011). Profiles were conducted using Mplus version 8 (Muthén & Muthén, 1998–2017). Students' individual mean scores were used to conduct LCA. Mplus robust maximum likelihood (MLR) estimated the profile solutions from one to five profiles of personal meaning. The Mplus analysis option "mixture" was used for the nested design of the data.

Regarding profiling classes, I used listwise deletion of responses with incomplete data (cases with ≥ 2 missing values of 17 personal meanings). The number of samples to be analysed (after listwise deletion, N = 264 in Germany and N = 243 in Finland) was relatively small, but met the minimum requirements for performing an LCA (Nylund et al., 2007). Further missing values were handled by Mplus FIML (Asparouhov & Muthén, 2010). To avoid local maxima in the bootstrap associated with suboptimal results (Geiser, 2011), 3000 random sets of start values and 100 initial stage iterations were set. Local maxima tended to be more common as the number of classes increased. This issue was considered when setting the LRTSTARTS (Mplus analysis option, Geiser, 2011).

Moreover, to identify the optimal amount of classes several goodness-of-fit indicators were used, the information criteria used were: Akaike's Information Criterion (AIC), the Bayesian Information Criterion (BIC), sample-sized-adjusted BIC (aBIC), and the Vuong–Lo–Mendell–Rubin likelihood ratio test (VLMR), as well as the inference statistical results of the model-comparing bootstrap likelihood ratio test (BLRT). Furthermore, it is preferable that the average latent class assignment probabilities on the main diagonal should be >.80 (Rost, 2006, p. 278) and that accuracy of classification entropy should be near 1 (values close to 0 represent a low level of accuracy). To conclude a meaningful interpretation, LCA models should not have groups with < 5 % of the cases (Marsh et al., 2009).

The final and most important decision as to the "best" amount of profiles was made by examining the class interpretability in terms of the theoretical conclusions (Kleemann et al., 2013). Therefore, it should be taken into account whether the profiles of personal meaning can be well differentiated with relation to the substantial theory behind that concept (Marsh et al., 2009). What is more, the procedure suggested by Geiser (2011) entails a parsimonious approach according to which an LCA model with a small number of classes is preferred. As each LCA is very specific with respect to the scholar's research interest, there is no clear description of how to interpret and then validate the LCA output. To assess the quality of LCA models, I investigated the interpretability initially as follows: Response values should be either high or low (i.e., not medium) for all factors in the profile (Geiser, 2011). The 4-point Likert scale as a whole gives clues, ranging from 0 (strongly disagree) to 3 (strongly agree). In this present study, first I examined whether each profile can be clearly interpreted by considering its mean scores (≥ 1.8 = constructed personal meanings and ≤ 1.2 = non-constructed personal meanings). Average values are probably made up of different components and therefore they do not provide any information when clarifying the profiles (Geiser, 2011). Therefore, I ignored the personal meanings with moderate mean values in all LCA models across countries. After detecting the best LCA solution for each country, I added the regulatory styles to the LCA model. As distal outcomes, regulatory styles represent additional profile indicators without affecting the identified profiles of personal meaning (Muthén & Muthén, 1998–2017). Moreover, the replicability of the latent class solution in a different data set, and coherence of profiles with external variables (here regulatory styles as covariates) represent support for a valid class solution (Geiser, 2011). To investigate the validity of the profile solution, further analyses (Mann–Whitney U test) in both countries were carried out to contrast the profile output. Furthermore, within the theory-driven analyses, I analysed and validated each profile with respect to this study's conceptual framework, which was built by coordinating theories (see section 4.2.2).

Mann–Whitney U test. This analysis is mainly used as a non-parametric alternative procedure to the independent *t–test*. The Mann–Whitney U test is used as a guide to test whether the profile output (the learners' profiles of personal meaning and their quality of motivation) varies between countries. For this analysis, IBM SPSS Statistics (version 27; IBM Corp., 2020) was used.

3.4 Résumé

This study's empirical work focuses on a classical survey (self-reported) that was carried out using standard quantitative research methods. As well as a new questionnaire (instrument for 17 personal meanings), I applied a standardized instrument (for five regulatory styles) to assess the latent variables under investigation. It is obvious that this study's properties (two theories behind 22 latent variables, investigated by means of two methodological approaches in two different cultural contexts) represent various methodical challenges. For this reason, an in-depth and critical examination of the measurement uncertainty and data analysis might be fruitful in terms of reaching the overall research goal (cf. 2.6). If we rely on the applied quantitative methods (including the coordination analysis method), it should be noted that this present quantitative study is more explorative than hypothesis-testing in its nature. Based on the methods used here, this study cannot explain causal relationships between learners' personal meanings and their quality of motivation.

4. Theory-Driven Analyses

As discussed above (cf. 3), I set out to study personal meaning's multimodal ontological natures (Radford, 2014, p. 359). This concept seems to exist both on an individual (through the lens of SDT) and a social level (through the lens of social constructivism). The empirical findings, self-report data gathered using quantitative statistical methods, captured only one of these aspects, namely personal meaning as an individual psychological phenomenon. Based on that weakness, an additional method was applied to reconstruct learners' social experiences in order to capture personal meaning as a social phenomenon. In this way, personal meaning could be considered from the viewpoint of both theoretical lenses, SDT and social constructivism, as a means of exploring how personal meaning interconnects the predicted social and biological regulatory mechanisms that seem to affect learners' motivation in mathematics (cf. 2.3.3). For this purpose, a research frame was necessary that networked the multimodal ontological natures of personal meaning, namely the social and biological regulatory mechanisms.

Stressing the philosophical ethos of triangulation, the networking strategy of coordinating represents a research practice that methodologically considers both theoretical approaches by respecting their "diversity as richness" (Prediger et al., 2008, p. 170). Before I describe the coordination analysis method, I introduce the philosophical background behind this research practice.

4.1 Methodological Considerations for the Coordination Analysis

As indicated earlier, the affective construct of personal meaning reflects the learner's perceived personal relevance with respect to the subject of mathematics in an educational setting. Theoretically, this construct may enhance our conceptual understanding of the relation between students' individual relevance systems and their quality of motivation, which could be of use in engaging students in learning. Considering the phenomenon as situated at the boundary of SDT and social constructivism can be useful in order to shed new light on the characteristics that make up personal meaning. Even though each of the two theoretical lenses provides valuable information about subjects' motivation when learning mathematics, a clarification of their interplay may provide sophisticated knowledge in terms of how they jointly interact while affecting the quality of motivation in mathematics lessons. In this spirit, I use the idea of theoretical triangulation to reflect on connecting both *epistemological viewpoints* (Sabena et al., 2014, p. 189).

4.1.1 Theoretical Triangulation

Triangulation is a good way to gain more convincing evidence in research practice. At first, the term referred to a trigonometrical concept applied in land surveying and geodesy (Denzin, 1989; Fielding & Fielding, 1986; Flick, 2006; Kelle & Buchholtz, 2015). In the sense of the term first used by Denzin, triangulation expresses the combination of different perspectives used for an investigation (Denzin, 1989; Flick, 2008).

The knowledge of a theory is, at some point, exhausted in terms of examining an empirical object. Theoretical triangulation (also triangulation of different data sources, survey or analysis methods, and scholars' perspectives) is a research strategy used to deepen knowledge of the research phenomenon and to increase the explanatory power of the empirical evidence (Denzin, 1989; Flick, 2008). According to Denzin, theoretical or perspective triangulation works by

> approaching data with multiple perspectives and hypotheses in mind. Data that would refute central hypotheses could be collected, and various theoretical points of view could be placed side by side to assess their utility and power (…). Such strategies would permit sociologists to move away from polemical criticisms of various theoretical perspectives, since pitting alternative theories against the same body of data is a more efficient means of criticism. (Denzin, 1970, p. 303)

This research strategy is particularly supportive when different theories shed light on one empirical phenomenon; in such cases, it can be used to examine the data's theoretical coherence from different points of view (Flick, 2008, pp. 14–15). However, the application of perspective triangulation does not automatically lead to enhanced validity of the empirical analysis (Bikner-Ahsbahs & Prediger, 2014, p. 243). In order to benefit from this approach, and not least to compare the results of multiple theoretical lenses, theories should target common properties of the empirical phenomenon as well as share the same conceptual understanding of the researched phenomenon. Only if both perspectives examine the same research object from complementary angles can the results be compared, and triangulation lead to a dense and coherent understanding of the studied object under investigation (Bikner-Ahsbahs & Prediger, 2014, p. 243; Prediger et al., 2008).

Triangulation of theories is a well-known research practice in the field of mathematics education (Bikner-Ahsbahs & Prediger, 2014, p. 243). In qualitative empirical case studies this tradition of linking theoretical lenses is particularly widespread, but it is rarely realized in quantitative empirical investigations. This is not surprising, since qualitative research essentially aims to develop or explore (Bikner-Ahsbahs, 2015; Przyborski & Wohlrab-Sahr, 2014) theories, categories, or concepts (Kelle & Buchholtz, 2015) by following inductive procedures (Neuman, 2011). Nonetheless, connecting theories in quantitative studies would be enriching, not only to solve practical problems in research analysis but also to broaden one's mind with respect to (at least) implicit knowledge behind the applied theoretical approaches, for example by pointing out their methodical benefits and technical obstacles for future research (Prediger et al., 2008, pp. 175–176).

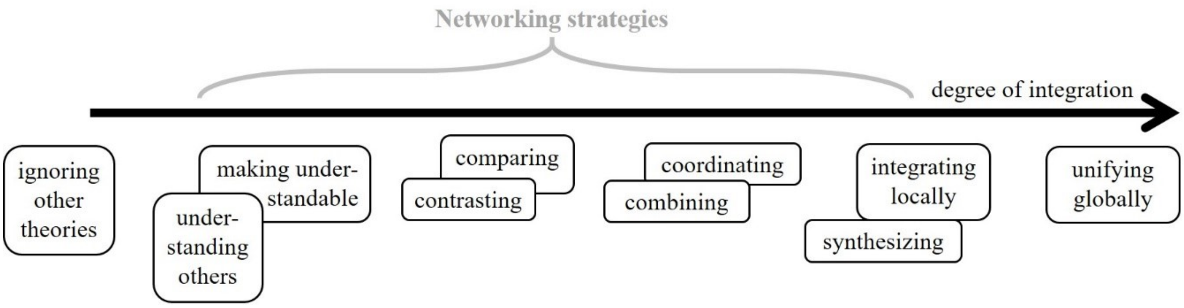
In this study's initial quantitative approach, highly structured statistical methods were used to measure the required affective constructs of personal meaning and regulatory styles. Now, I draw on the strengths of triangulating theories to compensate for the weakness that arose in quantitative mono-methodological research (Kelle & Buchholtz, 2015). That is, the main purpose of including theoretical triangulation is to bypass the methodological gap by systematically networking both theoretical perspectives (SDT and social constructivism); that is, to qualify the quantitative data (Schoonenboom & Johnson, 2017). In terms of triangulating theories, the research activity of networking theories not only supports the examination of the double ontological natures of personal meaning sequentially; it also helps to bring both (complementary) theoretical lenses into a dialogue concerning their individual and conjoint understandings of learners' multimodal (Radford, 2014) motivational experiences in the German and Finnish mathematics-classroom settings.

4.1.2 Approach of Networking Theories to Studying Personal Meaning's Social Nature

The members of a working group at the Congress of the European Society for Research in Mathematics Education (CERME) were the first scholars to discuss the idea of networking theories as a research practice. They elaborated several ways of scientifically linking multiple theoretical perspectives, and proposed strategies for networking theories in the field of mathematics education (cf. Bikner-Ahsbahs & Prediger, 2014; Prediger et al., 2008). These strategies provide the robustness to handle both lenses' specific conceptual identities without neglecting theoretical contradictions (Bikner-Ahsbahs & Prediger, 2014; Janßen, 2016, p. 80). The following figure (Figure 12) delineates all strategies on a scale that systematically places them in order of their connectivity, which ranges from *ignoring other theories* to *unifying globally*.

Figure 12

Networking Strategies for Connecting Theories in Mathematics Education (adapted from Prediger & Bikner-Ahsbahs, 2014).



With respect to the networking analysis carried out in this dissertation, I used the strategy of *coordinating*.

> Following the idea of triangulation, combining and coordinating means looking at the same phenomenon from different theoretical perspectives as a method for deepening insights into the phenomenon. The distinction between combining and coordinating is drawn according to the degree of integration of theory elements with respect to their compatibility. *Combining* theoretical approaches does not necessitate the complete compatibility of the theoretical approaches under consideration. Even theories with conflicting basic assumptions can be combined in order to get a multi-faceted insight into an empirical phenomenon in view. In contrast, we use the word *coordinating* when a conceptual framework (…) is built by fitting together elements from different theories for making sense of an

> empirical phenomenon. A conceptual framework is not a new theoretical approach but a pragmatic bricolage for the purpose of understanding empirical phenomena. (Prediger & Bikner-Ahsbahs, 2014, pp. 119–120)

Initially, I brought the theories together by *combining* them. However, simply juxtaposing both lenses (Sabena et al., 2014, p. 189), SDT and social constructivism, is insufficient to increase our knowledge about learners' quality of motivation in the mathematics classroom with respect to personal meaning. For my theory-driven analyses, I thus used a lens that emerged from coordinating (Prediger & Bikner-Ahsbahs, 2014) social and biological regulatory mechanisms through the medium of the motivational construct personal meaning.

In the following sections, I describe the applied coordination analysis method that I used to study the theoretical connections in Germany and Finland. To systematically structure the coordination analysis, coordinating the theories on quantitative data, I used three (adjusted) terminologies (essentially taken from the qualitative research paradigm, see section 4.2.1) that guided my interpretation procedure (see section 4.2.2): namely "identify with" (German: *Positiver Gegenhorizont*), "not identify with" (German: *Negativer Gegenhorizont*), and enacting potential (German: *Enaktierungspotential*). The use of such a methodological mixture (Kelle & Buchholtz, 2015, p. 330), using terms from qualitative research to interpret standardized data obtained by quantitative research methods, raises various questions; for example, how to interpret the empirical results (referring to the nature of the qualitative or quantitative paradigm) and where to anchor this mixed-research approach in the methodological research landscape. To avoid methodological risks and misinterpretations, I briefly elaborate the critical aspects associated with this mixture in order to clarify the depth to which these qualitative terminologies can lead by applying them to a body of quantitative data (see section 4.2.3).

4.2 Coordination Analysis Method

According to the working definition of coordinating theories, an application of this strategy can only be realized by considering theoretical lenses with *compatible cores* that *complement* each other (Prediger et al., 2008, p. 172). A theoretical assurance of this maxim can be done by highlighting the theories' commonalities and specificities. That emphasizes whether the different epistemic views (Sabena et al., 2014) are complementary or not. The existence of complementary views (*well-fitting elements*) enables the researcher not only to adopt different perspectives but also to constitute a *conceptual framework* to specifically target the research object under investigation (Prediger et al., 2008, p. 172).

Theoretically, I conceptualized personal meaning as located at the boundary of SDT and social constructivism (cf. 2.3.3). Thereby, the potential emerged to build a conceptual framework by coordinating both epistemic angles, which seem, in their natural state, to be compatible and complementary. On a theoretical level, I carefully elaborated how the theoretical boundaries between SDT and social constructivism can be bridged by considering personal meaning as a boundary object. Based on the level of principles (Radford, 2008a), I compared both theories' informative values in terms of quality of motivation in mathematics (cf. 2.3).

For the record, each theoretical angle shed light on different issues (Sabena et al., 2014, p. 188) surrounding learners' quality of motivation in the mathematics classroom. On the one hand, the solid framework of SDT is qualified to provide an insight into learners' regulatory styles (amotivation, external regulation, introjected regulation, identified regulation, intrinsic regulation) referring to their perceived basic psychological needs (for autonomy, competence, and relatedness). From an epistemic point of view (Sabena et al., 2014, p. 189), SDT stresses the importance of students' personal relevance in academic contexts but does not provide an insight into the "*what*" (kinds of, or patterns in the personal relevance of studying mathematics) that directs learners' behaviour in the mathematics classroom. The insight into learners' "what" may help to understand *why* they react in a certain way in class (controlled or autonomous behaviour). Moreover, although the dialectical conversation between the individual and their social context represents the nutrient medium for an organism's development, learners' social experiences are not likewise elaborated within SDT. At this point, SDT's principles (Radford, 2008a) show a theoretical limit.

Social constructivism, on the other hand, provides an insight into learners' experiences of social interaction in the classroom. Ernest proposed a model providing a micro insight into the "production of learning and of knowledge" (Ernest, 2010, p. 45) in a classroom setting that explains "how signs become appropriated by an individual through experiencing their public use" (Ernest, 2010, p. 44). Here, social constructivism is, from an epistemic point of view, limited to explaining learners' individual behaviour as a counterpart that associates with certain social experiences in the mathematics classroom.

The theoretical conceptualization of personal meaning (Vollstedt, 2011b; Vorhölter, 2009) seems to provide a basis on which to connect SDT and social constructivism's perspectives in order to jointly study learners' quality of motivation in class. Theoretically, when we consider the empirical phenomenon of personal meaning as a "bracket" at the boundary of two theoretical perspectives, the object "personal meaning" seems to link both theories in an interesting manner (cf. 2.3.3). In other words, through this joint view of social constructivism it may be possible to explain how the development of learners' individual relevance systems takes place; that is, explain learners' reactions to the outside world (referring to social regulatory mechanism). Further, SDT could explain how learners' inner worlds (referring to biological regulatory mechanism) react to the socially experienced individual relevance systems in the mathematics classroom. Since the theoretical cores of both perspectives refer to individuals' experiences, but perceived in different worlds, the perspectives of what (individual relevance system) and why (motivational behaviour) might complement and articulate each other in a useful way (Prediger et al., 2008, p. 172). The following coordination analysis of this conceptual framework, which considers both regulatory mechanisms as they relate to personal meaning, would not only provide an insight into the respective world (of social or biological regulation of personal meaning); if the assumed connection exists, the complementary understanding as a common view that has emerged through the conceptual framework would also clarify how learners' motivation develops in the German and Finnish mathematics classrooms.

Next, to interpret and relate these numerical data systematically with respect to the conceptual framework built by means of coordination, I used three (adjusted) terminologies: "identify with", "not identify with", and "enacting potential". Before I describe my coordination analysis procedure, I briefly introduce these terminologies and exemplify how they are commonly used within the qualitative research paradigm.

4.2.1 Terminologies for a Theory-Driven Interpretation of the Quantitative Data

In the qualitative research field, *group discussion* represents a research practice (amongst others) that is used to investigate and reconstruct ideal-typical (Weber, 1972) orientation frameworks constituted by a pattern of thought and action on the part of an observed group of actors with respect to a specific research context (Kleemann et al., 2013). According to Bohnsack (1989), this method aims to reconstruct the respondents' orientation framework (the so-called orientation pattern[24]). To confirm and validate the identified orientation frameworks of groups, a systematic comparative analysis is carried out in order to saturate the identified types. Finally, a series of contrasting comparisons of types aims to generate a typology that reflects the range of the multiple reconstructed orientation patterns of the researched participants (Kleemann et al., 2013). The typology illustrates multidimensional types of orientation, representing the generalization of orientation patterns that result in the investigated research context (Bohnsack, 2014; Kleemann et al., 2013).

Individuals rarely refer directly to their orientation pattern in group discussions (Kleemann et al., 2013). Their orientation pattern needs to be catalysed from their descriptions and examples (Kleemann et al., 2013, p. 161). Researchers use three structuring features that facilitate the excavation process of actors' orientation patterns: Positive counter-horizon (German: *Positiver Gegenhorizont*) and negative counter-horizon (German: *Negativer Gegenhorizont*), as well as enacting potential (German: *Enaktierungspotenzial*; Bohnsack, 1989, pp. 26–27; Bohnsack, 2014, p. 138). These three features provide "access and contours to their orientation pattern" (Kleemann et al., 2013, p. 161, translated by the author). A positive counter-horizon refers to the "point of reference (actions, people, attitudes, etc.)" that represents what "belongs to one's own will" (Kleemann et al., 2013, p. 238, translated by the author). It provides orientation regarding a certain context, and the offered area is accepted by the actor. On the contrary, the term negative counter-horizon describes the "reference point with which one distinguishes oneself and thus shows what does not belong to one's own will" (Kleemann et al., 2013, p. 238, translated by the author). It also provides orientation regarding a certain context, which is rejected by the actor. The restructured orientation therefore stands on a continuum between two opposing horizons. (Bohnsack, 1989, p. 27). Another key point of reference in group discussions is the "chance of practical realisation of one's own orientation" (Kleemann et al., p. 234, translated by the author). The chance of applying one's own basic orientation in the real world is known as enacting (Bohnsack, 1989, p. 26; Kleemann et al., 2013, p. 161). The individuals and their orientation patterns within their field of experience (i.e., that which provides, or fails to provide orientation) are situated in varying degrees of relation to these three structural components (Bohnsack, 1989, p. 27; Bohnsack, 199, 2014; Kleemann et al., 2013). Thus, these three components make up the orientation framework around the learner's field of experiences (Bohnsack, 1989, 1999). One example of

[24] This definition of action makes sense if it is based on the ideal-typical constructed pattern of orientation. In this context, orientation pattern is understood in the sense of Weber's ideal type (Bikner-Ahsbahs, 2015; Bohnsack, 2014, p. 146; Weber, 1972).

their use from a qualitative research study is provided by Kleemann et al. (2013), who investigated the importance of school and its significance in learners' lives:

> The students at the only grammar school (Gymnasium—for high achievers) in a small town developed despite all their differences (lower and upper secondary, girls and boys) a common sense of unity; this is displayed somewhat condescendingly towards students from other types of schools. The orientation framework here consists of an understanding of belonging to an elite. One's own school and the level of education it provides are the positive counter-horizon of self-worth, while peers with less education represent a negative counter-horizon. To be able to let the other students feel this evaluation through their self-presentation, is how the students exercise their enacting potential. (Kleemann et al., 2013, pp. 161–162, translated by the author)

The use of the three structuring terminologies[25] helps to capture the group's orientation pattern, and the comparison of these orientation frameworks enables us to generalize a multidimensional constructed typology. The configuration of these types demonstrates the level of difference between the reconstructed orientation frameworks and sheds light on the themes and concerns the group members are focused on (on the example shown above "the significance of attending a particular school", see Kleemann et al., 2013).

In contrast to the original application of these three terms within the qualitative approach, in the present study the empirical analysis helped to identify learners' individual relevance systems (patterns of personal meaning). To capture and compare the different profiles' mathematics-learning orientations, I used the structuring elements, adjusted as "identify with" (positive counter-horizon), and "not identify with" (negative counter-horizon). That is, I reorganized the *computed* profiles of personal meaning as orientation patterns that are constituted by these directions; that is, "identify with", and "not identify with". Hence, I followed the reverse order to the conventional procedure in qualitative research. Further, the adaptation of enacting potential made it possible to investigate the theoretical coherence between the social and biological natures of personal meaning, by connecting both ontological natures of personal meaning with respect to the conceptual framework. In the following section, I describe in more detail the systematic use of this procedure of structuring components to coordinate the analysis.

4.2.2 Coordination Analysis Procedure

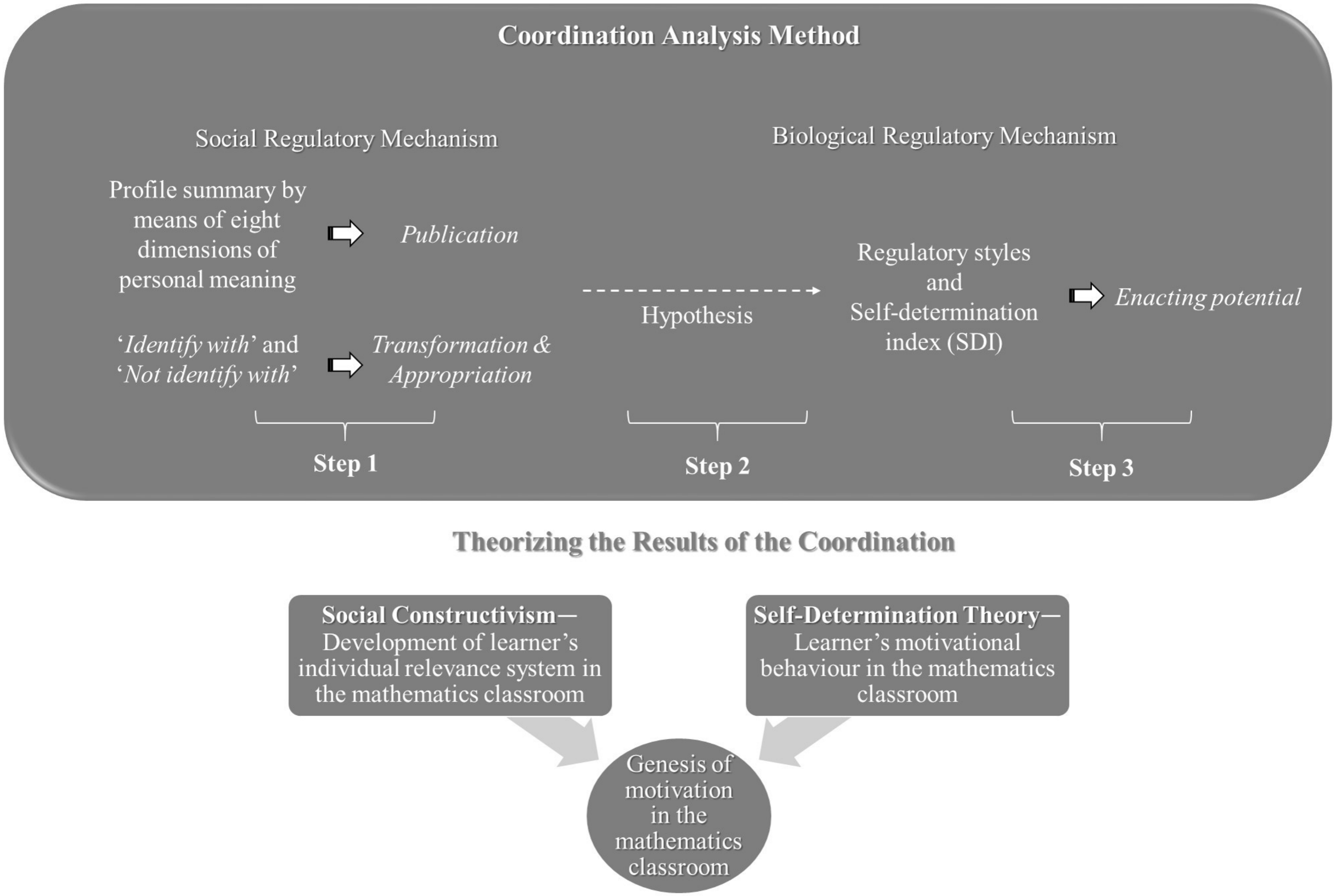
To investigate the hypothesized connection between social and biological regulatory mechanisms through personal meaning (conceptual framework built by coordinating theories), adjusted terminologies from qualitative research (identify with, not identify with, and enacting potential) were used to help structure the coordination analysis method systematically. In the previous section, I detailed how the structuring elements provide access to groups' orientation patterns, as extracted from interview data material within a qualitative research approach. Next,

[25] Of course, this was done in combination with other methodical steps that are not discussed here.

I used these terminologies in an adjusted manner to interpret the numerical data in line with this investigation's conceptual framework. While describing the coordinating analysis method, I define how I adapted these terms with respect to this study's research target. The following figure, Figure 13, provides an overview of the evaluation steps of the coordination analysis I conducted.

Figure 13

Overview of Present Investigation's Coordination Analysis Procedure.



The upper side (greyed out) of Figure 13 has two parts: the processes of the social regulatory mechanism (which refer to social constructivism) and those of the biological regulatory mechanism (which refer to SDT). The analysis sequences started first on the left-hand side, that of social regulations. To reconstruct[26] the learner's experiences of social interaction, I aimed at a systematic interpretation of the mean values with respect to the social regulatory mechanism. Again, based on the predicted social regulatory mechanism, I assume that the learner's individual relevance system is affected by all regulations (of appropriation, transformation, publication, and processes of conventionalization[27]) in the mathematics classroom. To explain *theoretically* how these individual relevance systems arise and unfold in the classroom (but also to examine whether the identified empirical profiles make sense on a theoretical level), I link these social processes (apart from processes of conventionalization) to each identified empirical profile of personal meaning.

Thereby, as indicated earlier (2.3.2), the publication stage provides the relevance system in which the learner produces their reactions to their social experiences in the mathematics-learning environment. As mentioned earlier, this very specific level of publication represents an outcome that eventuates after a series of experiences on the part of the learner, illustrated by the adapted cycle of appropriation, transformation, publication, and processes of conventionalization (the social regulatory mechanism of the individual relevance system, cf. Figure 6). By means of the classical survey instrument (self-report), I recorded what learners published by recounting to me; namely, their individual experiences dating from a certain point in the learner's life and not the numerous experiences that led to this production. Firstly, starting from the learner's constructed relevance system, I initially described the profile by means of the eight dimensions of personal meaning (cf. Figure 5) and called this the *publication* of the social regulatory mechanism. This procedure was helpful in terms of summarizing the profile outcome (regarding the constructed relevance system, $M \geq 1.8$ cut-off point; for non-constructed personal meanings, $M \leq 1.2$ cut-off point). Secondly, I evaluated the appearance of the eight dimensions of personal meaning in the students' relevance systems; that is, I studied whether the theoretical composition of the eight dimensions stayed conjoint for profile specification or diverged. This step, publication of social regulation, thus provides a summarized overview of each learner's individual relevance system in general.

Based on the general profile summary, I then worked out the profile's orientation pattern in more detail. I therefore used the adjusted terminologies to theoretically reconstruct the individual's social experiences in class as follows: Based on the numerical responses from their self-reporting (ticked boxes on the 4-point Likert scale), the individual's learning orientation in mathematics was identified by one scope that is accepted and another one that is rejected for one's own profile orientation. I interpreted these ticked boxes as *identify with* if personal meanings were constructed and *not identify with* when personal meanings were not constructed. Based on Bohnsack (1989, 1999, 2014), I used the term "identify with" for a scope that is

[26] This notion of reconstruction is not comparable with the reconstruction of qualitative data (see section 4.2.3).
[27] The operation "processes of conventionalization" triggers individuals' learning orientations in the classroom; that is, it influences their orientation towards "identify with" or "not identify with". In order to reconstruct "processes of conventionalization", the detected profiles' classroom environments need to be observed. Empirical observations are not included in this study. For this reason, no clear statements can be made about this operation. Processes of conventionalization are thus excluded from the coordination analysis. However, when implicit knowledge regarding this process emerges in the evidence, it will be discussed later.

accepted as a means of personal identification, a pole that provides orientation and in the context of which the profile identifies with studying mathematics in the classroom. The term “not identify with” applies to the exact reverse of this situation. This refers to a scope that provides no orientation for the learner and is avoided; thus, the profile is that of an individual who does not identify with that pole. For the record, the profile’s poles were reconstructed from the individual’s self-reports (biological regulatory mechanism) in terms of which scopes were accepted and which were rejected for their own relevance system as a means of providing orientation towards mathematics learning in a social classroom setting. The consideration of the opposing poles also helps validate the meaningfulness of the statistical profiles; that is, by showing whether they are contrariwise and mutually representing opposite of one another. In this connection, I specifically noted the learner’s reaction to the three specific personal meanings (EXPERIENCE OF AUTONOMY, COMPETENCE, and SOCIAL RELATEDNESS). I related the reconstructed profile’s orientation to the regulations *transformation* and *appropriation* as part of the social regulatory mechanism.

Then in step two, based on the profile’s orientation I formulated a theory-compliant hypothesis about what the learner’s motivational tendency in class may look like. In other words, after *theoretically describing the social nature* of personal meaning, I built a bridge between social and biological regulation in order to connect the outer social regulations within the classroom context with the learner’s inner biological regulation.

In step three, I analysed to what extent the theoretical explanation of the learner’s social regulation could be reflected within the empirically measured quality of motivation (covariates of each profile). This step not only reflects the theoretical coherence (between social regulatory mechanism and biological regulatory mechanism) but also evaluates whether the different perspectives consistently complement each other within the predicted conceptual framework (Prediger et al., 2008). This is because, if we can finally use this social regulatory mechanism to explain how these empirical profiles have developed, then we would have good reasons to accept this specific link—between biological and social regulatory mechanisms—as meaningful. The ability to biologically experience one’s own relevance system (orientation pattern) in practice is referred to here as *enacting potential* (Bohnsack, 1989).

After following these three steps, I interpreted the results from the German and Finnish data with respect to the conceptual framework behind them (bottom half of Figure 13). Thus, theorization of the coordination of the results was carried out in order to understand how quality of motivation develops in the German and Finnish mathematics classrooms. Therefore, the coordinating view integrated both theoretical lenses to capture the resulting theoretical interaction. On the basis of the obtained interaction between theories across countries, I interpreted these indicators with respect to learners’ educational systems in Germany and Finland. Finally, after critically reflecting the predicted links within the conceptual framework, I explored the extent to which both epistemic views contribute to the elucidation of learners’ motivation development in the mathematics classroom.

4.2.3 Critical Reflection on Applying Terms from the Qualitative Research Tradition

This study's empirical work solely focuses on quantitative research methods and the obtained standardized data is subsequently interpreted by means of adapted terminologies ("identify with", "not identify with", and "enacting potential"). Basically, these terminologies stem from the qualitative research tradition. Due to this mixture of approaches, that is, the use of qualitative terms to analyse numerical data, it is necessary to clarify how these terms are understood in the present methodology. For this purpose, I briefly outline the general methodological standards of qualitative and quantitative research that go along with the application of these structuring categories. While highlighting the differences in these standards, I simultaneously point out which relevant methodological properties are applied in this study.

The quantitative and qualitative research traditions use different standards: within the qualitative research paradigm, the group discussion method is applied to examine narrative interview material (Bohnsack, 2014). This material is closer to the actions of each participant and focuses on the respondent's point of view. By applying the structuring terms to non-standardized material and through the flexible process of reconstruction, the investigator aims to capture the respondent's holistic view by treating them as a subject (and not as an object; Lamnek & Krell, 2016).

In contrast, in the quantitative tradition, selected items that trace back to other students' views are prefabricated by scholars. For instance, the items used for profile identification can only consider the theoretical knowledge specified within the classical survey instrument of personal meaning. That is, by reconstructing students' mathematics-learning orientations, I limit respondents' actions to the given scales of personal meaning. Hence, the given items will only record the knowledge that stems from the 17 kinds of personal meaning. An insight into additional facets of the 17 kinds or other kinds that differ from Vollstedt's (2011b) results cannot be captured; for example, if personal meanings or facets exist that relate to Finnish culture specifically. For this reason, this reconstruction method (based on numerical data) is not comparable to the complex and flexible reconstructive work within the qualitative tradition where the investigator absorbs the perspective of the researched actor until theoretical saturation is reached.

Furthermore, the understanding of typology differs between the research traditions with reference to their genesis: a typology that arises from qualitative research can consist of ideal types. In this tradition, types correspond with the ideal-typical understanding in the sense of Max Weber (1972), in which an observed action is ideal-typically constructed[28] based on the participant's individual biography (Bohnsack, 2014). In contrast, quantitative methods generate average types (here: profiles based on learners' mean values of the 17 personal meanings) that are strongly bound in time and space (Weber, 1972). In accordance with the quantitative tradition, this study thus aims for representative data (although it is very limited here due to the relatively small sample size), and not the representation of the social meaning (ideal types; Bikner-Ahsbahs, 2015; Weber, 1972).

[28] A detailed discussion about the construction of ideal types is provided by Bikner-Ahsbahs (2015).

Despite the use of these terminologies from the qualitative paradigm, the study carried out here belongs to the quantitative field. Consequently, while the ideas of these adapted terms can meaningfully support the coordination analysis process, they cannot, however, reach the same level of analysis as that which they would traditionally produce when applied to qualitative data.

4.3 Résumé

Altogether, this investigation's theory-driven activity tries to counterbalance the limitations of quantitative mono-methodology in two ways. Firstly, this networking strategy helps to capture the social nature of the construct personal meaning (challenge/support the adapted model of social regulatory mechanism with the explanation/interpretation of how these profiles have developed). Secondly, the research activity of coordinating theories shows promise as a means of interpreting and validating the empirical profiles of personal meaning. Following an explorative approach, the whole theory-driven analysis will thus lead to a specification of the evidence-based effects across the biological and social worlds of personal meaning. Based on these evidence-based indicators, found across countries, I will critically review the conceptual framework as regards the conceptual interplay between individual relevance systems and motivation in the mathematics classrooms of Germany and Finland.

5. Empirical Results

In this chapter, I present this study's empirical results; discussion of these results follows in the next chapter, 6. Initially I investigated the psychometric properties of personal meaning and regulatory styles assessment in Germany and Finland to see whether the scales could be used for a mean comparison concerning the latent variables across countries (reliability through composite scale reliability ρ, factor structure through confirmatory factor analysis (CFA), measurement invariance, and latent mean differences). Subsequently, I identified and compared students' profiles of personal meaning and their motivational outcomes in Germany and Finland (latent class analysis and Mann–Whitney U test).

The present investigation aimed at understanding how learners' motivation arises in German and Finnish mathematics classrooms with respect to the construction of students' individual relevance systems. As an initial step to this end, I had three research questions (RQs) that I examined successively when preparing the empirical database by means of classical survey instruments.

Concerning RQ 1, I determined the psychometric properties of personal meaning and regulatory styles assessment to see how these properties compare cross-culturally (see section 5.1). Therefore, I examined the scale reliability and factor structure of both constructs. For personal meaning, I tested for empirical support for each of the predicted eight dimensions of personal meaning in separate measurement models. In the next step, I investigated whether the eight dimensions could also be differentiated within one comprehensive model. Concerning regulatory styles, I tested a five-factorial model including all regulatory styles. Moreover, I studied the intercorrelations between personal meaning and regulatory styles in both groups for the theoretically assumed links (cf. 2.3.1). In addition, I investigated the measurement invariance across countries to delineate the generalizability of the underlying constructs. For personal meaning, I based this test on the resulting best factor model (eight separate models). These analyses helped to avoid biasing effects (i.e., cultural bias) in the outcome and to consider any methodological issues concerning the new (personal meaning) and established (regulatory styles) instruments applied here.

The need to translate instruments can be associated with a reduction in the quality of psychometric properties and inconsistencies with the original settings, even in cases where the translation is of a high standard (Bofah & Hannula, 2015). For this reason, with respect to the survey instruments used, I assumed there would be challenges concerning the use of the survey instruments in Finland, as both scales had been developed for German-speaking countries (Thomas & Müller, 2011; Vollstedt & Duchhardt, 2019), although they resulted in good psychometric properties in the country of origin. Regarding the overall model of personal meaning, I assumed there would be problems of measurement when estimating a too-complex model combined with a new theory upon which it was based. Concerning the intercorrelations between both affective constructs, I expected support for the predicted strong interrelations between the 17 personal meanings and regulatory styles (cf. 2.3.1) and relatively similar correlation patterns for German and Finnish data that relate to the cultural properties of Western European countries (cf. 2.4).

Regarding RQ 2, I inspected potential differences between German and Finnish learners' latent means of personal meanings and regulatory styles (see section 5.2). In line with previous research (based on item response theory (IRT); Suriakumaran et al., 2019), I expected to find cultural differences that reflect the respective mathematics curricula.

In relation to RQ 3, I identified profiles (individual relevance systems) of personal meaning showing distinct mathematics-learning orientations (see section 5.3). Moreover, I added the regulatory styles as covariate outcome variables. With respect to their cultural diversity, I conducted a separate latent class analysis in order to detect the best class solution for each country. Analogously to the second research question, I expected to find different response patterns of personal meaning that refer to the specific mathematics-learning experiences that are inherent in the German and Finnish educational systems.

On the basis of these statistical analyses, I developed empirical data that provided the foundation for the subsequent coordination analysis in the theory-driven work (see Chapter 7). In the following sections, I report the empirical results in more detail.

5.1 Factor Structure of Personal Meaning and Regulatory Style Across Countries

Separate measurement models. As indicated earlier, for each scale of personal meaning and regulatory styles the composite reliabilities ρ (Bollen, 1989; Geldhof et al., 2014; Raykov, 2012) were rated as acceptable to good for both countries (see 'Reliability of Scales' in section 3.3.3 for more). In order to further examine the internal psychometric properties and the factor structure of the 17 personal meanings, I initially computed separate measurement models for each of the dimensions of personal meaning. Thus, I tested to what degree the theoretical differentiation of the dimensionality of personal meaning holds empirically in Germany (GER) and Finland (FIN).

As indicated earlier (cf. 2.3.1), I expected the following theoretical separation in terms of the dimensionality of personal meaning: "purism of mathematics" was hypothesized to be one-dimensional, whereas "self-development" was assumed to comprise three factors (kinds of personal meaning): active practice of mathematics, cognitive challenge, and self-perfection. For "freedom of action", I predicted two separable factors: EXPERIENCE OF AUTONOMY and EXPERIENCE OF COMPETENCE. The dimension of "professional qualification" was hypothesized to be three-dimensional: examination, marks, and vocational precondition. For "application in life", two different factors were predicted: general knowledge and reference to reality. "Emotional well-being in class" was expected to have four factors: classroom management, emotional-affective relation to the teacher, EXPERIENCE OF SOCIAL RELATEDNESS, and support by the teacher. Moreover, I expected "positive image" and "duty" to be one-dimensional.

To support the predicted separation of the eight dimensions of personal meaning, I compared the fit values of the assumed multifactor latent models with the alternative one-factor models for each multifaceted dimension respectively, following Satorra-Bentler's (Satorra & Bentler, 2001) scaled chi-square difference testing. These comparisons indicated the measured goodness-of-fit values of the expected multidimensional models. For the alternative one-factor models (Kleinke et al., 2017), the multidimensional models with correlated factors were each combined into one factor (model with no separation). Nonetheless, these one-factor models entail a theoretical understanding of the constructs that is different from that of the postulated models. Apart from the goodness-of-fit indication, the model comparisons empirically support the meaningfulness of each theoretically developed multifactor model. For personal meaning, Table 3 (GER) and Table 4 (FIN) show the estimated fit statistics for the CFA measurement models in Germany and Finland.

Table 3.
Fit Statistics for Confirmatory Factor Analysis of Dimension of Personal Meaning Models Separately and all Together in German Students.

GER Model	χ^2	*df*	CFI	TLI	RMSEA	SRMR
Purism of mathematics						
One Factor	23.83	14	.983	.974	.050	.029
Self-development						
One Factor	97.04	35	.914	.889	.080	.052
Three Factors	47.23	32	.979	.970	.042	.036
Freedom of action						
One Factor	87.54	20	.763	.668	.111	.077
Two Factors	24.82	19	.980	.970	.033	.036
Emotional well-being in class						
One Factor	392.78	104	.650	.597	.100	.093
Four Factors	145.56	98	.942	.929	.042	.052
Professional qualification						
One Factor	528.28	44	.468	.334	.200	.139
Three Factors	66.61	41	.972	.962	.048	.041
Application in life						
One Factor	272.35	20	.455	.237	.214	.151
Two Factors	27.03	19	.983	.974	.039	.035
Positive image						
One Factor	3.66	2	.989	.967	.055	.022
Duty						
One Factor	5.18	2	.986	.957	.076	.024
Overall models						
2^{nd} order	3546.484	2168	.804	.794	.048	.091

Note. CFI =comparative fit index; TLI = Tucker–Lewis index; RMSEA = root mean square error of approximation; SRMR = standardized root mean square residual.

Table 4.
Fit Statistics for Confirmatory Factor Analysis of Dimension of Personal Meaning Models Separately and all Together in Finnish Students.

FIN Model	χ^2	*df*	CFI	TLI	RMSEA	SRMR
Purism of mathematics						
One Factor	28.35	14	.977	.965	.063	.030
Self-development						
One Factor	80.04	35	.940	.923	.071	.043
Three Factors	58.56	32	.965	.951	.057	.036
Freedom of action						
One Factor	48.52	20	.922	.890	.075	.053
Two Factors	20.45	19	.996	.994	.017	.037
Emotional well-being in class						
One Factor	360.44	104	.766	.729	.098	.085
Four Factors	159.04	98	.944	.932	.049	.052
Professional qualification						
One Factor	388.94	44	.608	.511	.175	.111
Three Factors	69.91	41	.967	.956	.052	.040
Application in life						
One Factor	341.47	20	.459	.243	.252	.169
Two Factors	20.77	19	.997	.996	.019	.031
Positive image						
One Factor	3.05	2	.995	.984	.046	.023
Duty						
One Factor	5.10	2	.989	.966	.078	.020
Overall models						
2^{nd} order	3554.515	2168	.830	.821	.050	.082

Note. CFI =comparative fit index; TLI = Tucker–Lewis index; RMSEA = root mean square error of approximation; SRMR = standardized root mean square residual.

Overall, in the German sample the hypothesized (multi-)factor models (i.e., eight dimensions of personal meaning) yielded a good model fit (fit statistics ranging from CFI = [.942; .989], TLI = [.929; .967], RMSEA = [.033; .076], SRMR = [.022; .052]) whereas the alternative one-factor models (of the multifaceted five dimensions) all performed poorly where applicable (fit statistics ranging from CFI = [.455; .915], TLI = [.237; .889], RMSEA = [.80; .214], SRMR = [.052; .151]). For more details see Table 3. In Germany, each latent model recorded a good fit for the predicted eight dimensions of personal meaning. All factors had at least three manifest variables that loaded on the factor with loadings >.3.[29]

The same holds for the Finnish sample, with fit statistics for the hypothesized (multi-)factor models all indicating a good fit (fit statistics ranging from CFI = [.944; .997], TLI = [.932; .996], RMSEA = [017; .078], SRMR = [.020; .052]), whereas the alternative one-factor models (of the multifaceted five dimensions) all performed poorly where applicable (fit statistics ranging from CFI = [.459; .940], TLI = [.243; .923], RMSEA = [.071; .252], SRMR = [043; .169]). For more details see Table 4. All factors had at least three indicators that loaded on the factor with loadings >.3.

In a nutshell, these results showed empirical support for the predicted dimensionality of personal meaning in both Germany and Finland.

According to the underlying self-determination theory (SDT), the latent model of regulatory styles was expected to be five-dimensional: amotivation, external regulation, introjected regulation, identified regulation, and intrinsic regulation. Due to the robustness of SDT, I disclaimed a comparison of the five-factor model with an alternative model. Regarding regulatory styles, Table 5 gives an account of the fit statistics for the CFAs in Germany and Finland.

[29] I also estimated the standardized factor loadings (Appendix B) and intercorrelations (Appendix C1–C2) in the 17-factor model with all personal meanings together (see the additional files in the Appendix). As the further results will show (cf. second-order model with eight dimensions), this model did not fit the data across countries. However, all factor loadings were higher than the cut-off mark >.3.

Table 5.
Fit Statistics for Confirmatory Factor Analysis of Regulatory Style Models in German and Finnish Students.

Model	χ^2	*df*	CFI	TLI	RMSEA	SRMR
Regulatory styles						
Five Factors (GER)	185.59	109	.966	.958	.050	.044
Five Factors (FIN)	169.60	109	.968	.960	.047	.053

Note. CFI =comparative fit index; TLI = Tucker–Lewis index; RMSEA = root mean square error of approximation; SRMR = standardized root mean square residual.

In the German sample,[30] the theoretical five-factor solution resulted in a good fit of data (χ^2 = 185.59, *df* = 109, CFI = .966, TLI = .958, RMSEA = .050, SRMR = .044). Furthermore, all standardized factor loadings were higher than the cut-off mark >.3. The correlations (see Table 8) between the regulatory styles varied from |*.21*| (intrinsic regulation and external regulation) to |.54| (intrinsic regulation and amotivation).

Accordingly, in the Finnish sample the theoretical five-factorial measurement model fitted the data and recorded reasonable goodness-of-fit for the robust SDT underlying it (χ^2 = 169.60, *df* = 109, CFI = .968, TLI = .960, RMSEA = .047, SRMR = .053). Furthermore, all standardized factor loadings were higher than the cut-off mark >.3. The correlations (see Table 9) between the regulatory styles varied from |*.23*| (introjected regulation and amotivation) to |.73| (intrinsic regulation and amotivation).

In summary, the CFA results in Germany and in Finland supported the theoretical dimensionality of personal meaning for each predicted dimension. Moreover, the results supported the factor structure of the regulatory styles in a five-dimensional model across countries.

Second-order model with eight dimensions of personal meaning. Subsequently, I estimated a second-order model with all the dimensions together in an overall model. This specification examined more closely the predicted dimensionality of personal meaning. Therefore, I tested whether the differentiation of personal meanings into eight dimensions holds empirically in one comprehensive model. Thereby, I intended not only to study the associations between the eight predicted dimensions, but also to find an adequate model for further analysis (test of measurement invariance). In this second-order model, the first-order factors were depicted by kinds of personal meaning that loaded on the respective second-order factor, i.e., on the respective dimension of personal meaning. By means of the second-order model, I estimated the associations between all eight dimensions of meaning: application in life, duty, emotional well-being in class, freedom of action, positive image, professional qualification, purism of mathematics, and self-development. In this present investigation, the second-order model recorded a poor fit both in Germany (χ^2 = 3546.484, *df* = 2168, CFI = .804, TLI = .794, RMSEA = .048, SRMR = .091) and in Finland ($\chi 2$ = 3554.515, *df* = 2168, CFI = .830, TLI = .821, RMSEA = .050, SRMR = .082). Tables 6 (GER) and 7 (FIN) show the results from these second-order CFA models with factor loadings of the first-order factors and the resulting intercorrelations between the second-order dimensions. However, due to the poor overall model fit, these estimated model parameters are to be interpreted with care.

[30] In addition, Appendix D1–D4 report the standardized factor loadings with respect to the corresponding country (GER/FIN; see the additional file in the appendix).

Table 6.
Second-Order Confirmatory Factor Analysis: Factor Loadings of the First-Order Personal Meanings and Intercorrelations Between Dimensions of Personal Meaning in German Students.

Personal meaning	I	II	III	IV	V	VI	VII	VIII
Purism of mathematics	1.00							
Active practice of mathematics		.91						
Self-perfection		.86						
Cognitive challenge		.86						
Experience of competence			.70					
Experience of autonomy			.78					
Experience of social relatedness				.68				
Emotional-affective relation to the teacher				.77				
Classroom management				.51				
Support by the teacher				.88				
Marks					.84			
Examination					.64			
Vocational precondition					.50			
General education						.92		
Reference to reality						(.28)		
Positive image							1.00	
Duty								1.00
Purism of mathematics	-							
Self-development	.93	-						
Freedom of action	.78	.88	-					
Emotional w-b in class	(.19)	(.20)	.78	-				
Professional qualification	.40	*.47*	.80	.64	-			
Application in life	.78	.91	.86	(.42)	.63	-		
Positive image	*.35*	*.35*	.72	.63	.85	(.34)	-	
Duty	-.66	-.75	-.46	(.14)	(-.11)	-.52	(-.07)	-

Note. I = Purism of mathematics, II = Self-development, III = Freedom of action, IV = Emotional well-being (w-b) in class, V = Professional qualification, VI = Application in life, VII = Positive image, VIII = Duty.
Correlations in parentheses were not statistically significant; correlations printed in italics were significant at $p < .01$; all other correlations reported were significant at $p < .001$.

Table 7.
Second-Order Confirmatory Factor Analysis: Factor Loadings of the First-Order Personal Meanings and Intercorrelations Between Dimensions of Personal Meaning in Finnish Students.

Personal meaning	I	II	III	IV	V	VI	VII	VIII
Purism of mathematics	1.00							
Active practice of mathematics		.93						
Self-perfection		.94						
Cognitive challenge		.90						
Experience of competence			.93					
Experience of autonomy			.79					
Experience of social relatedness				.85				
Emotional-affective relation to the teacher				.95				
Classroom management				.61				
Support by the teacher				.88				
Marks					.72			
Examination					.62			
Vocational precondition					.75			
General education						.92		
Reference to reality						.32		
Positive image							1.00	
Duty								1.00
Purism of mathematics	-							
Self-development	.95	-						
Freedom of action	.69	.91	-					
Emotional w-b in class	.31	.43	.71	-				
Professional qualification	.63	.83	.94	.72	-			
Application in life	.84	.97	.85	.54	.94	-		
Positive image	.45	.57	.72	.61	.83	.60	-	
Duty	-.62	-.68	-.45	(-.07)	-.47	-.60	*-.19*	-

Note. I = Purism of mathematics; II = Self-development; III = Freedom of action; IV = Emotional well-being (w-b) in class; V = Professional qualification; VI = Application in life; VII = Positive image; VIII = Duty.
Correlations in parentheses were not statistically significant; correlations printed in italics were significant at $p < .01$; all other correlations reported were significant at $p < .001$.

In Germany, apart from reference to reality, all factor loadings of the first-order factors were significant and scored values higher than the cut-off mark >.3. The factor loadings ranged from .50 (vocational precondition) to .91 (active practice of mathematics). The correlations between the eight dimensions were not weak, ranging from *.35* ($p < .01$) (positive image and purism of mathematics, positive image and self-development) to .93 ($p < .001$) (self-development and purism of mathematics).

Regarding Finland, the first-order factor loadings were all significant and higher than the cut-off mark >.3. The loadings varied from .32 (reference to reality) to .95 (emotional-affective relation to the teacher). Here, most of the variance was well explained. Regarding the relations between the dimensions, the correlations between the higher-order dimensions ranged from .31 ($p < .001$) (emotional well-being in class and purism of mathematics) to .97 ($p < .001$) (application in life and self-development).

Altogether, these results indicated that this model structure did not fit the data. In both independent data sets, the fit statistics were behaving in a consistent way: the CFI fit resulted in a poor index, indicating that the variables used were not highly correlated (Brown, 2006; Maydeu-Olivares, 2019; Shi et al., 2019). Although the items were assigned to the respective factor, the drop in TLI and CFI suggested a need to consider modification of the measurement model by revising the theoretical factor structure (Shi et al., 2019). Moreover, although RMSEA (Shi et al., 2020) was supported, the weak SRMR fit reflected that even despite the large amount of variables (and high *df*), certain important relations between a couple of variables were still not measured appropriately (Brown, 2006; Maydeu-Olivares, 2019; Savalei, 2012). In conclusion, these results showed that the estimated model is too parsimonious to record essential connections between some factors (Maydeu-Olivares, 2019), and the model did not record the data well (Brown, 2006; Shi et al., 2019). Thus, next to the huge number of variables, the theoretical relations between the dimensions (Backhaus et al., 2015) decreased the fit statistics.

In a nutshell, it was not possible to test the separation of the dimensions in one complete model.[31] In the second-order model the complex composition of model structure, but certainly also the theoretical correlations between the factors, induced these poor results. For personal meaning, the eight separate measurement models for each dimension of personal meaning yielded the best

[31] By revising the factor structure, I also estimated an overall model with all 17 personal meanings together. The computation of a 17-factor model with 68 items is risky (Brown, 2006). As assumed from a literature review, in both groups the 17-factor model did not record a reasonable fit: Germany ($\chi2$ = 3140.704, *df* = 2074, CFI = .848, TLI = .834, RMSEA = .043, SRMR = .061), Finland ($\chi2$ = 3243.324, *df* = 2074, CFI = .857, TLI = .843, RMSEA = .047, SRMR = .062). Next to the huge number of variables, the theoretical relations among the 17 factors (Backhaus et al., 2015) decreased the fit statistics: In both independent data sets, the CFI fit resulted in a poor index, indicating that the variables used were not highly correlated. Across countries, SRMR values improved in comparison to the second-order model, indicating that the model based on the average of the residual correlations (Hu & Bentler, 1999) captures the data well (Shi et al., 2019). Although the items were assigned to the respective factors, the drop in TLI and CFI suggests a need to consider a modification of the measurement model by an alternative factor structure with respect to the theory (Brown, 2006; Savalei, 2012; Shi et al., 2019, 2020).

fit compared to both comprehensive models tested. As a consequence of these CFA results, I decided to continue the measurement invariance analyses with the eight separate measurement models of personal meaning and the five-dimensional model of motivational regulations separately.

Intercorrelations between personal meaning and regulatory styles. To investigate the intercorrelations between the theoretically assumed eight dimensions of personal meaning and the five regulatory styles, I initially studied the relations between the 17 kinds of personal meaning and regulatory styles. This step was necessary to inspect how the 17 personal meanings associate individually and subsequently as dimensions with the regulatory styles. Table 8 (GER) and Table 9 (FIN) report the intercorrelations between the 17 personal meanings and regulations. In the next step, I examined the relations between dimensions of personal meaning and regulatory styles. Table 10 (GER) and Table 11 (FIN) provide the associations between the dimensions of personal meaning and regulatory styles. In the following section, I refer solely to the significant relations ($p < .01$ and $p <. 001$).

Table 8.
Intercorrelations Between Regulatory Styles and Personal Meanings in German Students.

	Act	Cla	Cog	Dut	Emo	Exa	Au	Co	So	Gen	Mark	Pos	Puri	Rea	Sel	Supp	Voc	1	2	3	4
1 Am	-.63	-.54	-.51	.57	(-.18)	-*.33*	-.57	-.36	-.42	-.55	-.41	-*.25*	-.47	(-.11)	-.51	(-.18)	-.37				
2 Ext	-*.20*	(.00)	(-.10)	.41	(.15)	*.19*	(-.12)	(.05)	(-.04)	(-.06)	*.23*	.37	(-.17)	(.09)	(-.02)	.31	(-.02)	.34			
3 Intro	.38	*.28*	.38	-*.14*	.29	.37	.42	.54	(.19)	.42	.57	.86	.41	(.21)	.49	.35	.41	-.24	.28		
4 Iden	.23	*.22*	.32	-*.17*	(.17)	(.14)	*.32*	(.12)	(.10)	.51	.30	*.26*	.24	(.17)	.40	(.09)	.87	-*.30*	(.04)	.33	
5 Intr	.94	(.24)	.81	-.70	(.03)	(.19)	.70	*.23*	(.23)	.65	(.20)	*.23*	.79	(.12)	.69	(-.15)	.46	-.54	-*.21*	.31	.28

Notes. N = 276; Act = Active practice; Cla = Classroom management; Cog = Cognitive challenge; Dut = Duty; Emo = Emotional-affective relation to the teacher; Exa = Examination; Au = EXPERIENCE OF AUTONOMY; Co = EXPERIENCE OF COMPETENCE; So = EXPERIENCE OF SOCIAL RELATEDNESS; Gen = General education; Mark = Marks; Pos = Positive image; Puri = Purism of mathematics; Rea = Reference to reality; Sel = Self-perfection; Supp = Support by the teacher; Voc = Vocational precondition; Am = Amotivation; Ext = External regulation; Intro = Introjected regulation; Iden = Identified regulation; Intr = Intrinsic regulation.

Correlations in parentheses were not statistically significant; correlations printed in italics were significant at $p < .01$; all other correlations reported were significant at $p < .001$.

Table 9.
Intercorrelations Between Regulatory Styles and Personal Meanings in Finnish Students.

	Act	Cla	Cog	Dut	Emo	Exa	Au	Co	So	Gen	Mark	Pos	Puri	Rea	Sel	Supp	Voc	1	2	3	4
1 Am	-.82	-.50	-.68	.79	*-.24*	-.40	-.66	-.60	*-.31*	-.66	-.40	-.29	-.63	(-.13)	-.67	(-.16)	-.56				
2 Ext	*-.27*	(-.18)	*-.19*	*.33*	(.02)	(-.05)	(-.24)	-.28	(-.06)	(-.11)	(-.02)	(.23)	(-.15)	(-.06)	(-.20)	(.04)	(-.07)	.41			
3 Intro	.37	.31	.41	(-.13)	.37	.32	.38	.49	.40	.27	.29	.58	.35	*.17*	.36	(.25)	.23	*-.23*	(.14)		
4 Iden	.44	*.23*	.47	-.37	.33	.45	.45	.55	(.30)	.60	.53	.50	.38	(.08)	.49	*.23*	.84	-.35	(-.04)	*.24*	
5 Intr	.95	.42	.80	-.67	*.19*	.34	.68	.56	.32	.67	.33	.36	.81	(.17)	.77	(.06)	.50	-.73	-.27	.39	.33

Notes. $N = 256$; Act = Active practice; Cla = Classroom management; Cog = Cognitive challenge; Dut = Duty; Emo = Emotional-affective relation to the teacher; Exa = Examination; Au = Experience of autonomy; Co = Experience of competence; So = Experience of social relatedness; Gen = General education; Mark = Marks; Pos = Positive image; Puri = Purism of mathematics; Rea = Reference to reality; Sel = Self-perfection; Supp = Support by the teacher; Voc = Vocational precondition; Am = Amotivation; Ext = External regulation; Intro = Introjected regulation; Iden = Identified regulation; Intr = Intrinsic regulation.

Correlations in parentheses were not statistically significant; correlations printed in italics were significant at $p < .01$; all other correlations reported were significant at $p < .001$.

Table 10.
Intercorrelations of Second-Order Multifaceted Dimensions and Regulatory Styles in German Students.

	Amotivation	External	Introjected	Identified	Intrinsic
Self-development	-.63	-.15	.44	.31	.94
Freedom of action	-.63	(-.04)	.64	*.31*	.64
Emotional w-b in class	-.37	.20	.39	(.18)	(.03)
Professional qualification	-.53	(.08)	.62	.91	.52
Application in life	-.58	(-.05)	.46	.56	.69

Note. Correlations in parentheses were not statistically significant; correlations printed in italics were significant at $p < .01$; all other correlations reported were significant at $p < .001$.

Table 11.
Intercorrelations of Second-Order Multifaceted Dimensions and Regulatory Styles in Finnish Students.

	Amotivation	External	Introjected	Identified	Intrinsic
Self-development	-.78	*-.24*	.40	.50	.90
Freedom of action	-.70	-.31	.52	.59	.68
Emotional w-b in class	-.29	(-.00)	.38	.33	*.22*
Professional qualification	-.66	(-.08)	.36	.92	.57
Application in life	-.69	(-.12)	*.30*	.62	.70

Note. Correlations in parentheses were not statistically significant; correlations printed in italics were significant at $p < .01$; all other correlations reported were significant at $p < .001$.

Intercorrelations found in German data. As a first step, I examined which of the 17 personal meanings each of the regulatory styles recorded the strongest positive relation to. To gain a first overview, for each regulatory style I identified the highest correlating kind of personal meaning. In Germany, intrinsic regulation had the highest positive relation with the active practice of mathematics ($r = .94, p < .001$), whereas identified regulation scored the highest relation with vocational precondition ($r = .87, p < .001$). For introjected regulation, I found the highest positive association with positive image ($r = .86, p < .001$). External regulation ($r = .41, p < .001$) and amotivation ($r = .57, p < .001$) had the strongest positive reference with duty.

With respect to the multifaceted dimensions, intrinsic regulation scored the strongest positive correlation with self-development ($r = .94, p < .001$), whereas identified regulation correlated highly positively with professional qualification ($r = .91, p < .001$). Introjected regulation recorded the strongest positive association with freedom of action ($r = .64, p < .001$) and external regulation recorded only a moderate association with emotional well-being in class ($r = .20, p < .001$). For amotivation, I found no positive association regarding the multifaceted dimensions.

Intercorrelations found in Finnish data. In Finland, I also examined which of the 17 personal meanings each of the regulatory styles recorded the highest positive relation to. Intrinsic regulation had the highest positive relation with the active practice of mathematics ($r = .95, p < .001$), whereas identified regulation correlated highly positively with vocational precondition ($r = .84$, $p < .001$). For the style of introjected regulation, I found the highest positive association with positive image ($r = .58$, $p < .001$). External regulation ($r = .33$, $p < .01$) and amotivation ($r = .79$, $p < .001$) associated positively with duty.

Regarding the positive relations to the multifaceted dimensions of personal meaning, intrinsic regulation scored a strong positive correlation with self-development ($r = .90$, $p < .001$) and identified regulation correlated highly positively with professional qualification ($r = .92$, $p < .001$). Introjected regulation associated strongly positively with freedom of action ($r = .52$, $p < .001$). For external regulation and amotivation, I found no positive association in relation to the multifaceted dimensions.

Altogether, the correlation analyses in both independent data sets came up with interesting insights. In Germany, regarding the predicted strongest positive relations (cf. 2.3.1) between the 17 personal meanings and regulatory styles, five personal meanings (duty, examination, EXPERIENCE OF COMPETENCE, general education, and marks) showed a different relation to quality of motivation. Apart from general education, the personal meanings recorded associations with controlled types instead of with the predicted self-determined regulations. Surprisingly, examination (correlating highly positive with introjected regulation and not at all with identified regulation) was closer to controlled motivation than to self-determined motivation. In general, almost all personal meanings showed intercorrelations with regulations from bordering regions, apart from the active practice of mathematics and purism of mathematics (both correlated with intrinsic and introjected regulations, but not with identified regulation). As theoretically assumed, this result suggested that personal meanings do not refer to just one specific type of motivation. On the contrary, as expected, most of the personal meanings had one positive correlational emphasis and additional associations with regulations that were close to the prioritized positive correlation. Moreover, reference to reality did not yield any significant relations, neither positive nor negative, with types of motivation. EXPERIENCE OF SOCIAL RELATEDNESS did not show any positive reference to types of motivation.

On the level of dimensions, the predicted intercorrelations could be partly replicated in the German sample. Surprisingly, freedom of action and application in life showed the strongest correlations with regulations other than those that had been predicted. Here, freedom of action exhibited the same strong relation with both autonomous and controlled types of motivation. Apart from application in life (expected strongest relation with identified regulation) and freedom of action (expected to relate only with intrinsic regulation or self-determined regulation), all dimensions related to the regulations as predicted. For application in life and freedom of action, consideration of the facets of dimensions explains the respective correlation. Concerning application in life, reference to reality resulted in no significant correlation, whereas general education had the strongest relation with intrinsic regulation. With respect to freedom of action, both factors, EXPERIENCE OF AUTONOMY and EXPERIENCE OF COMPETENCE, recorded moderate to strong relations with intrinsic and introjected types of motivation. In summary, in contrast to the dimensions, consultation of the correlation analysis with the 17 personal meanings provided a more detailed insight into the interrelations between personal meaning and quality of motivation in Germany.

In Finland, almost all personal meanings recorded the predicted association structure with regulatory styles. With respect to the theoretically assumed highest positive relations among the 17 personal meanings and regulatory styles, five personal meanings, classroom management, duty, general education, reference to reality, and support by the teacher, showed a different relation to regulatory styles. Apart from duty and reference to reality, the personal meanings yielded relations with (more) self-determined regulatory styles. Interestingly, in Finland certain personal meanings also resulted, next to the association with introjected regulation, in minimum moderate positive relations with self-determined types of motivation: classroom management, emotional-affective relation to the teacher, examination, EXPERIENCE OF COMPETENCE, EXPERIENCE OF SOCIAL RELATEDNESS, positive image. In general, most personal meanings showed intercorrelations with regulatory styles of neighbouring types, apart from classroom management and EXPERIENCE OF SOCIAL RELATEDNESS (both correlated with intrinsic and introjected regulations). Also here, the theoretical assumptions were mirrored in the empirical data. Most of the personal meanings not only referred strongly to one certain type of motivation, but also to regulations that were close to the prioritized positive correlation. For all personal meanings, I found at least a low positive relation to the regulatory styles.

On the level of dimensions, the assumed intercorrelations could mostly be replicated. Regarding the predicted strongest correlations, only application in life resulted in a rather self-determined association, showing the highest correlation with self-determined regulatory styles rather than with identified regulation, as had been expected. For all other dimensions, the expected strongest association with each respective regulation was confirmed. Again, in contrast to the dimensions, consideration of the correlation analysis with the 17 personal meanings provided a more in-depth insight into the interrelations between the constructs of personal meaning and quality of motivation in Finland.

On a correlational level, results from the two countries were similar, although not identical. In order to further compare both countries, analyses of measurement invariance were conducted to determine the validity of such a comparison.

Measurement invariance across countries. To inspect the latent mean structure of the 17 personal meanings and five regulatory styles across countries, I estimated two-group CFAs. As personal meanings could only be measured by eight separate measurement models, eight two-group CFAs recorded to what extent the eight dimensions of application in life, duty, emotional well-being in class, freedom of action, positive image, professional qualification, purism of mathematics, and self-development can be generalized across both countries. For regulatory styles, one two-group CFA was conducted to estimate a certain level of measurement invariance regarding the five styles.

As previously stated, to test for measurement invariance, I made a comparison between the model fit indices for models with dissimilar sets of invariance constraints. As outlined by Meredith

(1993), the constraints ranked from the baseline model (no restrictions for parameters – configural invariance) to strong factorial invariance (constraints on measurement intercepts and factor loadings – scalar invariance) that represent the precondition for latent mean comparisons over groups (Meredith, 1993). For each two-group CFA model, I consistently implemented the appropriate constraints from the configural model (M1) over the metric (M2) to the (partial) scalar measurement model (M3/M3p). Following an exploratory approach, I evaluated the decrease in goodness-of-fit (Chen, 2007; Cheung and Rensvold, 2002) that resulted with the respective change in parameter constraints. Table 12 reports the model fit statistics of the measurement invariance test.

Table 12.
Fit Statistics for Each Measurement Invariance Model of the Nine Higher-Order Dimensions of Personal Meaning and Regulatory Style.

Tested Model	χ^2	*df*	CFI	Δ_{CFI}	TLI	RMSEA	Δ_{RMSEA}	SRMR	Δ_{SRMR}
Purisms of mathematics									
M1: configural invariance	54.32	28	.978		.967	.059		.030	
M2: weak measurement invariance	59.55	34	.978	.000	.973	.053	.006	.035	.005
M3: strong measurement invariance	72.02	40	.973	.005	.972	.055	.002	.039	.004
Positive image									
M1: configural invariance	8.04	4	.988		.963	.062		.022	
M2: weak measurement invariance	12.76	7	.982	.006	.970	.056	.006	.036	.006
M3: strong measurement invariance	28.06	10	.945	.037	.934	.083	.027	.050	.014
M3p: partial strong measurement invariance	14.87	9	.982	.000	.976	.050	.006	.042	-.006
Duty									
M1: configural invariance	10.29	4	.987		.962	.077		.022	
M2: weak measurement invariance	13.69	7	.987	.000	.977	.060	.017	.029	.007
M2p: partial weak measurement invariance	13.42	6	.985	.002	.970	.068	.009	.029	007
M3p: partial strong measurement invariance	17.46	9	.983	.002	.978	.060	008	.033	.004
Emotional w-b in class									
M1: configural invariance	293.30	196	.948		.936	.043		.052	
M2: weak measurement invariance	317.88	208	.941	.007	.932	.045	002	.059	007
M3: strong measurement invariance	367.44	220	.921	.020	.914	.050	.005	.066	-.007
M3p: partial strong measurement invariance	341.83	218	.934	.007	.927	.046	.001	.062	.003
Professional qualification									
M1: configural invariance	123.00	82	.976		.968	.043		.040	
M2: weak measurement invariance	133.80	90	.974	.002	.968	.043	.000	.046	.006
M3: strong measurement invariance	175.41	98	.954	-.020	.949	.054	.011	.052	.006
M3p: partial strong measurement invariance	149.19	97	.969	.005	.965	.045	.002	.048	.002
Self-development									
M1: configural invariance	96.09	64	.978		.969	.043		.036	
M2: weak measurement invariance	103.57	71	.978	.001	.972	.042	-.002	.042	-.003
M3: strong measurement invariance	156.88	78	.946	-.032	.938	.062	.020	.054	.012

M3p: partial strong measurement invariance	111.09	76	.976	.002	.972	.042	.000	.045	.003
Application in life									
M1: configural invariance	47.19	38	.991		.987	.030		.033	
M2: weak measurement invariance	56.54	44	.988	.003	.985	.033	.003	.042	.009
M3: strong measurement invariance	77.61	50	.973	-.015	.970	.046	.013	.051	.009
M3p: partial strong measurement invariance	68.97	49	.981	.007	.978	.039	.006	.049	.007
Freedom of action									
M1: configural invariance	46.07	38	.988		.982	.028		.036	
M2: weak measurement invariance	53.33	44	.986	.002	.982	.028	.000	.047	.011
M3: strong measurement invariance	74.95	50	.962	-.024	.958	.043	.015	.048	.001
M3p: partial strong measurement invariance	64.16	49	.977	.009	.974	.034	.006	.046	.001
Regulatory style									
M1: configural invariance	335.92	218	.971		.964	.045		.049	
M2: weak measurement invariance	359.62	230	.968	.003	.962	.046	.001	.052	001
M3: strong measurement invariance	443.48	242	.951	.017	.944	.056	010	.057	005
M3p: partial strong measurement invariance	405.06	240	.960	.008	.954	.051	.005	.055	.003

Notes. CFI = comparative fit index; TLI = Tucker–Lewis index; RMSEA = root mean error of approximation; SRMR = standardized root mean squared residual.

For all two-group CFAs, the initial configural model across countries showed a good fit to the data (M1). This reflected that all models fit the data across groups by estimating the same loading structure of factors.

Next, I compared the model fit from configural invariance (M1) with metric invariance (M2), for which factor loadings are constrained. Here, I found support for metric invariance for all two-group CFAs apart from duty. In line with Chen (2007), the two-group CFA for model fit for duty did not reveal an acceptable cut-off limit (ΔRMSEA 0.015); whereas CFI and SRMR supported the model fit, metric invariance was not supported for duty. This indicated that the measurement properties of the scale were not manifested equally over countries and duty cannot be generalized to measure the same latent construct in Germany and Finland. According to Chen (2008), non-metric invariance would probably result by importing a scale from one linguistic and cultural setting to another that indicated difference in conceptualization and meaning of construct. Moreover, a failure in loading invariance may result when respondents tend to tick or avoid extremes on scales (Chen, 2008).

To ensure that this scale was manifested in the same way in Germany and Finland, I examined the partial metric model, following the guidelines in the modification indices (Byrne et al., 1989; Cheung & Rensvold, 1998; Rensvold & Cheung, 1998). Here, partial measurement invariance allowed certain factor loadings to be freely estimated and some to be invariant across groups. As a consequence, I allowed one out of four items of the scale "duty" to be non-invariant across countries. As the majority of factor loadings were invariant across groups, the resulting model supported partial metric invariance for factor loadings. Thus, all investigated scales (application in life, duty (partial), emotional well-being in class, freedom of action, positive image, professional qualification, purism of mathematics, self-development, and regulatory styles) have invariant factor loadings and thus showed that in both countries the constructs are manifested similarly (Millsap & Olivera-Aguilar, 2012).

To explain the latent factors on a mean level, scalar invariance requires items' factor loadings (M2; weak measurement invariance) and items' intercepts to be equal across groups. Scalar invariance (M3; strong measurement invariance), thus, shows that potential differences of variable means are a result of the influence of common factors. As the scale for duty only reached partial metric invariance, I proceeded to also test partial scalar invariance for this variable. With regard to scalar invariance (M3), the results of application in life, emotional well-being in class, professional qualification, and regulatory styles revealed a drop in fit that was not within the acceptable cut-off limits of CFI (ΔCFI > 0.010). While RMSEA showed an acceptable drop in fit, CFI did not result in a reasonable fit. Further, the resulting model 3 for freedom of action, positive image, and self-development did not yield an acceptable drop of both CFI (ΔCFI 0.010) and RMSEA (ΔRMSEA 0.015). The results of model 3 for purism of mathematics and duty (partial) fit the data adequately when comparing scalar invariance to metric invariance. Thus, scalar invariance (partial scalar for duty) was supported only for those two, and not for the other seven models.

As indicated earlier (3.3.3), scalar invariance allows us to compare the same constructs across different groups and to interpret the mean differences as attributable to valid cultural differences, thus avoiding measurement bias (Chen, 2008). Here, various parameters might cause the lack of scalar invariance, such as translation inaccuracy and norms of social desirability (Hui & Triandis, 1985) within learners' responses (Chen, 2008). However, scalar invariance, or rather the invariance of item intercepts, is crucial to drawing reliable conclusions with regard to group differences on a mean level.

For this purpose, I have investigated partial invariance (e.g., Byrne et al. 1989; Steenkamp & Baumgartner, 1998) for the item intercepts in all seven models. Again, partial scalar invariance models allow some item intercepts to be invariant and others to be estimated freely. Following an exploratory approach proposed by Byrne et al. (1989) and Cheung & Rensvold (1998), I examined the possibilities of employing partial invariance models. As recommended (cf. "Test of measurement invariance" in section 3.3.3), for every scale I freely estimated one item of each subfactor and constrained all remaining items to display equal intercepts.

Altogether, all nine models yielded an equivalent estimated factor loading structure and supported metric invariance across groups. Thereby, for eight models I found support for full metric invariance and for duty, I found partial metric invariance by allowing one certain factor loading to be freely estimated. The attainment of (partial) metric invariance indicated that each item of the scale loaded to the specified latent factor in a similar way and with equivalent magnitude over groups. Alongside the inspection of equal factor loadings, I tested whether the item intercepts were invariant in both data sets. At least two items of each factor supported full metric and full scalar invariance (Byrne et al., 1989; Steenkamp & Baumgartner, 1998). This guaranteed a meaningful and valid interpretation of group differences for the measured constructs. However, the conclusions on potential group differences should be carefully interpreted, since 11 indicators (DIF items) out of 85 are biased.

5.2 Latent Mean Differences in Personal Meanings and Regulatory Styles

Latent mean differences in personal meanings and regulatory styles. After reaching (partial) scalar invariance, I examined the latent mean structure for the nine factor models over countries. Here, I chose the German sample as the reference group where the factor means were constrained at zero and the latent means for the Finnish group were freely estimated (Kleinke et al., 2017). A positive value in difference thus reflected a higher latent mean value for the Finnish group and a negative value mirrored a higher mean score for the reference group Germany. As asserted by Hancock (2001, pp. 375–377), these differences that emerged between the latent mean values are comparable to Cohen's d effect size values where $d = .2$ (but not 0; Cohen, 1988) can be interpreted as a "small", $d = .5$ as a "medium", and $d = .8$ as a "large" effect size (Cohen, 1988, pp. 24–27). Table 13 illustrates the results for the standardized latent mean differences over countries.

Table 13.
Latent Mean Differences in Personal Meanings and Regulatory Styles Between German and Finnish Students.

Factor	*M*	*SE*	*p*
Personal meaning			
Active practice	+0.75	0.12	<.001
Classroom management	-0.02	0.10	.814
Cognitive challenge	+0.26	0.10	.013
Duty	-0.20	0.09	.026
Emotional-affective relation to the teacher	-0.22	0.09	.020
Examination	+0.21	0.10	.033
EXPERIENCE OF AUTONOMY	+0.00	0.10	.987
EXPERIENCE OF COMPETENCE	-0.04	0.10	.697
EXPERIENCE OF SOCIAL RELATEDNESS	+0.01	0.10	.899
General education	+0.19	0.10	.053
Marks	+0.41	0.11	<.001
Positive image	+0.39	0.11	<.001
Purism of mathematics	+0.06	0.09	.465
Reference to reality	-0.17	0.10	.095
Self-perfection	+0.66	0.11	<.001
Support by the teacher	-0.21	0.09	.026
Vocational precondition	+0.57	0.11	<.001
Regulatory style			
Amotivation	+0.24	0.10	.017
External regulation	+0.28	0.12	.022
Identified regulation	+0.49	0.11	<.001
Intrinsic regulation	+0.13	0.09	.168
Introjected regulation	+0.07	0.09	.427

Notes. M = mean difference German – Finnish students.

The comparison of personal meanings and regulatory styles on the latent mean level produced results in which Finnish students were in a stronger position. Regarding personal meanings, significant mean differences between German and Finnish learners were found for five personal meanings ($p < .001$). The Finnish group perceived a higher personal relevance in learning mathematics through the active practice of mathematics ($M = +0.754$, $p < .001$), marks ($M = +0.408$, $p < .001$), positive image ($M = +0.386$, $p < .001$), self-perfection ($M = +0.663$, $p < .001$), and vocational precondition ($M = +0.573$, $p < .001$). For classroom management ($M = -0.024$, $p = .814$), EXPERIENCE OF AUTONOMY ($M = +0.002$, $p = .987$), EXPERIENCE OF COMPETENCE ($M = +0.039$, $p = .697$), EXPERIENCE OF SOCIAL RELATEDNESS ($M = +0.013$, $p = .899$), and purism of mathematics ($M = +0.063$, $p = .465$) German and Finnish learners ascribe similar relevance. I found no significant ($p < .001$) mean differences concerning the personal meanings of cognitive challenge ($M = +0.256$, $p = .013$), duty ($M = -0.198$, $p = .026$), emotional-affective relation to the teacher ($M =

−0.215, $p < .020$), examination ($M = +0.207$, $p = .033$), general education ($M = +0.186$, $p = .053$), reference to reality ($M = −0.167$, $p = .095$), and support by the teacher ($M = −0.208$, $p = .026$).

The mean comparison for regulatory styles in relation to learning mathematics resulted in a significantly positive score for Finnish students regarding identified regulation ($M = +0.489$, $p < .001$). Both groups revealed relatively the same amount for introjected regulation ($M = +0.074$, $p = .427$), whereas no significant mean differences were yielded with respect to amotivation ($M = +0.243$, $p = .017$), external regulation ($M = +0.278$, $p = .022$, and intrinsic regulation ($M = +0.125$, $p = .168$).

The reported standardized mean differences among German and Finnish students showed substantial differences, with Finnish learners scoring higher latent mean values for meanings related more strongly to the subject of mathematics than German students. These test results were also reflected in that the Finnish group perceived higher identified regulation than German learners. Both groups showed self-determined motivational behaviour, whereas the overall self-determination index[32] (SDI) was somewhat higher in Finland (SDI = 1.79) than in Germany (SDI = 1.37). These preferences and the attitude towards mathematics learning might relate to the curriculum setting of the respective culture. In terms of these differences, a discussion with respect to the subject-specific tasks in Germany and Finland might be fruitful (see Chapter 6).

Although partial scalar invariance allowed a fair comparison of the latent factors on a mean level, the results from previous analyses (cf. correlation analyses and measurement invariance) indicated the existence of cultural differences in learners' responses (Chen, 2008). Similar to those results, the significant differences in latent means again showed considerable differences between both groups. It seems that aspects from both countries' educational systems and mathematics curricula contribute to different preferences and perceptions of mathematics. To understand these differences, I provide tentative explanations for these results by referring to both countries' mathematics curricula (see section 6.2). As a consequence of this empirical evidence, the following latent class analysis considered both groups in a separate analysis in order to respect their cultural diversity. That is, for each country I conducted an individual latent class analysis to see which group solution is the best for the respective country. In the following section, I present the results of the latent class analysis that followed an exploratory approach and yielded surprising results.

[32] Self-determination index (SDI; Levesque et al., 2004; Vallerand et al., 1992) is calculated as follows: SDI = (2 × intrinsic motivation) + identified regulation – introjected regulation – (2 × external regulation). The SDI can therefore summarize self-determined behaviour (positive scores) or controlled motivation (negative values).

5.3 Profiling Classes of Personal Meaning and Their Outcomes in Motivation

Latent class analysis. Since I was interested in the patterns of personal meaning, I used mean values on each of the German and Finnish students for the 17 personal meanings.[33] I considered several important goodness-of-fit indices proposed by Geiser (2011), Marsh et al. (2009), and Nylund et al. (2007), for the assessment of the quality of the latent class analysis (LCA) models. Table 14 presents the goodness-of-fit indexes of the conducted LCA in Germany.

Table 14.
Fit Indices of Latent Class Analysis Based on the 17 Personal Meanings in the German Sample.

	p	*LL*	*AIC*	*BIC*	*aBIC*	*VLMR*	*BLRT*	*Entropy*
1-Class	34	-4528.309	9124.619	9246.201	9138.404	-	-	-
2-Class	69	-4118.395	8374.790	8621.531	8402.766	.0550	≤.001	.878
3-Class	104	-3954.775	8117.550	8489.449	8159.716	.1840	≤.001	.873
4-Class	139	-3814.390	7906.779	8403.836	7963.136	.2043	≤.001	.887
5-Class	174	(-3720.146)	7788.291	8410.507	7858.838	.3745	≤.001	.904

Note. $N = 264$; Scaling = scaling factor associated with MLR log likelihood estimates; AIC = Akaike's Information Criterion; BIC = Bayesian Information Criterion; aBIC = sample-size adjusted BIC; VLMR = Vuong–Lo–Mendell–Rubin; BLRT = Bootstrap Likelihood Ratio Test.

Regarding Germany, the fit statistics of the LCA models AIC, BIC, and aBIC dropped as the class amount increased. From four- to five-group solutions, AIC and aBIC only decreased and BIC increased. Based on their simulation study, Nylund et al. (2007) proposed using the BIC index in particular to determine the best class solution and thus the four-class solution was the best (Rost, 2006). The VLMR results were not significant for all models. However, the simulation study by Nylund et al. (2007) additionally recommended considering the bootstrap likelihood ratio test (BLRT) as this is a superior tracer for statistical model comparisons with the VLMR test. Therefore, I additionally consulted BLRT to statistically compare the LCA models. For all models, BLRT was ≤ .001. In all models, entropy was close to 1 and thus reflects an overall high level of classification immunity (Geiser, 2011). The average class assignment probabilities for the most likely class membership were greater than the cut-off mark .80 (Geiser, 2011) for each class on the main diagonal. Further, all solutions had profiles with more than 5% of the cases (Marsh et al., 2009). Aside from that, examination of log likelihood (LL) values reflected a rapid decrease from the two-class solution to the four-class model. For the five-class solution the best LL could not be replicated and the preliminary report (not trustworthy) of LL increased compared to the four-class model. According to the parsimonious principle (Geiser, 2011), the class solution should preferably represent the "new knowledge" by means of a minimal number of classes. Finally, by examining the interpretability (as the most important criterion; Geiser, 2011) of the class solutions, the

[33] I was interested in students' response patterns based on the 17 personal meanings. Therefore, I did not use students' mean values on the dimension level for the class profiling. However, appearance of the eight dimensions of personal meaning and the corresponding motivational outcome were part of the theory-driven analyses (cf. Chapter 7).

interpretation amount increased from the two-class solution, whereas the additional class in the five-group solution did not increase the interpretation value compared to the four-profile solution. Effectively, a class solution that is easy to interpret should have mainly high (here ≥ 1.8) or low (here ≤ 1.2) values, and none from the average range (Geiser, 2011). Taking all of this into account, I identified the four-group solution as the best LCA model for Germany. According to this criterion, all profiles in the four-class solution were easy to interpret.

Following this, I included the regulatory styles (intrinsic, identified, introjected, external motivation, and amotivation) within the four-LCA model as the distal outcomes. As external validation criteria (Geiser, 2011), the regulatory styles (as covariates) also showed theory-compatible connections (see Chapter 7) to the four-class solution when I compared the differences between the profiles with respect to the five regulatory styles. The external validation criteria, using regulatory styles as covariates, and the reflection of the individual class properties among the profiles, provided usefulness and certainty for the identified four-class model (Geiser, 2011). Figure 14 illustrates the identified four profiles of personal meaning and Table 15 reports the characteristics of the profile membership in the regulatory styles in Germany. Additionally, Figure 15 reflects learners' differences in their biological natures (standardized characteristics of the empirical profiles on the regulatory styles) within the empirical individual relevance systems.

For better traceability, I provide a detailed description and interpretation of the profiles' motivational outcomes (the biological regulatory mechanism) in the following chapter (Chapter 7) in detail in order to check both perspectives, biological and social regulations, for coherence. I provide here only the significant differences[34] in numerical format without reporting them comprehensively.

[34] The differences were the result of Mplus chi-square statistics (Muthén & Muthén, 1998–2017).

Figure 14

Four Profiles of Personal Meaning in German Students.

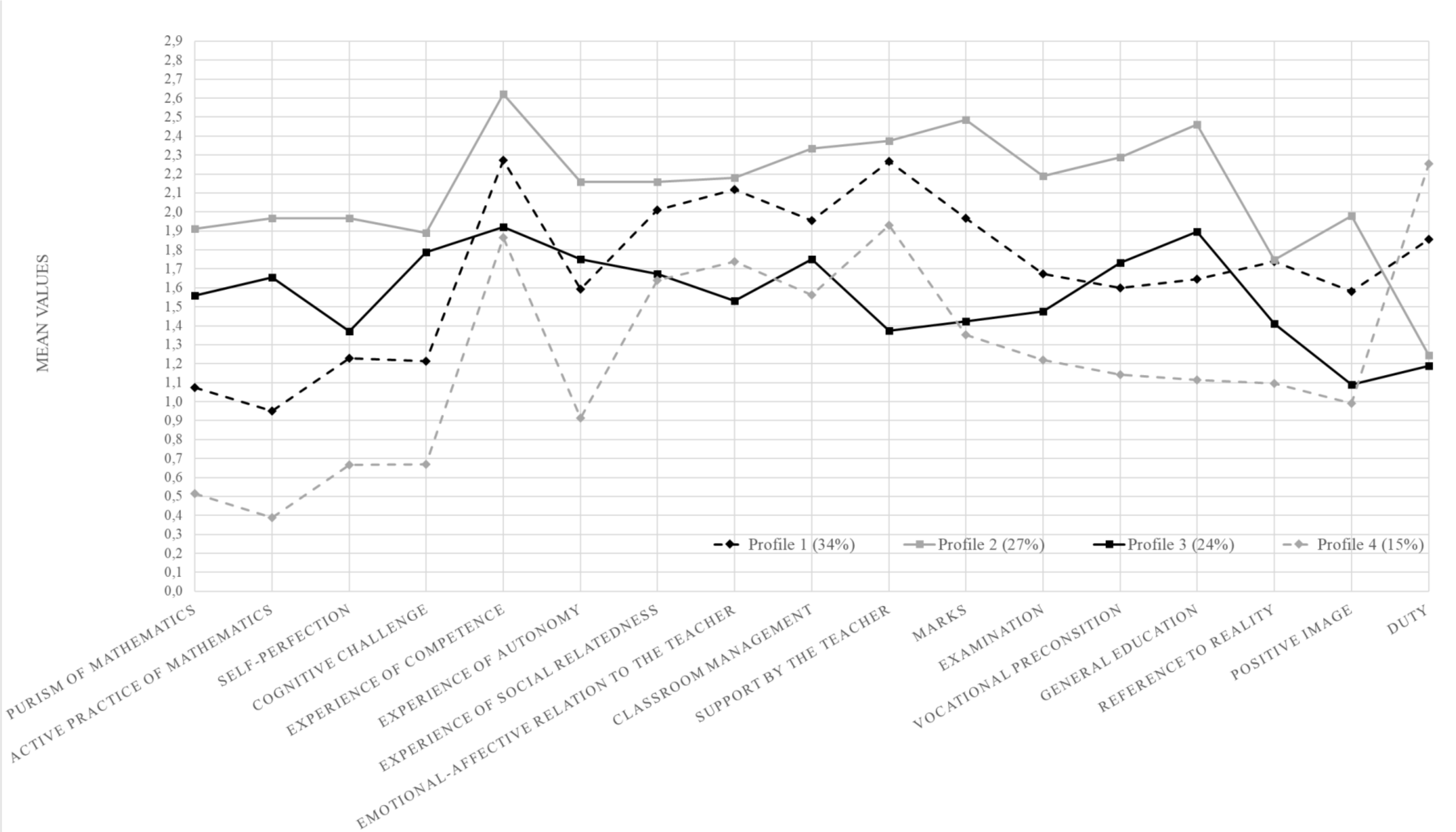
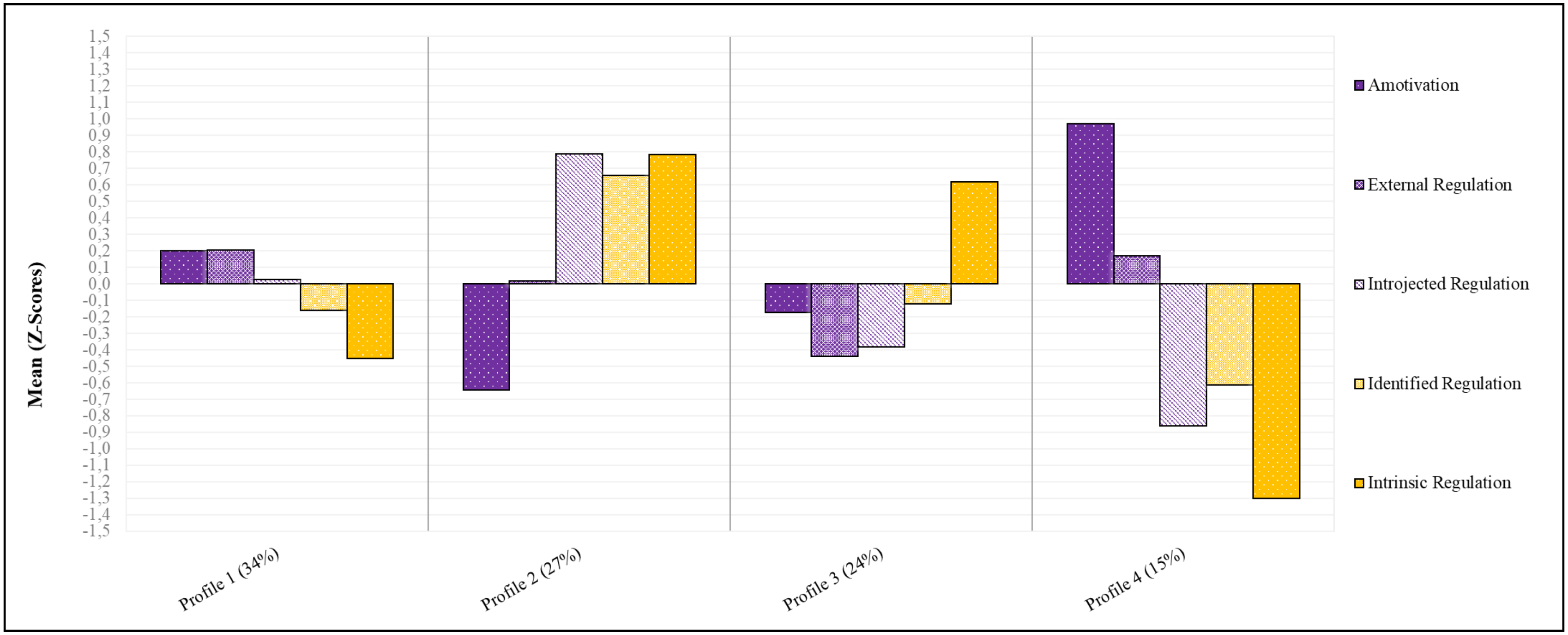


Table 15.
Relationship Between LCA Membership and Outcome Variables in German Students.

	M (SE)				Differences between
Regulatory style	Profile 1	Profile 2	Profile 3	Profile 4	Profiles
Intrinsic regulation	0.93 (.05)	1.95 (.08)	1.81 (.09)	0.23 (.06)	2=3>1>4
Identified regulation	1.84 (.08)	2.52 (.08)	1.87 (.10)	1.47 (.14)	2>1=3>4
Introjected regulation	1.24 (.08)	1.87 (.09)	0.90 (.09)	0.50 (.09)	2>1>3>4
External regulation	1.18 (.07)	1.04 (.09)	0.70 (.10)	1.15 (.13)	1=4=2>3
Amotivation	0.94 (.06)	0.37 (.05)	0.68 (.07)	1.46 (.14)	4>1>3>2

Figure 15

Characteristics of the Latent Classes in the Outcome Variables Regulatory Styles in German Students.



Note. The results were standardized to help in the interpretation of this histogram.

Table 16 reports the fit indexes of the conducted LCAs in Finland.

Table 16.
Fit Indices of Latent Class Analysis Based on the 17 Personal Meanings in the Finnish Sample.

	p	*LL*	*AIC*	*BIC*	*aBIC*	*VLMR*	*BLRT*	*Entropy*
1-Class	34	-4396.477	8860.96	8979.72	8871.94	-	-	-
2-Class	69	-3769.548	7677.10	7918.12	7699.40	≤.001	≤.001	.942
3-Class	104	-3576.896	7361.79	7725.07	7395.41	.0367	≤.001	.931
4-Class	139	-3450.538	7179.08	7664.61	7224.00	.3009	≤.001	.930
5-Class	174	(-3370.602)	7089.20	7697.10	7145.44	.2357	≤.001	.949

Note. *N* = 243; Scaling = scaling factor associated with MLR log likelihood estimates; AIC = Akaike's Information Criterion; BIC = Bayesian Information Criterion; aBIC = sample-size adjusted BIC; VLMR = Vuong–Lo–Mendell–Rubin; BLRT = Bootstrap Likelihood Ratio Test.

Concerning Finnish learners, the goodness-of-fit indexes of the LCA models AIC, BIC, and aBIC dropped as the class numbers rose. From four- to five-group solutions AIC and aBIC only decreased but BIC increased. The VLMR results were significant for the two-class and marginally significant for the three-class model. For the four- and five-class solutions, VLMR resulted in no significant value. Similar to the LCA on the German data set, I consulted BLRT to statistically compare the LCA models. For all models, BLRT was ≤ .001. In all models, entropy was close to 1 and thus reflected an overall high level of classification certainty (Geiser, 2011). For each class solution, the average class assignment probabilities for the most likely class membership were greater than the cut-off mark .80 (Geiser, 2011; Rost, 2006). Moreover, almost all class solutions had profiles with more than 5% of the cases, but one class of five-class model was close to the cut-off mark (*n* = 15 from *N* = 243). Aside from that, examination of LL values reflected a rapid decrease from the two-class to the four-class solution. For the five-class solution, the best LL could not be replicated. Finally, by examining the interpretability of the class solutions, the interpretation amount increased from the two-class solution, whereas the additional class in the five-group solution did not increase the interpretation value compared to the four-profile solution. Taking everything into account (including the parsimonious principle; Geiser, 2011), I concluded that in the case of Finland, the four-group solution was the optimal LCA model. All the profiles in the four-class model were well interpretable. Figure 16 illustrates the four profiles of personal meaning. Having ascertained the best class solution for Finland, I included the regulatory styles (intrinsic, identified, introjected, external motivation, and amotivation) as covariates in the final LCA model. Table 17 reports the characteristics of the profile membership on the regulatory styles in Finland. Additionally, Figure 17 reflects learners' differences in terms of their biological natures (standardized characteristics of the empirical profiles on the regulatory styles) within the empirical individual relevance systems.

Similar to the LCA in Germany, as a support for an external validation (Geiser, 2011), the regulatory styles as covariates also showed theory-compatible connection to the four-class solution as I compared the difference between the profiles with respect to the five regulatory styles (see

Chapter 7 for more). This aspect and the reflection of the individual profile characteristics between the four classes highlighted usefulness and certainty for the identified four-class solution in Finland (Geiser, 2011).

Figure 16

Four Profiles of Personal Meaning in Finnish Students.

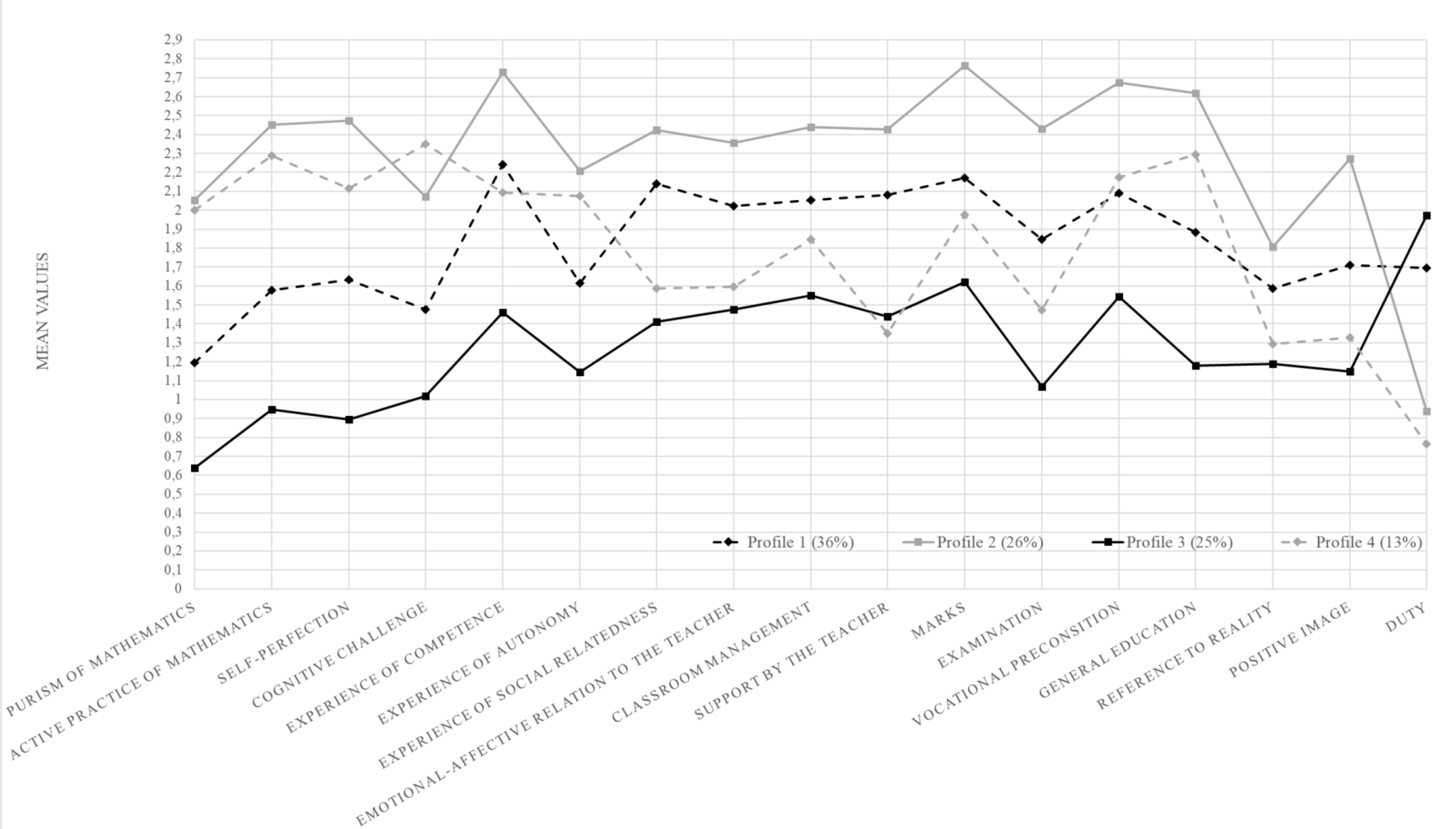
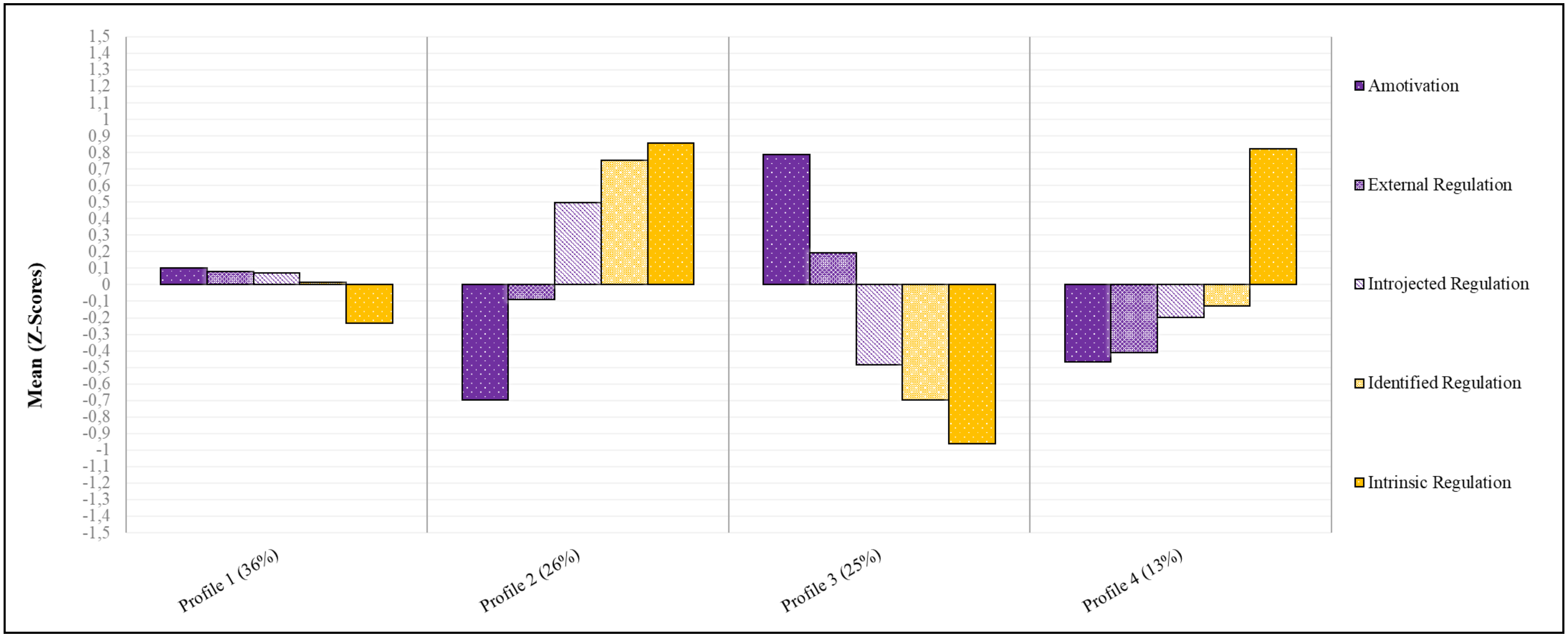


Table 17.
Relationship Between LCA Membership and Outcome Variables in Finnish Students.

	M (SE)				Differences Between
Regulatory style	Profile 1	Profile 2	Profile 3	Profile 4	Profiles
Intrinsic regulation	1.28 (.07)	2.21 (.06)	0.66 (.07)	2.19 (.13)	2=4>1>3
Identified regulation	2.33 (.06)	2.85 (.03)	1.82 (.10)	2.23 (.14)	2>1=4>3
Introjected regulation	1.23 (.07)	1.57 (.11)	0.79 (.07)	1.01 (.16)	2>1=4>3
External regulation	1.19 (.06)	1.07 (.09)	1.26 (.08)	0.84 (.13)	2=1=3>4
Amotivation	1.02 (.06)	0.48 (.07)	1.48 (.08)	0.63 (.09)	3>1>2=4

Figure 17

Characteristics of the Latent Classes in the Outcome Variables Regulatory Styles in Finnish Students.



Note. The results were standardized to help in the interpretation of this histogram.

Contrary to expectations (existing cultural differences may lead to different class solutions), in both countries a four-class solution resulted as the best. The class solution determined by exploration in Germany could be similarly replicated with a new or independent data set from Finland. In line with Geiser (2011), these results underpin the replicability of the four-class solution (Geiser, 2011). A brief overview of the numerical profile characteristics in Germany and Finland reflected certain similarities and differences in the data. To see whether the class profiling resulted in the exact same results in both countries, I compared the profile outputs that scored similar high and low values. To compare the profile outputs, I conducted Mann–Whitney U tests. More detailed analyses of all profiles confirmed that the German and Finnish classes follow similar learning orientations, but they do encompass unique characteristics with respect to their cultural settings (see Chapter 7). To support the upcoming interpretation about the differences between German and Finnish students' mathematics-learning orientations, I statistically identified the central tendency on which the profiles with similar response patterns differ. Although the profiles showed corresponding results across countries, Mann–Whitney U tests revealed specific significant differences.

Mann–Whitney U test. As indicated earlier, I detected four profiles of personal meaning in Germany and Finland that showed similar response patterns. An overview of all eight profiles (based on high and low values of each of the German and Finnish classes) indicated that Profile 1–GER and Profile 1–FIN (Table 18), Profile 2–GER and Profile 2–FIN (Table 19), Profile 3–GER and Profile 4–FIN (Table 20), and Profile 4–GER and Profile 3–FIN (Table 21) had many similarities, but also certain differences. On the basis of Mann–Whitney U tests, I compared these profiles (pairwise) in order to see to what extent these profiles corresponded to each other and whether they showed any significant differences with respect to their preferences (see Tables 18–21). Again, a detailed description and interpretation of all profiles follow in Chapter 7. Now, I provide only a report of the statistically significant (Bonferroni adjusted p-value) differences when a kind of personal meaning/regulatory style was realized (≥ 1.8 mean values) in one group but not at all (≤ 1.2 mean values) in the other, or if one group perceived more relevance for the constructed personal meaning or a rather higher value for the regulatory style than the other group.

Table 18.
Mann–Whitney U Test Statistics for Mean Difference of Personal Meanings and Regulatory Styles in Profile 1 From Germany and Profile 1 From Finland.

	M		*U*	*Z*	*p*	*r*
	GER	FIN				
Active practice of mathematics	0.95	1.58	6387.50	7.36	<.001	.55
Classroom management	1.96	2.05	4306.50	1.15	.249	.08
Cognitive challenge	1.21	1.48	4964.00	3.34	<.001	.25
Duty	1.86	1.69	3338.00	-1.71	.087	-.12
Emotional-affective relation to the teacher	2.12	2.02	3328.00	-1.74	.081	-.13
Examination	1.68	1.85	4438.00	1.55	.121	.11
EXPERIENCE OF AUTONOMY	1.59	1.62	4112.00	0.58	.559	.04
EXPERIENCE OF COMPETENCE	2.27	2.24	3668.00	-0.74	.459	-.05
EXPERIENCE OF SOCIAL RELATEDNESS	2.01	2.14	4457.00	1.61	.106	.12
General education	1.65	1.88	4979.50	3.17	.002	.23
Marks	1.97	2.17	4598.50	2.05	.040	.15
Positive image	1.58	1.71	4410.00	1.46	.143	.11
Purism of mathematics	1.07	1.19	4442.00	1.55	.120	.11
Reference to reality	1.74	1.59	3280.50	-1.88	.060	-.14
Self-perfection	1.23	1.63	5879.00	5.84	<.001	.43
Support by the teacher	2.27	2.08	3061.50	-2.56	.010	-.19
Vocational precondition	1.60	2.09	5660.50	5.31	<.001	.40
Amotivation	0.94	1.03	4267.50	1.18	.235	.08
External regulation	1.19	1.19	4011.00	0.28	.778	.02
Introjected regulation	1.24	1.24	4065.00	0.44	.659	.03
Identified regulation	1.85	2.34	5343.00	4.29	<.001	.32
Intrinsic regulation	0.93	1.29	5320.00	4.18	<.001	.31

Note. GER = Germany; FIN = Finland.

Profile 1 from Germany versus Profile 1 from Finland. The comparison between the first two profiles indicated that Finnish learners perceived relevance in vocational precondition (M = 2.09, SD = .48), whereas German students did not (M = 1.60, SD = .59), U = 5660.50, p = <.001, r = .40. Regarding types of motivation, Finnish students had higher experience in identified regulation (M = 2.34, SD = .58) than German learners (M = 1.85, SD = .72), U = 5343.00, p = <.001, r = .32.

Table 19.
Mann–Whitney U Test Statistics for Mean Difference of Personal Meanings and Regulatory Styles in Profile 2 From Germany and Profile 2 From Finland.

	M		*U*	*Z*	*p*	*r*
	GER	FIN				
Active practice of mathematics	1.97	2.45	3198.50	4.35	. <.001	.37
Classroom management	2.34	2.44	2528.00	1.31	.190	.11
Cognitive challenge	1.89	2.07	2741.00	2.28	.022	.19
Duty	1.24	0.94	1609.00	-2.81	.005	-.24
Emotional-affective relation to the teacher	2,18	2.36	2652.50	1.87	.061	.16
Examination	2.19	2.43	2771.50	2.43	.015	.20
EXPERIENCE OF AUTONOMY	2.16	2.21	2338.00	0.46	.645	.03
EXPERIENCE OF COMPETENCE	2.62	2.73	2580.00	1.58	.112	.13
EXPERIENCE OF SOCIAL RE-LATEDNESS	2.16	2.42	2906.50	3.02	.002	.26
General education	2.46	2.62	2753.00	2.35	.019	.20
Marks	2.49	2.76	2961.50	3.50	<.001	.30
Positive image	1.98	2.27	2924.00	3.098	.002	.26
Purism of mathematics	1.91	2.05	2496.50	1.31	.187	.11
Reference to reality	1.75	1.81	2297.50	0.27	.784	.02
Self-perfection	1.97	2.47	3665.00	6.44	<.001	.55
Support by the teacher	2.37	2.43	2363.00	0.57	.563	.04
Vocational precondition	2.29	2.67	3095.00	3.88	<.001	.33
Amotivation	0.37	0.49	2494.00	1.19	.230	.10
External regulation	1.05	1.07	2278.00	0.18	.852	.01
Introjected regulation	1.87	1.58	1854.00	-1.71	.086	-.14
Identified regulation	2.52	2.86	2784.00	2.77	.006	.23
Intrinsic regulation	1.95	2.22	2734.50	2.24	.025	.19

Note. GER = Germany; FIN = Finland.

Profile 2 from Germany versus profile 2 from Finland. The Mann–Whitney U test between the two second profiles of both countries showed a significantly greater mean value for Finnish students ($M = 2.45$, $SD = .37$) with respect to the personal meaning active practice of mathematics than German learners ($M = 1.97$, $SD = .68$), $U = 3198.50$, $p = <.001$, $r = .37$). Moreover, Finnish learners had a higher mean score in marks ($M = 2.76$, $SD = .43$) than German learners ($M = 2.49$, $SD = .58$), $U = 2961.50$, $p = <.001$, $r = .30$. Further, Profile 2 in Finland had a higher mean value in self-perfection ($M = 2.47$, $SD = .33$) than Profile 2 in Germany ($M = 1.97$, $SD = .42$), $U = 3665.00$, $p = <.001$, $r = .55$). In addition, the Finnish learners perceived more relevance in vocational precondition ($M = 2.67$, $SD = .35$) than German learners ($M = 2.29$, $SD = .60$), $U = 3095.00$, $p = <.001$, $r = .33$).

Table 20.
Mann–Whitney U Test Statistics for Mean Difference of Personal Meanings and Regulatory Styles in Profile 3 From Germany and Profile 4 From Finland.

	M		*U*	*Z*	*p*	*r*
	GER	FIN				
Active practice of mathematics	1.66	2.29	1559.50	4.74	<.001	.48
Classroom management	1.75	1.85	1026.50	.40	.686	.04
Cognitive challenge	1.79	2.35	1459.00	3.92	<.001	.15
Duty	1.19	0.77	615.00	-2.92	.003	-.30
Emotional-affective relation to the teacher	1.53	1.60	1005.50	0.23	.814	.02
Examination	1.48	1.47	939.00	-0.30	.760	.03
EXPERIENCE OF AUTONOMY	1.75	2.08	1303.00	2.66	.008	.27
EXPERIENCE OF COMPETENCE	1.92	2.09	1142.50	1.35	.175	.13
EXPERIENCE OF SOCIAL RELATEDNESS	1.67	1.59	871.00	-0.85	.392	-.08
General education	1.90	2.30	1421.50	3.62	<.001	.37
Marks	1.42	1.98	1410.50	3.53	<.001	.36
Positive image	1.09	1.33	1164.00	1.51	.129	.15
Purism of mathematics	1.56	2.00	1481.50	4.25	<.001	.44
Reference to reality	1.41	1.30	911.00	-0.53	.595	-.05
Self-perfection	1.37	2.11	1755.50	6.33	<.001	.65
Support by the teacher	1.38	1.35	849.00	-1.04	.295	-.10
Vocational precondition	1.73	2.17	1369.50	3.17	<.001	.32
Amotivation	0.69	0.64	929.50	-0.38	.700	-.03
External regulation	0.70	0.85	1083.00	0.87	.384	.08
Introjected regulation	0.90	1.02	1014.00	0.30	.76	.03
Identified regulation	1.88	2.23	1258.00	2.30	.021	.23
Intrinsic regulation	1.82	2.19	1324.00	3.14	.002	.32

Note. GER = Germany; FIN = Finland.

Profile 3 from Germany versus Profile 4 from Finland. The third comparison between Profile 3 from Germany and Profile 4 in Finland indicated higher mean values for Finnish learners for several personal meanings. In Profile 4 – FIN, Finnish students perceived relevance in the active practice of mathematics (M = 2.29, SD = .42), but German learners did not see this as relevance (M = 1.37, SD = .44), U = 1755.50, p = <.001, r = .65. For cognitive challenge, Profile 4 – FIN scored a higher mean value (M = 2.35, SD = .50) than class 3 in Germany (M = 1.79, SD = .464), U = 1459.00, p = <.001, r = .15. Additionally, the profile in Finland perceived more relevance in general education (M = 2.30, SD = .42) than German learners did (M = 1.90, SD = .48), U = 1421.50, p = <.001, r = .37). Regarding marks, Finnish learners had a higher mean score in marks (M = 1.98, SD = .83) than German students (M = 1.42, SD = .68), U = 1410.50, p = <.001, r = .36. In Profile 4 – FIN, the learner constructed purism of mathematics (M = 2.00, SD = .44), whereas German student did not (M = 1.56, SD = .51), U = 1481.50, p = <.001, r = .44. Further, Profile 4 in Finland had a higher mean value in self-perfection (*M* = 2.11, *SD* = .44) than Profile 3 in Germany (*M* = 1.38, *SD* = .44), *U* = 1755.50, *p* = <.001, *r* = .65). In Profile 4 – FIN, learners saw more relevance in vocational precondition (*M* = 2.17, *SD* = .55) than German students (*M* = 1.73, *SD* = .61), *U* = 1369.50, *p* = <.001, *r* = .32.

Table 21.
Mann–Whitney U Test Statistics for Mean Difference of Personal Meanings and Regulatory Styles in Profile 4 From Germany and Profile 3 From Finland.

	M		*U*	*Z*	*p*	*r*
	GER	FIN				
Active practice of mathematics	0.39	0.95	1870.00	4.30	<.001	.42
Classroom management	1.56	1.55	1288.50	0.26	.794	.02
Cognitive challenge	0.67	1.02	1563.50	2.96	.003	.29
Duty	2.25	1.97	952.50	-2.05	.040	-.20
Emotional-affective relation to the teacher	1.74	1.48	979.50	-1.86	.063	-.18
Examination	1.22	1.07	1153.00	-0.67	.500	-.06
EXPERIENCE OF AUTONOMY	0.91	1.15	1609.00	2.47	.013	.24
EXPERIENCE OF COMPETENCE	1.87	1.46	739.00	-3.51	<.001	-.34
EXPERIENCE OF SOCIAL RELATEDNESS	1.64	1.41	931.00	-2.20	.028	-.21
General education	1.12	1.18	1288.00	0.25	.796	.02
Marks	1.35	1.62	1443.50	1.33	.182	.13
Positive image	0.99	1.15	1461.50	1.45	.146	.14
Purism of mathematics	0.51	0.64	1409.00	1.08	.276	.10
Reference to reality	1.10	1.19	1349.0	0.67	.498	.06
Self-perfection	0.67	0.90	1657.00	2.82	.005	.27
Support by the teacher	1,93	1,44	809.00	-3.03	.002	-.30
Vocational precondition	1.14	1.54	1675.50	2.91	.004	.28
Amotivation	1.47	1.49	1305.00	0.59	.551	.05
External regulation	1.16	1.27	1338.50	0.60	.544	.06
Introjected regulation	0.51	0.79	1536.00	1.99	.046	.19
Identified regulation	1.48	1.82	1542.00	2.01	.044	.19
Intrinsic regulation	0.23	0.66	1710.50	3.49	<.001	.34

Note. GER = Germany; FIN = Finland.

Profile 4 from Germany versus Profile 3 from Finland. Finally, I compared Profile 4 – GER and Profile 3 – FIN. This test indicated a greater mean value for German students for EXPERIENCE OF COMPETENCE (*M* = 1.87, *SD* = .69) than for Finnish students (*M* = 1.46, *SD* = .51), *U* = 739, *p* = <.001, *r* = −.34.

5.4 Summary of the Empirical Results

The assessment of the psychometric properties of both affective constructs produced similar results across countries. In this present investigation, data from both German and Finnish ninth graders showed acceptable to good reliabilities. Compared to previous studies (Vollstedt & Duchhardt, 2019), it was possible to measure classroom management by using an adapted scale (OECD, 2014b, p. 331). Moreover, CFA exhibited reasonable fit statistics supporting the factor validity of regulatory styles. These results emphasized further the robustness of SDT and the validity of the established instrument for regulatory styles (Thomas & Müller, 2011). Regarding personal meaning, the predicted dimensionality of personal meaning held empirically, though only in eight separate measurement models representing the respective dimensions. The estimation of all personal meanings in one comprehensive model (all dimensions within the second-order model) did not fit the data. In both countries, correlation analyses mostly supported the predicted links between the personal meanings and regulatory styles. Moreover, the intercorrelations between constructs varied in both groups. Generally, the Finnish correlation pattern was dominated by intercorrelations between personal meanings and self-determined types of motivation, whereas in Germany relations between meanings and controlled regulations preponderated. Advanced methods of multivariate statistical analysis ensured a reliable/valid interpretation of the nature of the differences in constructs' mean structure by supporting (partial) scalar invariance. In particular, considerable mean differences emerged for highly mathematics-related personal meanings and for identified regulation, where Finnish students scored significantly higher mean values than German learners. Despite remarkable cultural differences, profiling learners' individual relevance systems yielded four distinct profiles of personal meaning for both countries as the best solution. The four-class solutions had similar response patterns over groups, but also resulted in significant differences. The implications of the empirical work are of crucial importance for further coordination analysis. In the following discussion (see Chapter 6), I therefore reflect on the empirical results in more detail.

6. Discussion of the Empirical Results

The results recorded from my empirical work addressed three major research questions (RQs): RQ 1–What are the psychometric properties of the affective constructs personal meaning and regulatory styles across Germany and Finland? RQ 2–What cross-cultural differences exist on the level of personal meanings and regulatory styles in Germany and Finland? RQ 3–How many distinct profiles of personal meaning can be detected in Germany and Finland?

6.1 Psychometric Properties of Personal Meaning and Regulatory Style Across Countries

Initially, I studied the psychometric properties and evaluated the factor structure of both constructs before I identified the profiles of personal meaning. The estimated composite scale reliabilities ρ resulted in acceptable to good values over countries. In the previous study of Vollstedt and Duchhardt (2019), it was not possible to measure "classroom management" in the German data. In this present investigation, items from a modified scale (OECD, 2014b) helped to estimate classroom management so that all 17 meanings could be considered in this study. Even though Germany and Finland are distinct cultural settings, both imported scales (developed originally in the German language) lead to good construct reliabilities. Despite good translation, translated instruments may lead to poor psychometric properties and might not be the same as in the original settings. Therefore, yielding acceptable (to good) psychometric properties is not a matter of course, as shown by, for example, Bofah and Hannula (2015). The reliable Finnish scales can mostly be attributed to a concise and solid translation[35] of the German scales into the Finnish language and the revisions proposed in the cognitive lab by Finnish native speakers to avoid problems of operationalization and measurement (Kelle & Buchholtz, 2015).

With regard to construct validity, the factor structure of regulatory styles could be replicated across countries. This result also supported the robustness of SDT as well as of the established instrument that was applied (Thomas & Müller, 2011) to study the biological nature of personal meaning in the sense of convergent validity evidence. Although the instrument of personal meaning is new, the results of this construct's psychometric properties supported its robustness in measurement (Vollstedt & Duchhardt, 2019) and conceptual rationale (Vollstedt, 2011b). Concerning personal meaning, I assessed its dimensionality in separate measurement models. In both countries, I found empirical support for the predicted dimensionality. This in turn supported the theoretical technique of specifying the dimensionality by means of the two meta-factors that refer to the theoretical link between personal meaning and regulatory styles. Thus, this yielded implicit knowledge regarding the associations between the theoretical conceptualization of personal meaning and regulatory styles (SDT). Moreover, the differentiation of the 17 personal

[35] Thank you again, Jessica Salminen-Saari and Dr Eeva Haataja, for your very precise and quick translation work. I also wish to thank the Finnish students who helped work out the translation mistakes in the cognitive lab!

meanings justified the further statistical analysis (test of measurement invariance) in a meaningful way.

Nevertheless, measuring all eight dimensions together in one comprehensive model, within the second-order model, consistently yielded a weak fit across countries. As described in the literature (Brown, 2006), the complex higher-order structure recorded poor model fit indices. A detailed inspection of the fit values reflected a modification of the factor structure in order to reduce the complexity of the model and to revise the model's substantive basis, as the factors used did not exhibit a high correlation with each other. Where the sample is small, it is advised that parameter numbers be kept low so as to ensure parameter estimates are stable and the quality of the model is sufficient (Little et al., 2002). Having a moderate sample size, here especially for the second-order model,[36] fit statistics resulted in that crucial relations between some factors could not be estimated as the model did not capture the data well. That is to say, in the second-order model it was not possible to record all the important information among variables as the model was too parsimonious (Brown, 2006; Maydeu-Olivares, 2019; Savalei, 2012). These results also bring implications to the fore that refer to the conceptual rationale of personal meaning. A model is as robust as the theoretical basis of the individual factors and their intercorrelations (Brown, 2006, p. 165-166). These results indicated that the estimated model is too parsimonious to capture very important associations between some factors and therefore a revision is necessary to obtain a "clear rationale" for the model (Brown, 2006, p. 166).

These results possibly reflected the fact that the 17 kinds of personal meaning together did not completely account for all theoretically possible facets of the overall construct personal meaning. The previous study by Vollstedt (2011b) reconstructed the 17 personal meanings from data from Germany and Hong Kong. These different kinds refer to those data sets and thus reflect their cultural settings. The low intercorrelations between the 17 kinds may suggest that there are more kinds of personal meaning, that is, other kinds than Vollstedt's personal meanings, around the world that need to be detected to complete the rationale of this motivational construct. If so, future research will have to think about a robust methodological solution to consider all factors within one model.

Whereas the composite construct of reliabilities and the factor structure of constructs showed similar behaviour, the correlation patterns between personal meaning and regulatory styles were different in both groups. The correlation analyses among both constructs resulted in interesting insights into groups. While the correlation pattern in Germany displayed the most relations of personal meanings with types of controlled motivation, personal meanings in Finland had the most relations with regulations of self-determined motivation. In both data sets, from Germany and from Finland, the predicted relations (cf. 2.3.1) among the 17 personal meanings and regulatory styles fitted the data fairly well. In both countries, the kinds of personal meanings did not only correlate with one specific type of motivation. Instead, most of them displayed one specific strongest positive correlation with a certain regulatory style and at the same time further associations with neighbouring regulatory styles. These insights into groups supported the prediction that personal

[36] Similar results were indicated for the alternative 17-factor model of personal meaning.

meanings can be perceived in various ways (Deci & Ryan, 1993) but prioritize the one that is compliant with the respective cultural setting. In Finland, I found at least a low positive relation to regulatory styles for all personal meanings, but in Germany I detected no correlation whatsoever (positive or negative) for reference to reality. In a nutshell, the data of Finnish students recorded more stable (moderate or strong) relations among the 17 personal meanings and self-determined types of motivation than did the German data. One reason could be that these results indicate more complex inter-individual differences (Geiser, 2011) in the German data than in the Finnish data. This complexity in response structure might relate to the use of a German sample from different types of schools and federal states versus the Finnish data that comes from comprehensive schools located together in one Finnish region.

In summary, correlation analyses of data from Germany and Finland (West European countries) indicated empirical support for the expected positive linear connection between the motivational construct of personal meaning and quality of motivation. Concerning the dimensionality of personal meaning, the expected relations between the dimensionality of personal meaning and regulations were supported across countries, apart from a few exceptions. These results reflect that next to similarities, fine differences in cultural settings exist. Generally, the consideration of the single personal meanings provided a more detailed insight into constructs' interrelations than associations on the level of dimensions. This aspect further supported the decision to examine learners' individual relevance systems (profiles of personal meaning) as a pattern of the 17 kinds of personal meaning. Altogether, correlation analyses support an empirical link among *socially* experienced personal meanings and *biologically* perceived quality of motivation.

For both affective constructs, I could evaluate the measurement invariance analyses of groups with eight two-group CFA models for personal meaning and one five-factorial two-group CFA for types of motivation. Following an exploratory approach, I found for all two-group models support for (partial) scalar invariance, reflecting that the nature of latent mean differences in constructs can be interpreted. In order to make a meaningful interpretation of cross-cultural dissimilarities in mean structure, partial scalar invariance is essential (Byrne et al., 1989; Steenkamp & Baumgartner 1998). The detected non-invariant items may be caused by different conceptual understandings, small translation mistakes, or cultural preferences (Chen, 2008). In order to construct a fair and valid group comparison, all scales contained at least two indicators having metric and scalar invariance (Byrne et al., 1989; Steenkamp & Baumgartner, 1998). Moreover, the addressed biased items, compared to the total number of items, represented a relatively small amount to affect the mean comparison within the affective constructs (Chen, 2008; Steenkamp & Baumgartner, 1998). In a nutshell, the results from the measurement invariance test, similar to the distinct correlation structure within countries, support the idea that differences exist within countries.

6.2 Nature of Latent Mean Differences in Personal Meaning and Regulatory Style

Cultural differences arose in the mean factor structure of personal meaning and regulatory styles. In general, the differences that indicated that the Finnish students were in a stronger position. In comparison with German learners, Finnish students had higher mean scores for personal meanings that reflected a strong relation to the individual learner perspective (active practice of mathematics and self-perfection). Interestingly, in Finland, these personal meanings correlated highly with intrinsic motivation. These specific preferences might fit with certain aspects of the Finnish mathematics curriculum. Thereby, it seems that the mathematics class is organized in such a manner that learners receive support to develop a positive attitude regarding the subject of mathematics as well as a positive self-conception when dealing with it (Finnish National Board of Education, 2016, p. 402). This consistent focus on these specific attitudes to the subject during mathematics lessons might support Finnish students' construction of mathematics-related personal relevance. Unlike in the Finnish curriculum, there were hardly any detailed statements in the German mathematics core curriculum (KMK, 2005) on how to support individual learners' personal development (particularly through transversal competencies) in mathematics. Instead, the German educational standards (including content-related mathematical competencies and central mathematical ideas) strongly highlighted various mathematical competences.

Moreover, and significantly, Finnish learners placed more importance on marks, positive image, and vocational precondition than German students. Since in Finland students have to take a decision in the ninth grade about their upper secondary education (OECD, 2020a), choosing between academic or vocational tracks, the importance of mathematics and beneficial marks in mathematics as a major subject seem to matter. In Germany, "good" grades are usually the crucial factor in school reports in the tenth grade when most students transfer to further education tracks (OECD, 2020b). These differences in educational systems might explain the distinct importance in the relevance of mathematics for German and Finnish ninth graders. The latent cross-cultural mean structure further indicated that learners from both countries assign the same priority for classroom management, EXPERIENCE OF AUTONOMY, EXPERIENCE OF COMPETENCE, EXPERIENCE OF SOCIAL RELATEDNESS, and purism of mathematics. Surprisingly, all three specific personal meanings,[37] which seem to have a connection with basic psychological needs, have equivalent importance (no differences emerged on a latent mean level) independent of cultural setting.

Regarding personal meaning, the resulting differences showed similarities to the findings of a previous study based on *item response theory* (IRT), partial credit models, and differential item functioning (DIF) analysis (Suriakumaran et al., 2019). However, the resulting differences in the former IRT-based study and the present investigation showed few differences in the mean structure. A statistically significant (according to the Benjamini–Hochberg procedure on a 5% false discovery rate) difference in cognitive challenge could not be found in the present study, while positive image resulted in a difference ($p < .001$) that did not become apparent in the IRT-based study. Although both methodological approaches, *classical test theory* (CTT) and IRT, result

[37] The three specific personal meanings are EXPERIENCE OF AUTONOMY, COMPETENCE, and SOCIAL RELATEDNESS.

in comparable outputs (Brown, 2006; Meade & Lautenschlager, 2004), CTT and IRT models have differences within their estimation framework, to name but two: item parameterization (Macintosh & Hashim, 2003; Muthén et al., 1991) and standard error of measurement (Brown, 2006; De Ayala, 2009). Regarding model estimations, CTT aims to clarify the interrelations between items, whereas IRT seeks to model the relationship between latent trait and item responses (Brown, 2006; De Ayala, 2009; Hambleton et al., 1991). Although the exact same instrument of personal meaning was applied to the same data set, the effects of the measurement theories are visible. Since a detailed comparison of these measurement theories would go beyond the scope of this thesis, I have limited the explanations given thus far.

Concerning the regulatory styles, Finnish students scored a higher mean value for identified regulation than German learners. On the one hand, this fact might also go along with the circumstances (as mentioned above) that Finnish learners face in grade nine; namely the upcoming graduation and the transition to an academic or vocational track. On the other hand, the personal meanings (active practice of mathematics, self-perfection, marks, and vocational precondition) favoured by Finnish students reflected, in a previous analysis (cf. correlation analyses), strong positive correlations with self-determined regulatory styles (intrinsic and identified regulation). In general, these results and the fact that the Finnish learners scored a higher SDI than German students show that Finnish students perceive the motivation in mathematics to be less external. Interestingly, these empirical results underline previous PISA results in terms of mathematics anxiety. The PISA results highlighted that Finnish students had among the lowest anxiety levels, and in terms of the association between mathematics anxiety and performance, Finnish students had less anxiety than German students (OECD, 2015, p. 2). Moreover, the change between 2003 and 2012 in the mean index of intrinsic motivation to study mathematics increased in Finland over those years, whereas in Germany it decreased. The further coordination analyses of this study will show whether these findings remain consistent within countries' sub-groups (i.e., profiles found in Germany and Finland; see Chapter 7).

6.3 Identification of Profiles of Personal Meaning and Their Motivational Outcomes

The previous statistical analyses exhibited cross-cultural differences in correlation patterns, in latent mean structure, and in the test of measurement invariance. With respect to both groups' cultural diversity, I conducted two separate latent class analyses for each group to identify their best class solution. Accordingly, the scales of personal meaning provide a solid foundation to empirically map German and Finnish individuals to profiles of personal meaning by means of latent class analysis. Surprisingly, the comparison, followed by an exploratory procedure, of a one- to five-class solution resulted in a four-class model for both groups as the best.

Within the empirical individual relevance systems, certain factors from learners' biological natures are reflected. We assume this must be the case, because otherwise the empirical profiles would develop in the same way and have the same results and there would only be one profile. Since there are very different profiles, from a variety of classes, we can assume that students in the classes have completely different profiles as well, and this fits the picture of a biological regulatory mechanism. The four profiles detected in Germany and Finland recorded similar response patterns, and the comparison (Mann–Whitney U tests) revealed fine but significant differences in profiles of personal meaning and their motivational outcome. Most differences in nuance refer to the latent mean structure of the respective country. Potentially, the upcoming coordination analysis method could clarify to what extent these subtle differences have an effect with regard to the interaction of learners' social and biological regulatory mechanisms (see Chapter 7). I will now examine to what extent these profiles are found in the classroom and what this means for the classroom in Germany and Finland. The upcoming analyses are a step towards supporting this present dissertation's theoretical work in that I seek to strengthen the empirical profiles (i.e., by clarifying the social regulatory mechanism).

7. Results According to the Coordination Analysis Method

In this chapter, I present the theory-driven analyses according to coordinating (conceptual framework built by coordinating theories) biological (SDT) and social regulatory mechanisms (social constructivism) through the empirical phenomenon personal meaning. As mentioned earlier, this study's empirical work only captured one ontological essence of personal meaning, its individual nature. To capture its second ontological nature, personal meaning as a social phenomenon, as well as to evaluate the validity of the cyclical model concerning the social regulatory mechanism of the individual relevance system posited earlier (cf. Figure 6), it was necessary to draw hypothetical conclusions in order to examine the theoretical assumptions. Therefore, I used the empirically developed data. The data consist of the empirical profiles of personal meaning (individual relevance systems) from Germany and Finland, their corresponding regulatory styles, and the self-determination index (SDI) predicting learners' relative autonomy.

In sections 7.1 and 7.2, I report the results of the coordination analyses based on the empirical data from Germany and Finland. Subsequently, in section 7.4, I interpret the coordination of the results using the hypothesized conceptual framework (cf. Figure 7); this was constructed by linking SDT and social constructivism by means of the boundary object personal meaning, in order to propose a theoretical model explaining the genesis of motivation in mathematics. As an initial step to this end, I had three (further) research questions (RQs) that I examined successively when theorizing the coordination of the results.

Several issues were considered in order to address the interaction between individual relevance systems and motivation (RQ 4). The adapted model of the social regulatory mechanism considers the role of the three specific personal meanings (EXPERIENCE OF AUTONOMY, COMPETENCE, and SOCIAL RELATEDNESS) to describe how these individual relevance systems (patterns of personal meaning) arise and unfold in mathematics classrooms. Regarding the predicted connection between social constructivism and personal meaning, I assumed that the application of this adapted cycle would provide a meaningful tool with which to interpret the empirical profiles in light of the social regulations experienced by learners in mathematics lessons. Theoretically, the learner constructs a complex pattern of personal meanings regarding mathematics as a school subject (Vollstedt, 2011b; Vorhölter, 2009) that is personally relevant for them. Accordingly, I predicted that learners rarely construct all the factors of a dimension of personal meaning (cf. Figure 5) at the same time. Moreover, I expected to find indicators for the predicted inner feedback mechanism (cf. Figure 4), that is, theoretical connections between the three specific meanings and the basic psychological needs. Accordingly, I hypothesized that the effects of the social regulatory mechanism (learners' individual relevance systems) would be reflected in the respective biological regulatory mechanism (motivational outcome) of both countries in terms of theoretical coherence.

In the case of RQ 5, at first glance, the profiles from Germany and Finland have similar relevance systems. Yet, empirical results (Mann–Whitney U tests) indicated significant differences between them. Since learners' social regulations are also determined by culture, I expected to find cultural differences that became visible through theoretical conclusions (Kleemann et al., 2013), that is, the work of interpreting the numerical profiles through the predicted conceptual framework.

Regarding this study's basic assumption (RQ 6), I expected to find evidence for the predicted theoretical connections between the boundary object of personal meaning and the two theoretical perspectives in order to explain the genesis of motivation in mathematics classroom.

For the coordination analysis procedure, I followed the steps described in section 4.2.2. Each profile's results subsection thus effectively consists of three steps – step 1: capture the learner's reaction to the outside social world in mathematics class in terms of the social regulatory mechanism; step 2: construct a hypothesis on the complementary motivational behaviour (biological regulation), and step 3: analyse the learner's inner world with reference to biological regulatory mechanism.

The *publication* stage (the starting point of the coordination analysis in step 1) of the social regulatory mechanism represents the learner's individual relevance system. To understand the learner's profile output, I initially summarized the constructed meanings with respect to the strong indicators represented by the constructed personal meanings ($M \geq 1.8$ cut-off point) and non-constructed personal meanings ($M \leq 1.2$ cut-off point). In some profiles, I included personal meanings with thresholds that are close to the cut-off values and with a tendency towards (non-) construction (e.g., $M = 1.71$ tendency towards construction and $M = 1.29$ tendency towards non-construction). Where possible, I described the profile summary by means of the eight dimensions of personal meaning. This was for the purpose of examining whether the predicted dimensions (dis)appear in the same way (conjoint or divergent for instance as single factors) and to see whether certain dimensions are characteristic of German and/or Finnish profiles' learning orientations. Based on this publication stage (profile summary), I reconstructed the learner's individually owned orientation pattern with respect to the processes of *transformation* and *appropriation* of the social regulatory mechanism. Through the reconstruction of these processes of social regulations, I aimed to get closer to student's social experiences and in particular their perception of the three specific personal meanings (EXPERIENCE OF AUTONOMY, COMPETENCE, AND SOCIAL RELATEDNESS). Again, this theoretical reconstruction of social experiences only focuses on learners' individual responses (ticked boxes on the Likert scale), and thus these insights only apply to the individual's perception of classroom interaction underlying their produced individual relevance system (and not an impartial perspective on the mathematics-classroom situation in terms of objective hermeneutics; Kleemann et al., 2013).

Furthermore, based on their profile's orientation pattern, I subsequently hypothesized the direction of the learner's motivation (step 2). To provide the biological nature of personal meaning, in step 3 I described the learner's motivation in terms of the (non-)existing regulatory styles (same cut-off points as for personal meanings) and the SDI (motivational tendency in the mathematics classroom).

In the following sections, I report in more detail on the theory-driven analyses found in the German (7.1) and Finnish data (7.2) according to the coordination analysis method.

7.1 Mathematics-Learning Orientations in Germany

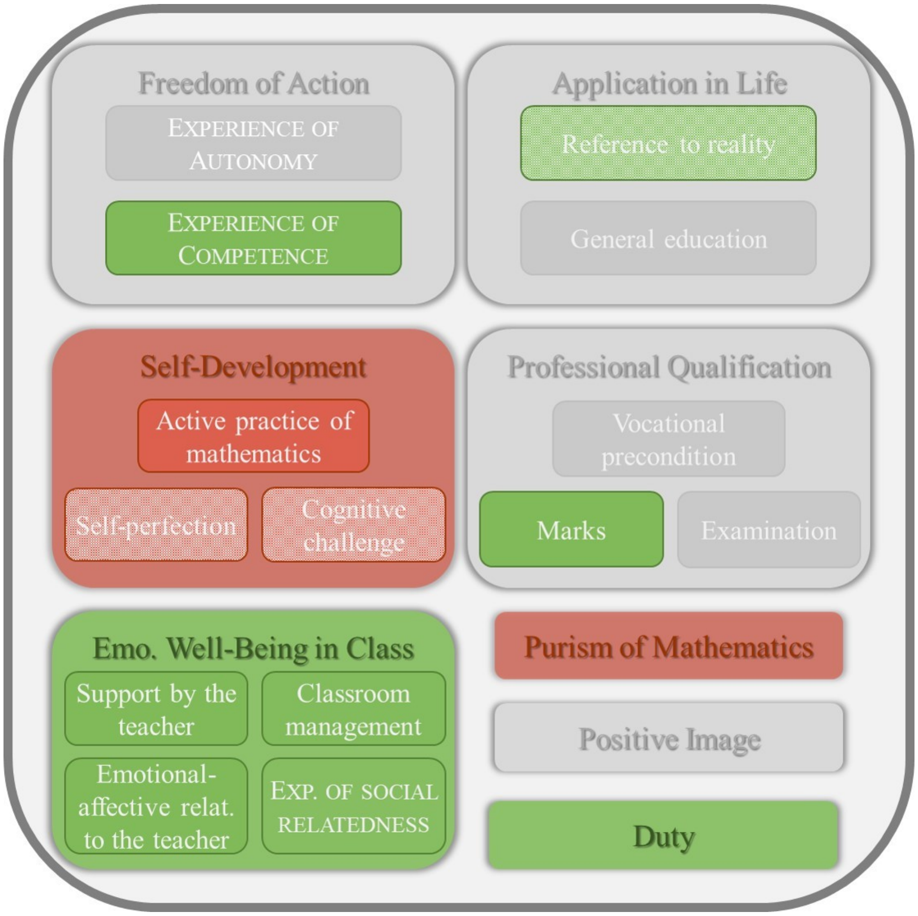
In this section, the four profiles found in Germany are described using the coordination analysis method. At the end of this section, the learning orientations and motivation of the four identified profiles are compared as regards their similarities and differences.

7.1.1 Profile 1: Emotional-Social Integration with a Focus on Seeking Acceptance from the Teacher

The following figure, Figure 18, provides an overview of Profile 1, found in Germany, with respect to the strong indicators represented by the constructed ($M \geq 1.8$ cut-off point) and non-constructed ($M \leq 1.2$ cut-off point) dimensions/kinds of personal meaning. This profile consisted of 34% of the sample ($N = 264$), of which 52% of the learners were female.

Figure 18

Profile 1 in Germany: Emotional-Social Integration With a Focus on Seeking Acceptance From the Teacher.



Note. Summarized overview of profile's pattern of personal meaning and the corresponding motivational behaviour. Profile 1 in Germany is depicted through the constructed and non-constructed kinds of personal meaning embedded within the corresponding dimension of personal meaning. The colour code (light) green is used for (a tendency towards) constructed personal meanings and (light) red is used for (a tendency towards) non-constructed personal meanings. Each dimension is coloured either green or red when all factors of the dimension behaved in a similar manner. Average values are greyed out. The colour code of the profile's frame reflects the profile's SDI, with orange for self-determined motivation, purple for controlled motivation, and grey when the SDI is balanced, i.e., neither positive nor negative.

Step 1. The *publication* stage (as the starting point of the coordination analysis) shows the learner's individual relevance system as having a mathematics-learning orientation that is focused on emotional-social integration in class. Referring to the learner's responses, I identified dimensions of personal meaning and single personal meanings that appeared as strong indicators with regard to construction: EXPERIENCE OF COMPETENCE and marks, together with two dimensions of personal meaning: emotional well-being in class and duty. I additionally considered reference to reality as part of the learner's relevance system, as this pointed to a tendency for construction

(M = 1.74, SD = .60). The dimension purism of mathematics appeared as a strong indicator for non-construction. The dimension of self-development arose partly as a strong indicator with regard to non-construction (self-perfection (M = 1.23, SD = .37), and cognitive challenge (M = 1.21, SD = .39) showed only a tendency, but the active practice of mathematics emerged as a strong indicator for non-construction). According to the results gathered about the publication stage, two out of the five multifaceted dimensions of personal meaning behaved in a similar manner. However, to understand the orientation pattern of this learner, I further examined the items behind the (non-)constructed personal meanings.

Based on strong indicators that manifested at the publication stage, it would appear that the learner studies mathematics for social reasons. Mathematics represents a compulsory subject (duty) in which they work for the teacher (dimension: emotional well-being in class). The teacher offers this emotional-social environment and allows the learner to experience competence (EXPERIENCE OF COMPETENCE) and make an effort in order to be rewarded with good grades (marks). These situations strengthen the learner's self-confidence. Further, the learner appreciates working on mathematics content that relates to everyday life (reference to reality). As a non-constructed personal meaning, the dimension purism of mathematics and self-development showed that the relevance of mathematics as a subject matter (German: *Sachgebiet*) and the prospect of personal development through mathematics did not play a role for the particular learner. Here, a profile occurred where mathematics as a socially mediated scope becomes important. Based on this relevance system and their social experiences in class, the individual constructed two out of the three specific meanings, namely EXPERIENCE OF COMPETENCE and EXPERIENCE OF SOCIAL RELATEDNESS. Thus, they did not construct EXPERIENCE OF AUTONOMY in social interaction. To conclude, the reconstruction of *transformation and appropriation* of social regulation showed that the learner *identifies with* social processes that focus on a teacher who lets them feel competent, whereas they do *not identify with* individual processes in mathematics. In this profile, the learner's biographical orientation thus moves between these two opposing poles, which mutually represent opposing scopes for one another. However, in this orientation pattern they did not perceive EXPERIENCE OF AUTONOMY when learning mathematics. Based on the learner's publication, I reconstructed the levels of transformation and appropriation. Then, I considered this orientation pattern as an effect of social interaction in class, to investigate the biological regulation that could correspond to the social experiences of such a person.

Step 2. On the basis of this specific perception of social interaction in class, the learner's sense of success in terms of action in the classroom seemed to depend on their teacher. In particular, they did not perceive EXPERIENCE OF AUTONOMY in mathematics; that is, the learner did not experience themselves as the centre of action. Therefore, I formulated the following hypothesis in view of their motivational behaviour in mathematics class: If the learner publishes a relevance system whereby they socially experience their mathematics class through social processes, then their motivation tends to be controlled.

Step 3. The motivational regulation (as *enacting potential*) of this learner showed a high level of identified regulation. The other regulatory styles were low (intrinsic regulation, introjected regulation, external regulation, and amotivation). Surprisingly, the SDI (0.09) tends towards self-determined motivation, so that there is a slight tendency towards self-determined motivation. However, this result is essentially balanced, that is, neither controlled nor self-determined. Here, my hypothesis (step 2) did not fit the data.

Summary of Profile 1. This relevance system represents the highest percentage of students (34%) out of the four profiles identified in Germany. Here, the difference in gender is balanced[38] (♀: 52%, ♂: 48%). This profile contains mostly average students (60 students had either grade 3 or 4), many high achievers (27 students had either grade 1 or 2), and fewer low achievers (which grade it was, 5 or 6). As a result of social interaction in class, the learner constructs a relevance system of personal meaning that basically focuses on emotional-social integration. The teacher is specifically important for their learning processes and enables them to EXPERIENCE COMPETENCE through their social involvement, even though they perceive this as a compulsory subject. In this context, they experience social integration. Here, mathematics is perceived as a socially mediated scope in which they study the subject less for individual than for social reasons. To conclude, this mathematics-learning orientation focusing on emotional-social integration scored a high level of identified regulation and showed motivational behaviour that was balanced, SDI = 0.09 (biological regulation), with a slight tendency towards self-determined motivation. This SDI is neither positive nor negative (for comparison only, the other German profiles had SDI values between -0.89 and 3.22). Since the learner's quality of motivation is well balanced, even though the specific meaning EXPERIENCE OF AUTONOMY was not constructed, a clarification of this biological experience is important (see section 7.4). Based on the profile orientation and the characteristic dimension of personal meaning (emotional well-being in class) that also emerged in a similar Finnish profile (but with a slightly different specification; see Profile 1–FIN in section 7.2.1), I termed this profile "Emotional-social integration with a focus on seeking acceptance from the teacher."

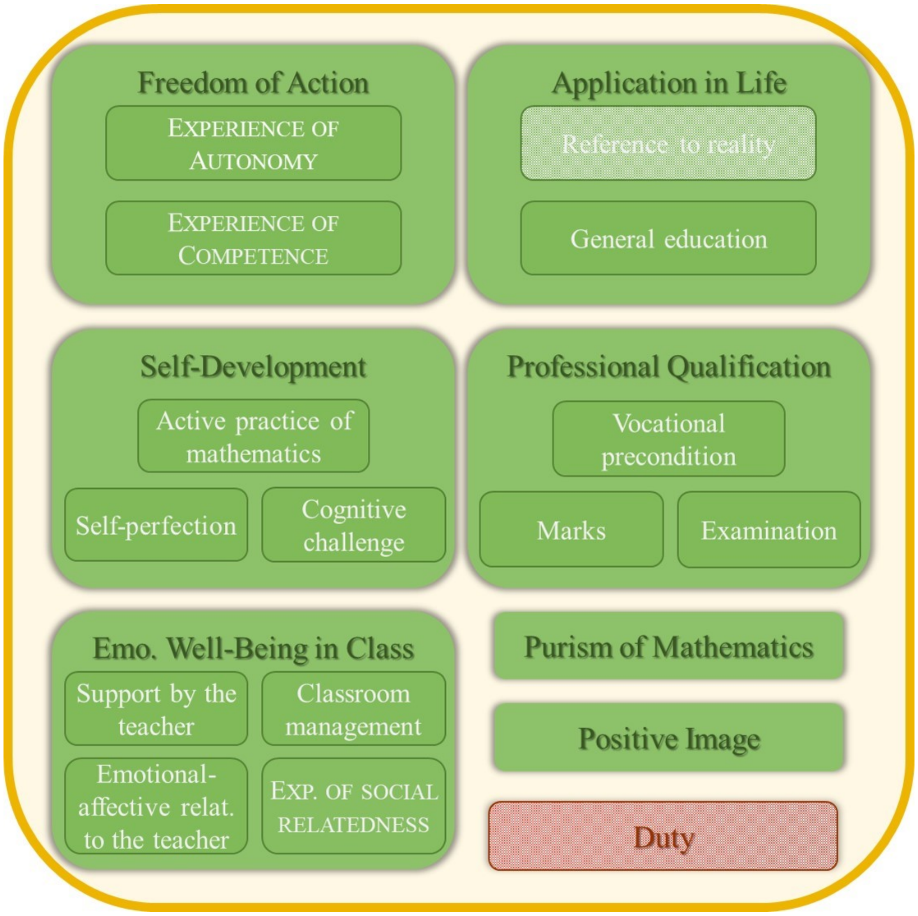
7.1.2 Profile 2: Enjoyment in Mathematics Learning

The following figure, Figure 19, provides an overview of Profile 2 found in Germany with respect to the strong indicators represented by the constructed ($M \geq 1.8$ cut-off point) and non-constructed ($M \leq 1.2$ cut-off point) dimensions/kinds of personal meaning. This profile consisted of 27% of the sample ($N = 264$), of which 41% of the learners were female.

[38] Gender difference was not statistically tested.

Figure 19

Profile 2 in Germany: Enjoyment in Mathematics Learning.



Note. Summarized overview of profile's pattern of personal meaning and the corresponding motivational behaviour. Profile 2 in Germany is depicted through the constructed and non-constructed kinds of personal meaning embedded within the corresponding dimension of personal meaning. The colour code (light) green is used for (a tendency towards) constructed personal meanings and (light) red is used for (a tendency towards) non-constructed personal meanings. Each dimension is coloured either green or red when all factors of the dimension behaved in a similar manner. Average values are greyed out. The colour code of the profile's frame reflects the profile's SDI, with orange for self-determined motivation, purple for controlled motivation, and grey when the SDI is balanced, i.e., neither positive nor negative.

Step 1. The learner produced an individual relevance system with a mathematics-learning orientation that focused on enjoyment. Regarding the profile summary (*publication*), I identified seven out of the eight dimensions of personal meaning that appeared conjoint as strong indicators with regard to construction: purism of mathematics, freedom of action, self-development, professional qualification, emotional well-being in class, and positive image. The dimension of application in life also appeared conjoint, but reference to reality showed a tendency for construction ($M = 1.75$, $SD = .69$). Only duty was not constructed and showed a tendency for non-construction (M = 1.24, SD = .94). On the basis of this profile summary, I sought to understand the learner's orientation framework. To capture their perceptions through social interaction, I reconstructed the transformation and appropriation processes of social regulations in mathematics class.

Based on strong evidence in the publication stage, the learner seemed to study mathematics for individual as well as for social reasons. The learner experienced mathematics as a socially mediated scope (dimensions: professional qualification, application in life, emotional well-being in class) in which individual development is possible (dimensions: self-development). The learner coped with learning processes independently and could use their skills without outside support (dimension: freedom of action). They worked with the teacher, and thereby mathematics was perceived as a school subject and at the same time as a scientific field (purism of mathematics). It was also important to them to get appreciation for their achievements in mathematics (positive image). Duty was a non-constructed personal meaning, which showed that the learner did not perceive mathematics-learning situations as an obligation. Based on these specific social experiences in mathematics class, the learner perceived all three specific meanings, namely EXPERIENCE OF AUTONOMY, COMPETENCE, and SOCIAL RELATEDNESS. With respect to the processes of *transformation* and *appropriation*, relevance as well as fun were *identified with* mathematics class, whereas duty was *not*. From these social experiences, the learner's biographical orientation pattern thus moves between these two opposing poles.

Step 2. Following the reconstructed social experiences in Step 1, the learner seemed to perceive themselves as self-directed and as the centre of action in mathematics lessons but did not connect classroom interaction with external pressure. Since they experienced all three specific meanings, I hypothesized this corresponding motivational behaviour: If the learner publishes a relevance system whereby they socially experienced their mathematics class as fun and as relevant, then their motivation tends to be self-determined.

Step 3. When considering the biological regulatory mechanism as enacting potential, the learner's motivation resulted in a high level of identified, intrinsic, and introjected regulation. The other types of regulation scored low mean values (external regulation and amotivation). As assumed, the SDI was positive, SDI = 2.45; that indicated self-determined motivation. Here the hypothesis (step 2) fits the data.

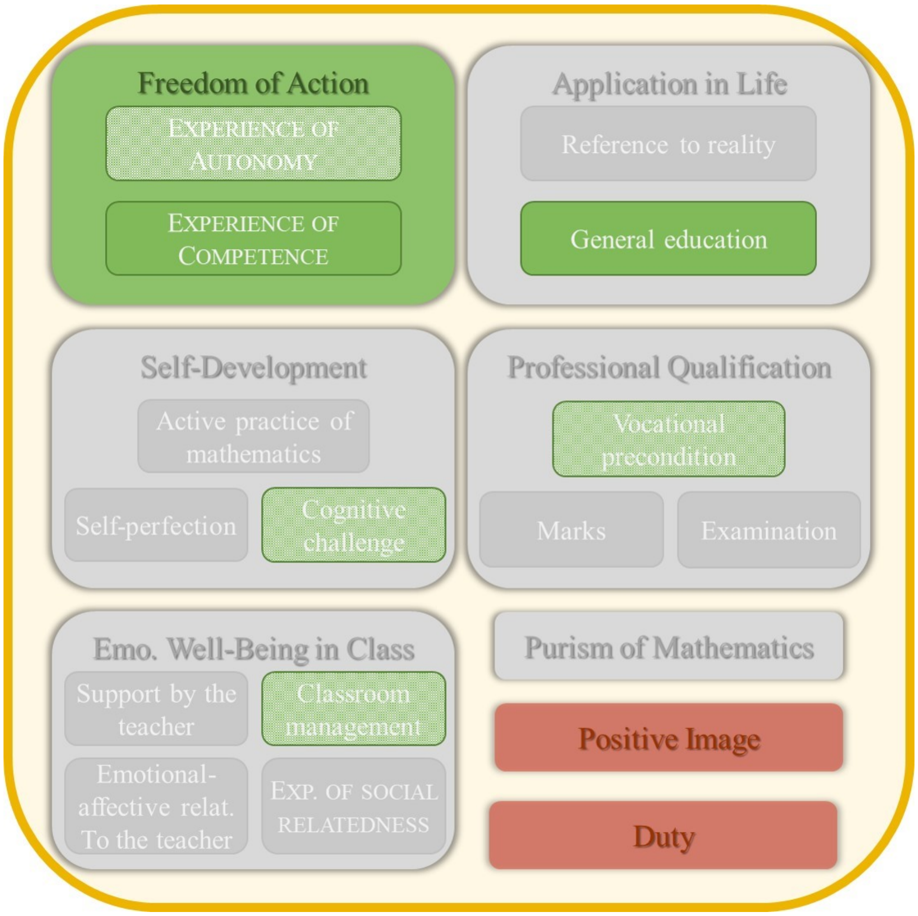
Summary of Profile 2. This relevance system is the second highest percentage of students out of the four profiles identified in Germany (27%). Here, the number of females is somewhat lower than the amount of males (♀: 41%, ♂: 59%). This profile attracted mostly high achievers (41 students had either grade 1 or 2), many average students (29 students had either grade 3 or 4), and fewer low achievers (which grade it was, 5 or 6). Based on the social interactions in mathematics class, the learner constructed a relevance system of personal meaning that focused on individual as well as on social processes. The support of the teacher was not perceived as an external pressure, but as a multifaceted support. The students constructed EXPERIENCE OF AUTONOMY, COMPETENCE, and SOCIAL RELATEDNESS through their social regulations. Here, mathematics is experienced as a socially mediated scope that offers a learning environment in which to deal with individual processes. This mathematics-learning orientation associated with a self-determined motivational behaviour, SDI = 2.45 (the other profiles in Germany had SDI values between -0.89 and 3.22) that scored a high level of identified, intrinsic, and introjected regulation. On the basis of the learner's orientation pattern and the characteristic dimensions of personal meaning (application in life, emotional well-being in class, freedom of action, positive image, purism of mathematics, professional qualification, self-development) that emerged in the same manner in the Finnish profile (see Profile 2–FIN in section 7.2.2), I termed this profile "Enjoyment in mathematics learning".

7.1.3 Profile 3: Self-Improvement with a Focus on Mastery

The following figure, Figure 20, provides an overview of Profile 3 found in Germany with respect to the strong indicators represented by the constructed ($M \geq 1.8$ cut-off point) and non-constructed ($M \leq 1.2$ cut-off point) dimensions/kinds of personal meaning. This profile consisted of 24% of the sample ($N = 264$), of which 40% of the learners were female.

Figure 20

Profile 3 in Germany: Self-Improvement With a Focus on Mastery.



Note. Summarized overview of profile's pattern of personal meaning and the corresponding motivational behaviour. Profile 3 in Germany is depicted through the constructed and non-constructed kinds of personal meaning embedded within the corresponding dimension of personal meaning. The colour code (light) green is used for (a tendency towards) constructed personal meanings and (light) red is used for (a tendency towards) non-constructed personal meanings. Each dimension is coloured either green or red when all factors of the dimension behaved in a similar manner. Average values are greyed out. The colour code of the profile's frame reflects the profile's SDI, with orange for self-determined motivation, purple for controlled motivation, and grey when the SDI is balanced, i.e., neither positive nor negative.

Step 1. The *publication* stage showed the learner's individual relevance system as having a mathematics-learning orientation focused on self-improvement. Regarding the learner's responses, I identified only one dimension of personal meaning (freedom of action) that appeared conjointly, but EXPERIENCE OF AUTONOMY showed a tendency for construction (M = 1.75, SD = .54). In addition, general education as a single personal meaning resulted as a strong indicator with regard to construction. Further, cognitive challenge (M = 1.79, SD = .64), classroom management (M = 1.75, SD = .64), and vocational preconditions (M = 1.73, SD = .61) showed a tendency for construction. The dimensions of duty and positive image appeared as strong indicators with respect to non-construction. Hence, one out of the five multifaceted dimensions of personal meaning arose conjointly, even though one factor of this dimension was not a strong indicator. Next, to understand the orientation framework of this learner, I evaluated the items behind the (non-)constructed personal meanings.

Based on the personal meanings constructed at the publication stage, the student learns mathematics for individual reasons. The learner experiences mathematics as a scope in which to develop their own cognitive skills (general education) and seek cognitive challenges (cognitive challenge). This is necessary to fulfil their own competence requirements (EXPERIENCE OF COMPETENCE) and to have better chances in their professional life (vocational preconditions). Further, they cope with learning processes independently and can use their skills with no support from outside (dimension: freedom of action). The teacher ensures a good working environment is provided (classroom management). In terms of duty and positive image as non-constructed personal meanings, it was shown that the learner did not perceive the subject of mathematics as an obligation, nor did they study mathematics to please others. It was not important that the learner did what others asked them to do or received social appreciation for what they did in order to receive feedback from significant others. This was not relevant to them. Based on this specific relevance orientation and their social experiences in class, the learner experienced two specific meanings, namely EXPERIENCE OF AUTONOMY and COMPETENCE. Regarding the regulations of *transformation* and *appropriation*, the learner in this profile *identified with* mathematics class to fulfil their own competency requirements and to seek cognitive challenges—that is, they identified with the statement “Mathematics fulfils my competence requirements”, whereas they did *not identify with* “I try my best in learning mathematics to please others”. From these social experiences, the learner’s biographical orientation pattern thus moves between these two contrary scopes, which mutually represent opposing poles for one another. In the next step, I made the following assumption, which may correspond to this orientation pattern in terms of the learner’s motivational regulation.

Step 2. Following the reconstructed social experiences in Step 1, the learner seemed to perceive mathematics lessons as a means of self-improvement in order to fulfil their competence requirements and to feel cognitively challenged. Here, they experienced two specific meanings (dimension: freedom of action). Accordingly, the individual did not act for external reasons (like social appraisal) but valued experiencing themselves as a centre of action. Therefore, I hypothesized this corresponding motivational behaviour: If the learner publishes a relevance system where they socially experience fulfilment of their competency requirements, then their motivation tends to be self-determined.

Step 3. When considering the biological regulatory mechanism as *enacting potential*, the learner’s motivation resulted in a high level of identified and intrinsic regulation. The other regulatory styles scored low mean values (introjected regulation, external regulation, and amotivation). As assumed, the SDI was positive, SDI = 3.22, which indicated self-determined motivation. Here the hypothesis (step 2) fits the data.

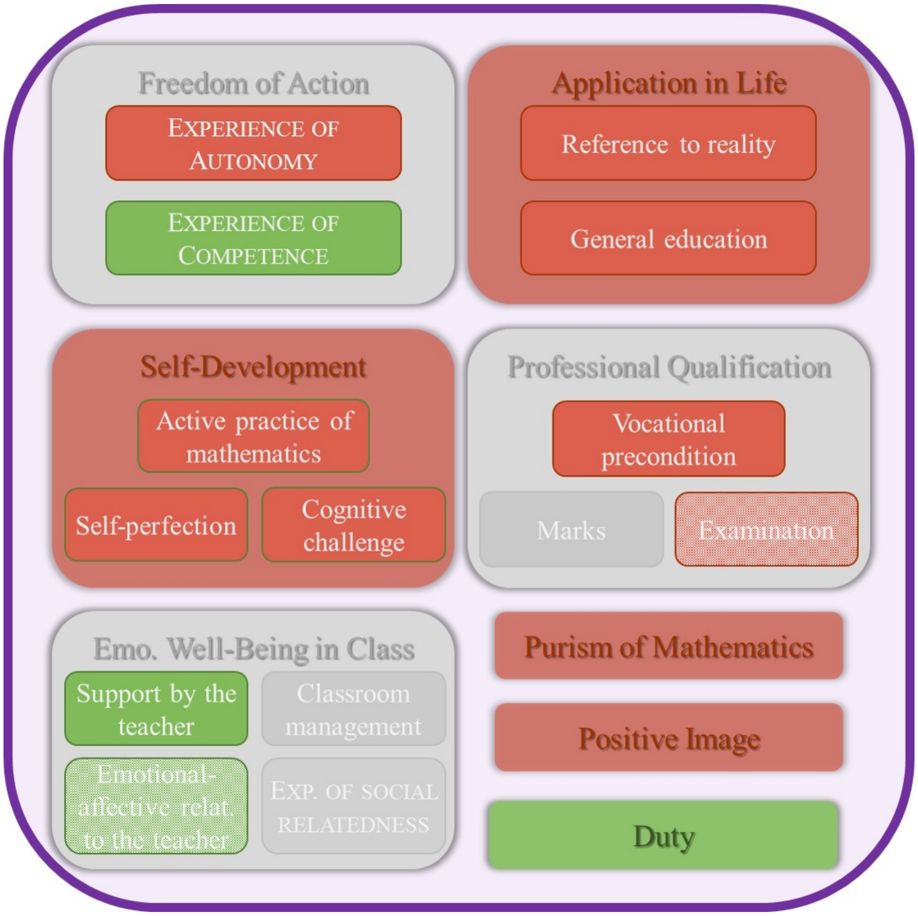
Summary of Profile 3. This relevance system is the third highest percentage of students out of the four identified relevance systems in Germany (24%). Here, the percentage of females is lower than that of males (♀: 40%, ♂: 59%). This profile attracted mostly high achievers (35 students had either grade 1 or 2) and many average students (27 students had either grade 3 or 4). Based on the social interactions in mathematics class, the learner constructed a relevance system that focused on individual processes. There was no dependency on the teacher, nor was a feeling of communality in class relevant. The teacher did not play such a big role in this profile. The teacher's support was important with respect to classroom management. The teacher gave the learners freedom to act and learn on their own. In this context, the learner constructed EXPERIENCES OF COMPETENCE and AUTONOMY and fulfilled their own competence requirements. Mathematics also had relevance for their future profession and allowed them to experience cognitive challenges. Mathematics was not perceived as irrelevant subject, nor was it studied to please others. This orientation (as a result of the social regulations) resulted in the highest self-determined motivational behaviour (SDI = 3.22; the other profiles in Germany had SDI values between -0.89 and 2.45) within Germany. Based on this specific relevance orientation in mathematics and the characteristic dimension of personal meaning (freedom of action) that also emerged in the similar Finnish profile (but with a different specification, see Profile 4–FIN in section 7.2.4), I termed this profile "Self-improvement with a focus on mastery".

7.1.4 Profile 4: External Pressure Focused on Adaption to the Teacher's Performance Requirements

The following figure, Figure 21, provides an overview of Profile 4, found in Germany, with respect to the strong indicators represented by the constructed ($M \geq 1.8$ cut-off point) and non-constructed ($M \leq 1.2$ cut-off point) dimensions/kinds of personal meaning. This profile consisted of 15% of the sample ($N = 264$), of which 54% of the learners were female.

Figure 21

Profile 4 in Germany: External Pressure Focused on Adaption to the Teacher's Performance Requirements.



Note. Summarized overview of profile's pattern of personal meaning and the corresponding motivational behaviour. Profile 4 in Germany is depicted through the constructed and non-constructed kinds of personal meaning embedded within the corresponding dimension of personal meaning. The colour code (light) green is used for (a tendency towards) constructed personal meanings and (light) red is used for (a tendency towards) non-constructed personal meanings. Each dimension is coloured either green or red when all factors of the dimension behaved in a similar manner. Average values are greyed out. The colour code of the profile's frame reflects the profile's SDI, with orange for self-determined motivation, purple for controlled motivation, and grey when the SDI is balanced, i.e., neither positive nor negative.

Step 1. The *publication* stage of the social regulatory mechanism showed the learner's individual relevance system to have a mathematics-learning orientation that focused on external pressure. In this relevance system, the dimension of duty and the single meaning EXPERIENCE OF COMPETENCE were the strong indicators with regard to construction. The dimension of emotional well-being in class partly occurred, whereas the personal meaning of support by the teacher was constructed as a strong indicator, and the emotional-affective relation to the teacher was a threshold for construction (M =1.74, SD =.71). The dimensions of purism of mathematics, self-development, application in life, positive image, and the single meaning EXPERIENCE OF AUTONOMY arose as strong indicators with regard to non-construction. The dimension of professional qualification partly emerged, whereas examination (*M* = 1.22, *SD* = .82) showed a tendency towards non-construction. At the publication stage, two out of the five multifaceted dimensions of personal meaning showed similar characteristics. Based on this profile summary, I approached the learner's orientation framework to reconstruct their social experience in mathematics class, that is, their transformation and appropriation level of social regulation.

Based on the constructed personal meanings that the individual published, the learner experienced mathematics through social processes. They experienced mathematics as an exclusively compulsory subject (dimension: duty) and the individual adapted to competence requirements (EXPERIENCE OF COMPETENCE) from the teacher (support by the teacher and emotional-affective relation to the teacher). In this learning environment, the individual was not involved in any individual processes supporting their individual development (non-construction of the dimension: self-development). They did not experience themselves as a centre of action (non-construction of EXPERIENCE OF AUTONOMY) or appreciate making a positive impression on others because of their achievement in mathematics (non-construction of positive image) while learning. Mathematics was neither perceived as an important school subject (non-construction of the dimension application in life, and the personal meanings examination and vocational precondition), nor as a specialist field (non-construction of purism of mathematics). Based on this specific perception of social interactions in class, the learner only experienced one specific meaning out of three; namely, EXPERIENCE OF COMPETENCE. In terms of the regulations of *transformation and appropriation*, the learner *identified with* mathematics as an adaption to the teacher's performance requirements, whereas they did *not identify with* mathematics as being interesting as an academic subject and as a specialist field. On the basis of these social experiences, the learner's biographical orientation framework thus exists between these two opposing poles. Next, I formulated the following assumption that may correspond with this orientation pattern in terms of the learner's motivational regulation in mathematics class.

Step 2. Based on the socially experienced relevance system in Step 1, the learner seemed to be dependent on the teacher and experienced mathematics lessons as an adaption to educational performance requirements. Here, they perceived only EXPERIENCE OF COMPETENCE as a specific meaning and did not construct EXPERIENCE OF AUTONOMY. Hence, I made the following assumption with respect to the learner's motivational behaviour: If the learner publishes a relevance system whereby they socially experience their mathematics class as an adaption to competency requirements from the teacher, then their motivation is tends to be controlled.

Step 3. When considering the biological regulatory mechanism as *enacting potential*, the learner's motivation did not result in any high mean values for regulatory styles. Identified regulation and amotivation scored moderate mean values and the others showed low mean values (intrinsic regulation, introjected regulation, and external regulation). However, the SDI was negative, SDI= -0.89, indicating a less controlled motivation. Here the hypothesis (step 2) fits the data.

Summary of Profile 4. This relevance system accounts for the smallest percentage of students out of the four identified profiles in Germany (15%). Here, the percentage of females is higher than that of males (♀: 54%, ♂: 44%). This profile contained mostly average students (28 students had either grade 3 or 4), several high achievers (eight students had either grade 1 or 2), and fewer low achievers (four students had grade 5 or 6). Based on the social interactions in the mathematics class, the learner constructed a relevance system of personal meaning that focused on duty and on academic performance requirements. This adaption and the EXPERIENCE OF COMPETENCE was mediated by the teacher. The learner did not EXPERIENCE AUTONOMY and SOCIAL RELATEDNESS through their social interactions. Here, mathematics was experienced as a socially mediated compulsory subject that was purely an obligation to them. This mathematics-learning orientation (as a result of social regulations) resulted in less controlled behaviour (SDI = -0.89; the other German profiles had SDI values between 0.09 and 3.22) with moderate levels of identified regulation and amotivation. Based on this specific relevance orientation pattern in mathematics and the characteristic dimension of personal meaning (duty), which also emerged in the similar profile from Finland (but with a different specification, see Profile 3–FIN in section 7.2.3), I termed this profile "External pressure focusing on adaption to the teacher's performance requirements".

7.1.5 Comparison of German Learners' Mathematics-Learning Orientations

The identified profiles in Germany showed differences with respect to their mathematics-learning orientations. In all four German profiles, learners constructed the personal meaning EXPERIENCE OF COMPETENCE.

Two out of the four profiles constructed relevance systems that referred to social processes in mathematics class (Profile 1 and Profile 4). In Profile 1 and in Profile 4, from the individual's point of view the teacher had an essential and dominant role. Teacher-mediated situations in class were where learners EXPERIENCED COMPETENCE and emotional-social support, but at the same time they perceived mathematics learning as an obligation. The EXPERIENCE OF COMPETENCE appeared in both profiles, but EXPERIENCE OF SOCIAL RELATEDNESS only arose in Profile 1. In both relevance systems, the learner did not publish EXPERIENCE OF AUTONOMY. Here, only Profile 1 scored a high value in identified regulation, while Profile 4 showed moderate values for amotivation and identified regulation. Here, the motivational regulations resulted in a balanced (Profile 1, SDI = 0.09) and a less controlled motivation (Profile 4, SDI = -0.89).

On the contrary, Profile 2 perceived mathematics lessons through social and individual processes, and Profile 3 studied mathematics for individual reasons. In both profiles, the teacher took care of classroom management, whereas in Profile 2 the teacher additionally seemed to mediate multifaceted roles. Both individual relevance systems showed high values in intrinsic and identified regulation, and in Profile 2 introjected regulation was also represented. Profile 2 (SDI = 2.45) experienced all three specific meanings, whereas Profile 3 (SDI = 3.22) perceived EXPERIENCE OF AUTONOMY and EXPERIENCE OF COMPETENCE. These two types of orientation pattern showed self-determined motivation.

Altogether, the inferred step, interpreting the numerical data into meaningful profiles, could be supported through the coordination analysis assisted by the three adjusted terminologies. In a nutshell, for Profile 1 the adapted model social regulatory mechanism of individual relevance system could not thoroughly explain the predicted motivational outcome. Although the learner perceived duty and did not construct EXPERIENCE OF AUTONOMY, this Profile 1 showed a tendency for self-determined regulated behaviour. It seems that these results highlighted certain crucial factors that shaped all profiles in view of their nature; these were all facets of the German educational culture. The (non-) construction of the specific meanings and their respective individual relevance patterns seem to stay in a specific relation to their corresponding motivational outcomes. To clarify these issues, as a last step, an interpretation of these results with regard to the educational system and the predicted conceptual framework would be fruitful, in particular to explore the interesting effects that emerged in Profile 1 (see section 7.4).

7.2 Mathematics-Learning Orientations in Finland

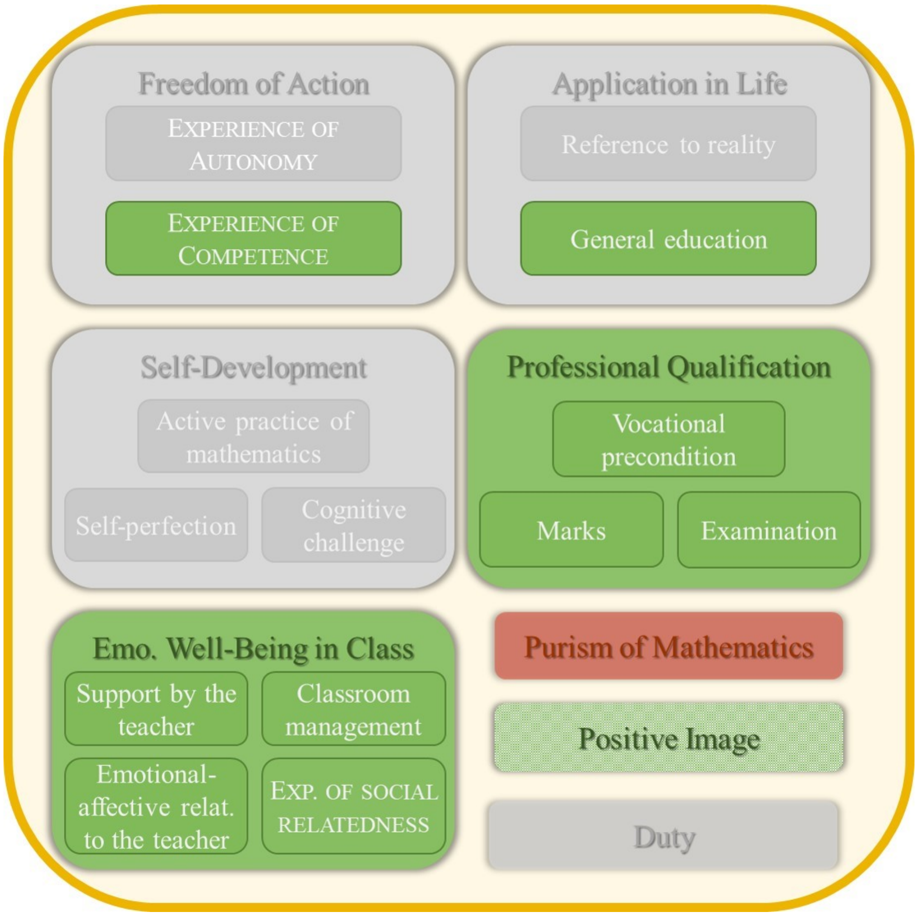
In this section, the four profiles found in Finland are described by using the coordination analysis method. At the end of this section, the four identified profiles' learning orientations and motivation are compared in terms of their similarities and differences.

7.2.1 Profile 1: Emotional-Social Integration Focused on Own Professional Qualification and Seeking Acceptance from the Teacher

The following figure 22 provides an overview of Profile 1 found in Finland with respect to the strong indicators represented by the constructed ($M \geq 1.8$ cut-off point) and non-constructed ($M \leq 1.2$ cut-off point) dimensions/kinds of personal meaning. This profile consisted of 36% of the sample ($N = 243$), of which 45% of the learners were female.

Figure 22

Profile 1 in Finland: Emotional-Social Integration Focused on Own Professional Qualification and Seeking Acceptance From the Teacher.



Note. Summarized overview of profile's pattern of personal meaning and the corresponding motivational behaviour. Profile 1 in Finland is depicted through the constructed and non-constructed kinds of personal meaning embedded within the corresponding dimension of personal meaning. The colour code (light) green is used for (a tendency towards) constructed personal meanings and (light) red is used for (a tendency towards) non-constructed personal meanings. Each dimension is coloured either green or red when all factors of the dimension behaved in a similar manner. Average values are greyed out. The colour code of the profile's frame reflects the profile's SDI, with orange for self-determined motivation, purple for controlled motivation, and grey when the SDI is balanced, i.e., neither positive nor negative.

Step 1. The *publication* stage (as the starting point of the coordination analysis) shows the learner's individual relevance system as having a mathematics-learning orientation that is focused on emotional-social integration. Referring to the learner's responses, I identified two dimensions of personal meaning (emotional well-being in class and professional qualification) and single personal meanings (EXPERIENCE OF COMPETENCE, and general education) that appeared as strong indicators with regard to construction. I additionally considered positive image in a learner's relevance system, as this showed a tendency for construction (M = 1.71, SD = .07). Purism of mathematics appeared as a strong indicator with regard to non-construction. According to the results gathered about the publication stage, two out of the five multifaceted dimensions of personal meaning appeared conjoint. However, to understand the orientation pattern of this learner, I examined the items behind the (non-) constructed personal meanings.

Based on the constructed personal meanings in the publication stage, the learner seemed to study mathematics for highly social reasons. Mathematics as a school subject was important to them with regard to their own future education (dimension: professional qualification and general education). Thereby the teacher was central to their learning process (dimension: emotional well-being in class). Further, it was important to them that they received appreciation and felt proud of their achievements in mathematics (EXPERIENCE OF COMPETENCE and positive image). As a non-constructed personal meaning, the dimension of purism of mathematics showed that the personal relevance of mathematics as a specialist field did not play a role for them. Based on this relevance system and their social experiences in class, the individual constructed two out of the three specific meanings, namely EXPERIENCE OF COMPETENCE and SOCIAL RELATEDNESS. Thus, they did not construct EXPERIENCE OF AUTONOMY in the classroom. To conclude, the reconstruction of *transformation* and *appropriation* of social regulation showed that the learner *identified with* social processes that focused on their further education and the teacher. However, they did *not identify with* mathematics as a specialist field. In this profile, the learner's biographical orientation thus moved between these two opposing poles, which mutually represent opposing scopes for one another. Based on the learner's publication, I reconstructed the levels of transformation and appropriation. Further, I considered this orientation pattern as a result of social interaction in class, in order to make the following assumption about the biological regulation that corresponded to these social experiences in mathematics class.

Step 2. On the basis of this specific perception of social interaction in class, the learner seemed to experience the teacher's support in terms of emotional-social support and vocational preparation. In particular, they did not perceive EXPERIENCE OF AUTONOMY in the mathematics classroom and therefore did not perceive themselves as a centre of action. Here, their sense of the success of their actions depends on their teacher. Therefore, I formulated the following hypothesis with respect to motivational regulation: If the learner published a relevance system whereby they socially experienced their mathematics class as emotionally-socially supportive for their occupational qualification, then their motivation was somewhat controlled.

Step 3. In terms of *enacting potential*, the motivational regulation of this learner showed a high level of identified regulation. The other regulatory styles were low (intrinsic regulation, introjected regulation, external regulation, and amotivation). Surprisingly, the SDI was clearly positive, SDI = 1.30, which indicated self-determined motivation. Here my hypothesis (step 2, similar to Profile 1 in Germany) did not fit the data.

Summary of Profile 1. This relevance system represents the highest percentage of students (36%) out of the four identified profiles in Finland. Here, the difference[39] in gender is balanced

[39] Gender difference was not statistically tested.

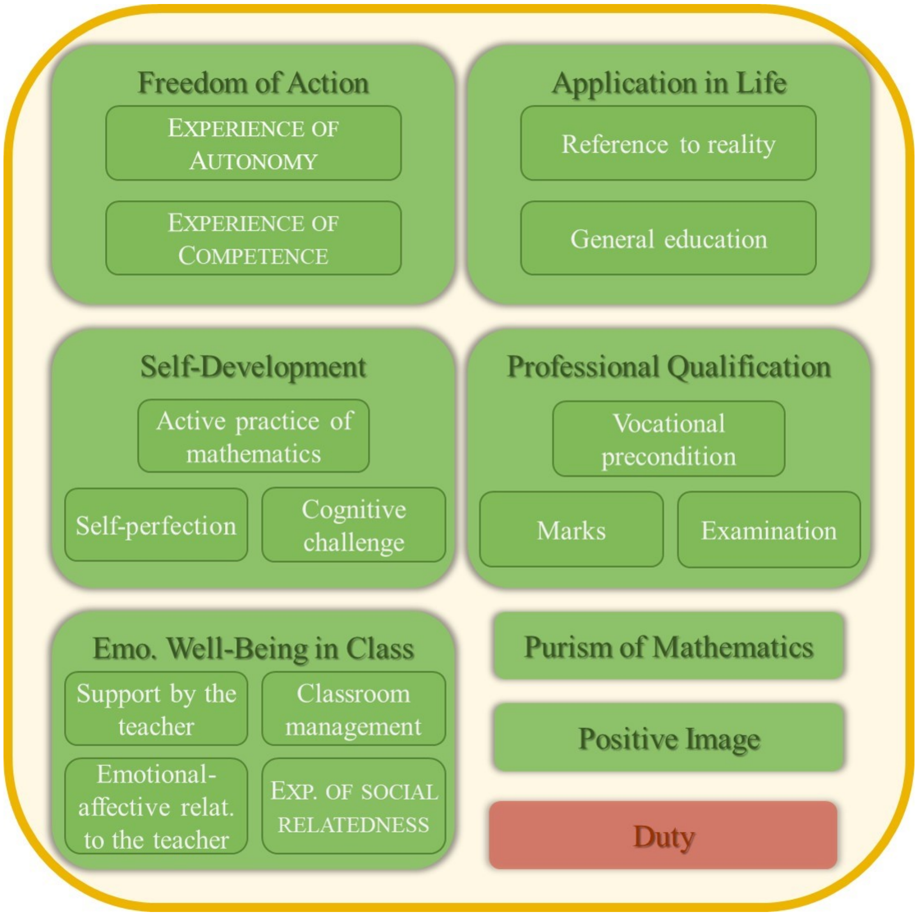
(♀: 45%, ♂: 49%). The students in this profile were high achievers (64 students had grades 10 or 8) and average students (18 students had grades 7 or 6). As a result of social interactions in class, the learner constructed a relevance system of personal meaning that basically focused on emotional well-being in class and on their own professional qualification. Specifically, the teacher was crucial to their learning process. They EXPERIENCED COMPETENCE through their social involvement and a supportive social environment. In this context, they experienced social integration. Here, mathematics was experienced as a socially mediated scope and as an important school subject (not as a compulsory subject) for the learner's occupational career. The social processes were more important than the individual processes, and therefore the learner adapted to the social ones. In a nutshell, this mathematics-learning orientation (as a result of the social regulation) focusing on emotional-social relations resulted in a self-determined motivational behaviour where SDI = 1.30 (biological regulation; the other Finnish profiles had SDI values between -0.19 and 3.89) and that scored high level of identified regulation. Hence, the learner's quality of motivation was autonomous even though the specific meaning EXPERIENCE OF AUTONOMY was not constructed. This surprising result needs to be clarified with respect to the classroom setting in Finland. Based on this specific relevance orientation pattern in mathematics and the characteristic dimension of personal meaning (emotional well-being in class) that also emerged in the similar profile from Germany (but with a different specification, see Profile 1–GER in section 7.1.1), I termed this profile "Emotional-social integration focused on own professional qualification and seeking acceptance from the teacher".

7.2.2 Profile 2: Enjoyment in Mathematics Learning

The following figure, Figure 23, provides an overview of Profile 2 found in Finland with respect to the strong indicators represented by the constructed ($M \geq 1.8$ cut-off point) and non-constructed ($M \leq 1.2$ cut-off point) dimensions/kinds of personal meaning. This profile consisted of 26% of the sample ($N = 243$), of which 63% of the learners were female.

Figure 23

Profile 2 in Finland: Enjoyment in Mathematics Learning.



Note. Summarized overview of profile's pattern of personal meaning and the corresponding motivational behaviour. Profile 2 in Finland is depicted through the constructed and non-constructed kinds of personal meaning embedded within the corresponding dimension of personal meaning. The colour code (light) green is used for (a tendency towards) constructed personal meanings and (light) red is used for (a tendency towards) non-constructed personal meanings. Each dimension is coloured either green or red when all the factors of the dimension behaved in a similar manner. Average values are greyed out. The colour code of the profile's frame reflects the profile's SDI, with orange for self-determined motivation, purple for controlled motivation, and grey when the SDI is balanced, i.e., neither positive nor negative.

Step 1. The learner produced (*publication*) an individual relevance system showing a mathematics-learning orientation that focused on enjoyment. To understand the learner's profile output, I initially summarized the appeared meanings with respect to construction and non-construction. In this profile, I identified seven out of the eight dimensions of personal meaning that emerged as strong indicators with regard to construction: purism of mathematics, freedom of action, self-development, professional qualification, application in life, emotional well-being in class, and positive image. Only duty occurred as a strong indicator with regard to non-construction.

Next, on the basis of this profile summary I intended to understand the learner's orientation framework. To capture their perception of social interaction, I reconstructed the transformation and appropriation levels of social regulation.

Based on the constructed personal meanings in the publication stage, the learner seemed to study mathematics for individual as well as for social reasons. The learner experienced mathematics as a socially mediated (dimensions: professional qualification, application in life, emotional well-being in class) scope in which individual development was possible (dimensions: self-development). The learner coped with the learning process independently and could use their skills without outside support (dimension: freedom of action). They worked with the teacher, and thereby mathematics was perceived as a school subject and at the same time as a specialist field (purism of mathematics). They also enjoyed feeling pride in their achievements in mathematics (positive image). Duty, as a non-constructed personal meaning, showed that the learner did not perceive mathematics-learning situations as an obligation. Based on this specific relevance orientation and their social experiences in class, the learner experienced all three specific meanings, namely EXPERIENCE OF AUTONOMY, COMPETENCE, and SOCIAL RELATEDNESS. In terms of the processes of *transformation* and *appropriation*, the learner *identified with* the association of the mathematics class with fun and relevance, whereas they *did not identify* with the statement that mathematics lessons are a duty. Responding to these social experiences, the learner's orientation pattern thus moves between these two opposing poles. Further, I made the following assumption, which may correspond to this orientation pattern in terms of the learner's motivational regulation.

Step 2. Following the reconstructed social experiences in Step 1, the learner seemed to perceive mathematics lessons as enjoyment and not as pressure. Here, they experienced all three specific meanings and perceived the mathematics lessons as self-directed and themselves as a centre of action without external pressure. Therefore, I hypothesized the following corresponding motivational behaviour: If the learner publishes a relevance system whereby they socially experience their mathematics class as fun and relevant, then their motivation is tends to be self-determined.

Step 3. When considering the biological regulatory mechanism as *enacting potential*, the learner's motivation resulted in high levels of identified regulation and intrinsic regulation. The other types of regulation scored moderate (introjected regulation) to low mean values (external regulation and amotivation). As assumed, the SDI was positive, SDI = 3.58, which indicated self-determined motivation. Here the hypothesis (step 2) fit the data.

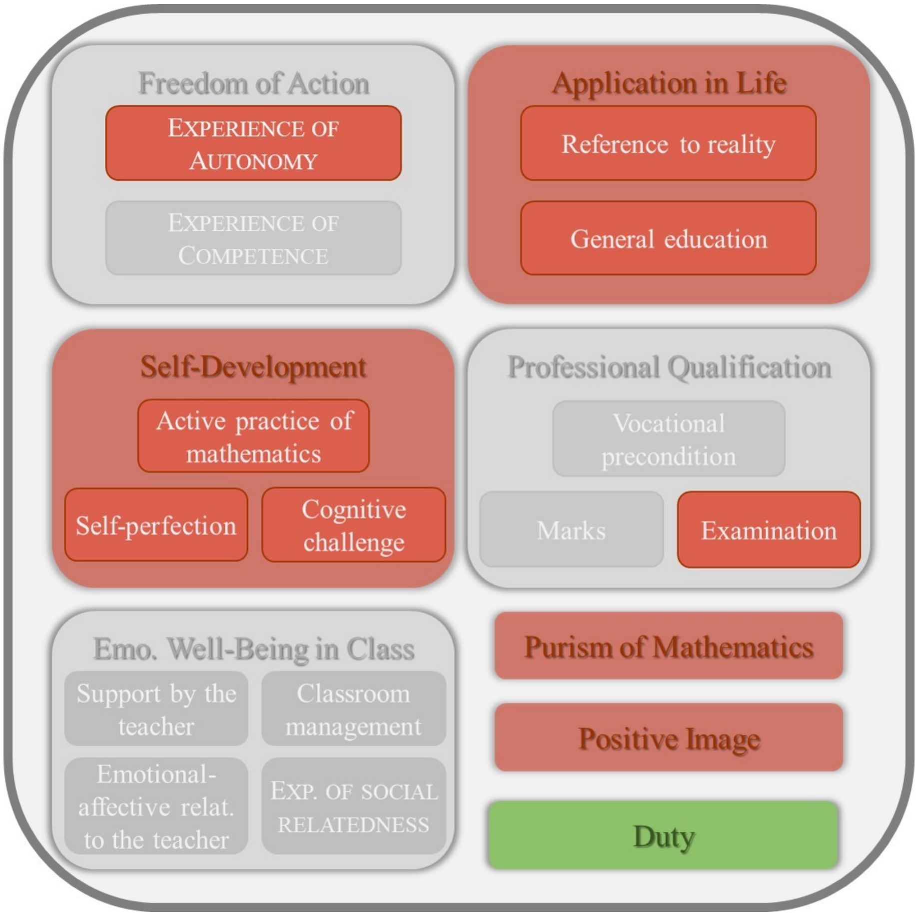
Summary of Profile 2. This relevance system has the second highest percentage out of the four identified profiles in Finland (26%). Here, the percentage of females is nearly twice that of males (♀: 63%, ♂: 33%). This profile attracted mostly high achievers (61 students had either grade 10 or 8) and fewer average students (which grade it was, 7 or 6). Based on the social interaction in mathematics class, the learner constructed a relevance system of personal meaning that focused on individual and social processes. The teacher's support was not perceived as a control, but as a multifaceted support. The learner constructed EXPERIENCE OF AUTONOMY, COMPETENCE, and SOCIAL RELATEDNESS through their social interactions. Here, mathematics was experienced as a socially mediated scope in parallel to the experience of the environment as supportive in terms of dealing with individual processes in mathematics. This mathematics-learning orientation (as a result of the social regulation) is one of enjoyment for the students and resulted in a self-determined motivational behaviour (SDI = 3.58; the other Finnish profiles had SDI values between -0.19 and 3.89) that scored highly in identified regulation and intrinsic regulation. On the basis of the learner's orientation pattern and the characteristic dimensions of personal meaning (application in life, emotional well-being in class, freedom of action, positive image, purism of mathematics, professional qualification, self-development) that also emerged similarly in the profile from Germany (see Profile 2–GER in section 7.1.2), I termed this profile "Enjoyment in mathematics learning".

7.2.3 Profile 3: External Pressure Focusing on Adaption to the Educational System

The following figure, Figure 24, provides an overview of Profile 3 found in Finland with respect to the strong indicators represented by the constructed ($M \geq 1.8$ cut-off point) and non-constructed ($M \leq 1.2$ cut-off point) dimensions/kinds of personal meaning. This profile consisted of 25% of the sample ($N = 243$), of which 43% of the learners were female.

Figure 24

Profile 3 in Finland: External Pressure Focused on Adaption to the Educational System.



Note. Summarized overview of profile's pattern of personal meaning and the corresponding motivational behaviour. Profile 3 in Finland is depicted through the constructed and non-constructed kinds of personal meaning embedded within the corresponding dimension of personal meaning. The colour code (light) green is used for (a tendency towards) constructed personal meanings and (light) red is used for (a tendency towards) non-constructed personal meanings. Each dimension is coloured either green or red when all the factors of the dimension behaved in a similar manner. Average values are greyed out. The colour code of the profile's frame reflects the profile's SDI, with orange for self-determined motivation, purple for controlled motivation, and grey when the SDI is balanced, i.e., neither positive nor negative.

Step 1. The *publication* stage of the social regulatory mechanism showed the learner's individual relevance system as having a mathematics-learning orientation that was focused on external pressure. In this profile output, the personal meanings that emerged with respect to construction were fewer in number than meanings relating to non-construction. The dimension duty was the only strong indicator with regard to construction. The dimensions of purism of mathematics, self-development, application in life, positive image, and the single meanings EXPERIENCE OF AUTONOMY and examination appeared as strong indicators with regard to non-construction. Based on this profile summary, I approached the learner's orientation framework in order to reconstruct their social experiences in the mathematics classroom, that is, their *transformation* and *appropriation* level of social regulatory mechanism.

Based on the constructed personal meanings that the individual published, the learner dealt with mathematics in class through social processes. They studied this subject out of obligation and the fear of sanctions. They experienced mathematics as an exclusively compulsory subject (dimension: duty) in which individual development was not possible (non-construction of dimension: self-development). They did not experience themselves as a centre of action (non-construction of EXPERIENCE OF AUTONOMY) or feel proud of their achievements in mathematics (non-construction of positive image) while learning. Mathematics was neither perceived as an important school subject (non-construction of the dimension application in life and examination) nor as a specialist field (non-construction of the dimension purism of mathematics). Surprisingly, there was no clear attitude towards the role of the teacher here, but the learner rejected learning mathematics. Based on this specific relevance orientation and their social experiences in class, the learner experienced none of the three specific meanings, namely EXPERIENCE OF AUTONOMY, COMPETENCE, and SOCIAL RELATEDNESS. Here, the learner *identified with* mathematics as a social adaption to the educational system, whereas they *did not identify with* mathematics lessons as interesting as an academic subject and as a specialist field. From these social experiences, the learner's orientation framework thus exists between these two opposing poles that mutually represent opposing scopes for one another. Further, I made the following assumption that may correspond to this orientation pattern in terms of the learner's motivational regulation.

Step 2. Based on the socially experienced relevance system in Step 1, the learner seemed to perceive mathematics lessons in the context of the pressure provided by the Finnish educational system. They experienced none of the three specific meanings and thus did not experience themselves as a centre of action or as involved in any individual or social processes (apart from duty). Therefore, I made the following assumption with respect to the learner's motivational behaviour: If the learner published a relevance system whereby they socially experienced their mathematics class as an adaption to the educational system, then their motivation tends to be controlled.

Step 3. When considering the biological regulatory mechanism as *enacting potential*, the learner's motivation showed a high level of identified regulation. The other regulatory styles scored moderate (amotivation) to low mean values (intrinsic regulation, introjected regulation, external regulation). Interestingly, the self-determination index (SDI) showed only a negative tendency, SDI = -0.19 of controlled motivation, but the value was not strong. Here my hypothesis (step 2) yielded an ambivalent result that needs to be interpreted with respect to the Finnish educational culture.

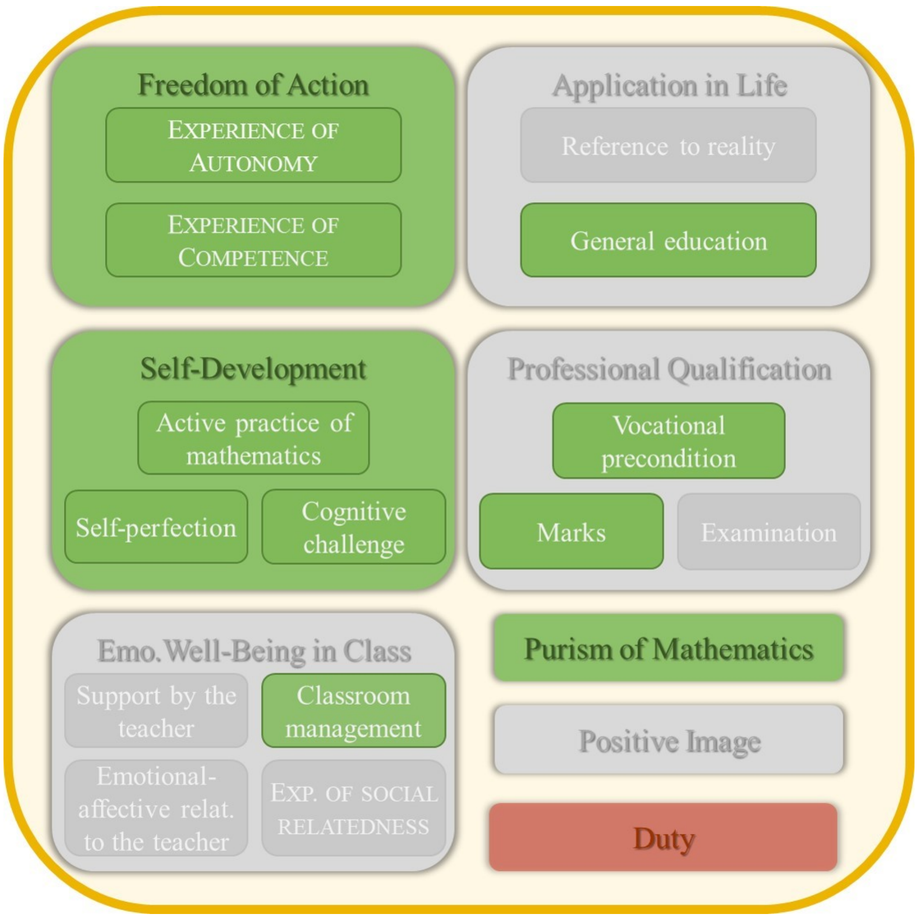
Summary of Profile 3. This relevance system has the third highest percentage of students out of the four identified profiles in Finland (25%). Here, the percentage of females is lower than that of males (♀: 43%, ♂: 54%). This profile attracted mostly high achievers (39 students had either grade 10 or 8), several average students (16 students had either grade 7 or 6), and fewer low achievers (which grade it was, 5 or 4). Based on the social interaction in the mathematics class, the learner constructed a relevance system of personal meaning with a focus on obligation. The role of the teacher was neither perceived as support nor as control. The learner did not experience any specific personal meaning through their social interactions. Here, mathematics was experienced as a socially mediated scope that was purely an obligation to them. This mathematics-learning orientation (as a result of the social regulation) resulted in balanced motivational behaviour with a tendency towards controlled motivational behaviour; SDI = -0.19. This is neither positive nor negative (for comparison only, the other Finnish profiles had SDI values between 1.30 and 3.89, which scored a high level of identified regulation. This surprising result needs to be clarified in the context of the mathematics curricula in Finland. Based on this specific relevance orientation pattern in mathematics and the characteristic dimension of personal meaning (duty) that also emerged in the similar profile from Germany (but with a different specification, see Profile 4–GER in section 7.1.4), I termed this profile "External pressure focusing on adaption to the educational system".

7.2.4 Profile 4: Self-Improvement with a Focus on the Individual

The following figure, Figure 25, provides an overview of Profile 4 found in Finland with respect to the strong indicators represented by the constructed ($M \geq 1.8$ cut-off point) and non-constructed ($M \leq 1.2$ cut-off point) dimensions/kinds of personal meaning. This profile consisted of 13% of the sample ($N = 243$), of which 39% of learners were female.

Figure 25

Profile 4 in Finland: Self-Improvement With a Focus on the Individual.



Note. Summarized overview of profile's pattern of personal meaning and the corresponding motivational behaviour. Profile 4 in Finland is depicted through the constructed and non-constructed kinds of personal meaning embedded within the corresponding dimension of personal meaning. The colour code (light) green is used for (a tendency towards) constructed personal meanings and (light) red is used for (a tendency towards) non-constructed personal meanings. Each dimension is coloured either green or red when all the factors of the dimension behaved in a similar manner. Average values are greyed out. The colour code of the profile's frame reflects the profile's SDI, with orange for self-determined motivation, purple for controlled motivation, and grey when the SDI is balanced, i.e., neither positive nor negative.

Step 1. The *publication* level of social regulation showed the learner's individual relevance system to have a mathematics-learning orientation focused on self-improvement. In this profile output, I identified the dimension purism of mathematics and the multifaceted ones (self-development, freedom of action) that appeared conjoint, partial (professional qualification), and single personal meanings (classroom management and general education) that resulted as strong indicators with regard to construction. The dimension of duty appeared as a strong indicator for non-construction. Hence, two out of the five multifaceted dimensions of personal meaning appeared conjoint. Next, to reconstruct the learner's perceptions in class, I evaluated the items behind the (non-) constructed personal meanings.

Based on the constructed personal meanings at the publication stage, the learner studied mathematics for individual reasons. The learner experienced mathematics as a scope in which to develop their personal cognitive skills (dimension: self-development and general education) and seek self-improving challenges in mathematics. It was also seen as important to give them better opportunities in their professional life (vocational precondition and marks). Further, they coped with learning processes independently and could use their skills without support from the outside (dimension: freedom of action). The teacher ensured a good working environment (classroom management) to support their individual work, which also supported their perception of mathematics as a specialist field (purism of mathematics). Duty was a non-constructed dimension of personal meaning, which showed that the learner did not perceive mathematics-learning situations as an obligation. Based on this specific relevance orientation and their social experiences in class, the learner experienced two specific meanings, namely EXPERIENCE OF AUTONOMY and COMPETENCE. Regarding the regulations of *transformation* and *appropriation*, the learner *identified with* the statement that mathematics class was a "self-improving challenge", whereas they did *not identify with* the statement that mathematics was a "non-relevant discipline". From this social experience, the learner's biographical orientation pattern thus moves between these two opposing poles. In the next step, I made the following assumption that may correspond to this orientation pattern in terms of the learner's motivational regulation.

Step 2. Following the reconstructed social experiences in Step 1, the learner seemed to perceive mathematics lessons as supporting their own personal self-improvement and did not perceive the subject as a non-relevant discipline. Here, they experienced two specific meanings that showed that their success of action was independent of the teacher and that by focusing on individual processes, they perceived themselves as a centre of action. Therefore, I hypothesized this corresponding motivational behaviour: If the learner publishes a relevance system whereby they socially experience individual self-improvement in class, then their motivation tends to be self-determined.

Step 3. When considering the biological regulatory mechanism as *enacting potential*, the learner's motivation showed a high level of identified and intrinsic regulation. The other regulatory styles scored low mean values (introjected regulation, external regulation and amotivation). As assumed, the SDI was positive, SDI = 3.89, which indicated self-determined motivation. Here the hypothesis (step 2) fits the data.

Summary of Profile 4. This relevance system has the lowest percentage of students out of the four identified profiles in Finland (13%). Here, the percentage of females is somewhat lower than that of males (♀: 12%, ♂: 17%). This profile attracted mostly high achievers (29 students had either grade 10 or 8) and fewer average students (one student had either grade 7 or 6). Based on the social interactions in mathematics class, the learner constructed a relevance system of personal meaning that focused on individual processes. There was neither a dependency on the teacher, nor a sense of community feeling in class. The teacher did not play a central role, but their support was important with respect to classroom management. The teacher left the learner free to act and they learnt for themselves. In this context, they constructed EXPERIENCE OF COMPETENCE and EXPERIENCE OF AUTONOMY. They sought self-improving challenges and experienced them in mathematics lessons. Therefore, mathematics was not perceived as a non-relevant subject. This mathematics-learning orientation (as a result of the social regulation) resulted in a self-determined motivational behaviour (SDI = 3.89; the Finnish profiles had SDI values between -0.19 and 3.58) that scored highly in identified regulation and intrinsic regulation. Based on this specific relevance orientation in mathematics and the characteristic dimension of personal meaning (freedom of action) that also emerged in the similar profile from Germany (but with a different specification, see Profile 3–GER in section 7.1.3), I termed this profile "Self-improvement with a focus on the individual".

7.2.5 Comparison of Finnish Learners' Mathematics-Learning Orientations

The identified profiles in Finland showed differences with respect to their mathematics-learning orientations. Two out of the four profiles were relevance systems that referred more to social processes in the mathematics class (Profile 1 and Profile 3). From the individual's point of view, the teacher and one's own professional qualification had an essential role in Profile 1. Teacher-mediated situations in class whereby the learner EXPERIENCED COMPETENCE and emotional-social integration were important. In Profile 3, the learner only participated in mathematics class to fulfil an obligation. The construction of EXPERIENCE OF COMPETENCE and OF EXPERIENCE OF SOCIAL RELATEDNESS resulted in Profile 1, whereas in Profile 3 none of the three specific meanings were perceived by the student. In both relevance systems, learners did not publish EXPERIENCE OF AUTONOMY. Here, Profiles 1 and 3 scored highly in identified regulation. Both profiles yielded surprising results with respect to their motivational regulation. The motivation in Profile 1 was self-determined, and the quality of motivation in Profile 3 showed only a tendency towards controlled motivation.

On the contrary, Profile 2 perceived mathematics lessons through social and individual processes, and Profile 4 was geared towards mathematics learning for individual reasons. In Profile 2 the teacher combined multifaceted roles, and in Profile 3 the teacher helped the learner to realize mathematics as a specialist field. Profile 2 experienced all three specific meanings, whereas Profile 4 perceived EXPERIENCE OF AUTONOMY and EXPERIENCE OF COMPETENCE. These two types of orientation pattern showed self-determined motivation. Both individual relevance systems showed high values in intrinsic and identified regulation.

In all four profiles, the students did not show clear controlled behaviour; only Profile 3 scored a tendency towards controlled motivation. In a nutshell, for two (Profile 1 and Profile 3) out of four cases the adapted model social regulatory mechanism of the individual relevance system could not thoroughly explain the hypothesized motivational outcome. Even though the learner did not construct EXPERIENCE OF AUTONOMY, Profile 1 showed self-determined regulated behaviour. Surprisingly, Profile 3, which only constructed duty, but none of the three specific meanings, resulted only in a tendency towards controlled behaviour in class (I expected the controlled behaviour to be more in evidence). The non-construction of the specific personal meanings and the respective individual relevance system showed no convincing connection to their corresponding motivational outcome in Profile 3. It seems that next to the three specific meanings, the mathematics curricula with the respective cultural settings that underpin them play an essential role in the social regulatory mechanism. Based on these results, the adapted model of social regulatory mechanism needs to include additional factors to clarify the social regulatory mechanism holistically. In particular, Profiles 1 and 3 seem to encompass interesting references that relate to a specifically Finnish classroom environment.

The results according to the coordination analysis of German and Finnish students made clear that the profile patterns from Germany and Finland that emerged had similar mathematics-learning orientations, but with some cultural variations. It seems that the subtle differences between the social regulatory mechanisms in classrooms not only play an essential role, but also result in significant differences in learners' motivational behaviour.

7.3 Summary of the Results According to Coordination Analysis

Based on the indicators found in the German and Finnish empirical data, personal meaning as a boundary object seems to have the characteristics to coordinate SDT and social constructivism. From the perspective of social constructivism, all the identified empirical profiles across countries could be interpreted as meaningful and valid empirical results by means of the adapted model, the social regulatory mechanism of the individual relevance system. In both countries, four different individual relevance systems could be identified that explain German and Finnish students' learning orientations towards emotional-social integration (characterized by the dimension: emotional well-being in class); enjoyment in mathematics learning (representative dimensions: all seven dimensions apart from duty); self-improvement (typical dimension: freedom of action); and external pressure (characterized by the dimension: duty) in mathematics classroom. The identified profiles showed similar patterns with differences in nuance, whereas the motivational outcomes varied across countries.

According to the findings, in both countries the profiles with emotional-social integration were most valued where learners focused more on social processes accompanied by the construction of EXPERIENCE OF COMPETENCE and SOCIAL RELATEDNESS. Whereas the German profile showed a more balanced motivation with a tendency towards self-determination, the Finnish relevance system scored a clearly positive SDI. The profiles of enjoyment in learning mathematics in which learners constructed all three specific meanings existed independently of culture, and, in both countries, showed strong self-determined motivation. In both groups, the learning orientations towards self-improvement with a strong focus on individual processes resulted in self-determined motivation when the learner constructed EXPERIENCE OF AUTONOMY and EXPERIENCE OF COMPETENCE. The mathematics-learning orientations towards external pressure reflected a tendency towards controlled motivation (Profile 3) in Finland and towards a less controlled motivation (Profile 4) in Germany. It became apparent that in particular the construction of EXPERIENCE OF AUTONOMY plays a central role within the three specific meanings.

The results from Finland, especially Profile 3 (external pressure focusing on adaption to the educational system), came up with surprising results. The socially experienced relevance system was not entirely reflected in the motivational outcome.

Although German and Finnish learners showed similar learning orientations in mathematics, their profile patterns seem to be specific to each country. It seems that next to the three specific meanings, the mathematics curricula with the respective cultural settings that underpin them play an essential role in the social regulatory mechanism. For this reason, additional mediators of social regulations exist that seem to affect the motivational outcome. These mediators should be added to the adapted model (social regulatory mechanism of the individual relevance system) in order to strengthen and balance the conceptual framework when coordinating SDT and social constructivism through the boundary object of personal meaning.

7.4 Theorizing the Results of the Coordination

The research activities considered in this dissertation are sequential-dependent (Schoonenboom & Johnson, 2017). For this reason, the coordination procedure, preceded by the quantitative procedure, should appropriately build on the previous empirical results. Based on the empirical results discussed in Chapter 6, I reported the results according to the coordination analysis (cf. 7.1 and 7.2). In the following sections, I interpret these results with respect to the main issue with which this dissertation is concerned; namely, the genesis of learners' motivation in the mathematics classroom.

To theorize the coordination of the results, I targeted three (further) RQs: RQ 4–How do the individual relevance systems and motivation interact in Germany and Finland, when viewed from a perspective coordinating SDT and social constructivism? RQ 5–How does the interaction found in German and Finnish data differ in view of learners' mathematics curricula? RQ 6–How does the conceptual framework built by coordinating SDT and social constructivism by means of the boundary object personal meaning explain the development of motivation in mathematics classroom?

By focusing the coordinating procedure on these questions, I reflect on the conceptual framework built by coordinating SDT and social constructivism in which personal meaning is conceptualized as situated at the boundary of both theoretical lenses (see section 7.4.1). This step intends to examine how far both theoretical lenses reflect one another. The theory-driven analyses emphasized that each country's relevance system has cultural tags that refer to their respective cultural setting. Therefore, I also contrasted the results from Germany (GER) and Finland (FIN) by referring to learners' cultural settings in the classroom. Through this step, I refined the interplay between theories and clarified surprising results whereby the adopted theoretical lenses were limited to explaining the coordination biological and social regulatory mechanism (see section 7.4.2). To support the interpretation procedure in view of comparing results from Germany and Finland, certain results (cf. 5.1–5.3) from quantitative statistical analyses are combined in section 7.4.2. Finally, I present the evidence from this study's coordination procedure in the shape of a theoretical model explaining the development of motivation by means of the linkage between biological and social regulatory mechanisms, which seem to converge within the boundary object of personal meaning (see section 7.4.3).

7.4.1 Interpreting the Interaction Between Biological and Social Regulatory Mechanisms

The theory-driven analyses based on German and on Finnish empirical data helped to describe how learners' individual relevance systems (the content as *what*, described by the perspective of social constructivism) arise and unfold in mathematics classroom as a counterpart of the empirical evidence produced by the individuals and their motivational behaviour (information processing as *why*, explained through the perspective of SDT). Coordinating theories provided the potential to examine the empirical phenomenon—profiles of personal meaning—from two different theoretical perspectives, as well as to consider two complementary epistemic views as well-fitting elements (Prediger et al., 2008, p. 172; Sabena et al., 2014). This was done as a means of improving our understanding of how learners' quality of motivation develops in the mathematics classroom. The use of two different lenses also made it easier to mutually validate the conceptual framework and to interpret data from two different cultural settings, through theoretical coherence and theoretical contradiction in terms of "what and why".

In both countries, the empirical profiles of personal meaning could be separated into four kinds of mathematics-learning orientation, that is: emotional-social integration (Profile 1–GER and Profile 1–FIN); enjoyment in mathematics learning (Profile 2–GER and Profile 2–FIN); self-improvement (Profile 3–GER and Profile 4–GER); and external pressure (Profile 4–GER and Profile 3–FIN). In the following, I theorize the interaction between the biological and social regulatory mechanisms by means of both theoretical lenses, SDT and social constructivism.

Emotional-Social Integration. Through the repeated social experiences in their social environment, learners were facilitated in the classroom to construct the specific personal meanings EXPERIENCE OF COMPETENCE and EXPERIENCE OF SOCIAL RELATEDNESS. Based on these social experiences they produced learning orientations across countries that focused on emotional-social integration (Profile 1–GER and Profile 1–FIN). As EXPERIENCE OF AUTONOMY was missing, I expected both profiles to indicate relatively controlled behaviour. In both countries, these profiles have *personal importance* (identified regulation) as a relevant regulatory process (Ryan & Deci, 2017, p. 193) that resulted in a balanced motivation showing a tendency for self-determination (GER) and self-determined motivation (FIN) in mathematics class. In this connection, learners' specific profile patterns across countries implied the fulfilment of certain basic psychological needs: From the perspective of SDT, the fulfilment of the basic psychological needs for competence and relatedness takes place when learners start internalizing their motivations for performing a compulsory, externally imposed task (Vansteenkiste et al., 2018). In this development process, they move in the direction of autonomy fulfilment and therefore they need social integration in order to complete the process of internalization and to become autonomous (Niemiec & Ryan, 2009; Vansteenkiste et al., 2020). Therefore, learners particularly need a strong bond with their teacher. This bond provides them with the confidence to perform in learning situations, since they have no complete experience of psychological freedom, i.e., fulfilment of autonomy. In return for the teacher's support, they do not want to disappoint the teacher who has let them feel effective. Accordingly, they show their loyalty and try to please the teacher in order to get further support (Bartholomew et al., 2018; Haerens et al., 2015; Vansteenkiste et al., 2018) in mathematics lessons. This view from SDT can be reflected in both patterns of personal meaning that the learner experienced in their social setting (orientation towards emotional-social integration).

Enjoyment in Mathematics Learning. In both countries the social regulations in mathematics class supported the construction of all three specific personal meanings, EXPERIENCE OF COMPETENCE, AUTONOMY, and EXPERIENCE OF SOCIAL RELATEDNESS, which produce learning orientations towards enjoyment in mathematics learning (Profile 2–GER and Profile 2–FIN). These relevance systems go along with self-determined motivation, in which the respective regulatory processes (Ryan & Deci, 2017, p. 193) are *enjoyment* (intrinsic regulation) and *personal importance* (identified regulation), but also *ego-involvement* (introjected regulation in GER). In learning situations, the satisfaction of all basic psychological needs provides the nutriments of an increasing internalization process, integration, and well-being (Deci & Ryan, 2000; Ryan & Deci, 2017). Next to the energizing factors of intrinsic motivation (basic need for autonomy and competence; Deci & Ryan, 2000), the need for relatedness shows that the learner feels a strong connection to their significant others (Deci & Ryan, 2000; Krapp, 2005), but especially to the teacher (Vansteenkiste et al., 2018), who guides the processes of conventionalization in the mathematics classroom. Students are able to take ownership of their learning if they are provided with an accommodating social situation in which their basic psychological needs are met (Vansteenkiste et al., 2018). In Germany, ego-involvement is a notable factor, as studies from the perspective of SDT postulate that it is a typical effect of internally controlled feelings (Ryan & Deci, 2020) in which learners' self-esteem depends on their mathematics-learning outcomes (Frank, 1941; Ryan, 1982). Also here, the view from SDT can be reflected in the patterns of personal meaning that learners experienced in their social context (orientation towards enjoyment in mathematics learning).

Self-Improvement. For Profile 3–GER and Profile 4–FIN, the social regulations in mathematics class supported the construction of the specific personal meanings EXPERIENCE OF AUTONOMY and EXPERIENCE OF COMPETENCE that produce mathematics-learning orientations that focus on self-improvement. From the perspective of SDT, these profiles have *enjoyment* (intrinsic regulation) and *personal importance* (identified regulation) as relevant causes (Ryan & Deci, 2017, p. 193) for their high self-determined motivation. In regard to the learner's personal growth, the basic psychological needs for autonomy and competence are deeply interwoven (Deci & Ryan, 2000; Krapp, 2005). In general, the learner's autonomy refers to their need to experience themselves as a centre of action, whereas competence consists of the need to be capable of acting (Krapp, 2005, p. 635) and to feel optimally challenged (Deci & Ryan, 2000). However, an individual wishes for liberty of action if they feel self-assured about their abilities (Tafarodi et al., 1999; Krapp, 2005) and are certain that they can complete the relevant task without needing assistance (Krapp, 2005). Thus, in order to experience a feeling of competence, it is necessary for the individual to desire autonomy (Krapp, 2005, p. 635). In these specific profile orientations, the two psychological needs, autonomy and competence, that are responsible for maintaining intrinsic motivation are addressed. Their satisfaction explains the individual's striving for self-improvement and to bring their competence to perfection (Krapp, 2005). However, relatedness, the less central need for the maintenance of intrinsic motivation (Deci & Ryan, 2000; Ryan & Deci, 2017), does not occur. Notably, the specific personal meaning EXPERIENCE OF SOCIAL RELATEDNESS was not constructed here. Also here, the view from SDT can be reflected in the patterns of personal meaning that learners experienced in their social context (orientation towards self-improvement).

External Pressure. In contrast, the social environment that supports the sole construction of EXPERIENCE OF COMPETENCE, Profile 4–GER, produced a learning orientation focusing on external pressure. As a result of this focus in the learner's relevance system, they show a motivational behaviour that directs them towards controlled behaviour. None of the regulatory styles show a clear regulation, but here the SDI indicates less controlled motivation. Analogously to Profile 1–GER, the teacher ensures the learner's basic need for competence is satisfied. The learner's performance is not completely internalized and as a result, their actions in class is neither autonomous nor fully committed (Vansteenkiste et al., 2018). Instead, their attention is centred on social integration, especially on gaining their teacher's approval in the classroom. The interpretation through SDT's lens finds theoretical coherence in the learner's socially regulated profile of personal meaning. Accordingly, the learner constructs personal meanings that reflect their adaption to external pressure.

Through the long-term mathematics lessons, Profile 3–FIN constructed none of the three specific personal meanings, and the social regulations in class supported a mathematics-learning orientation that focused on external pressure. Quite contrary to my hypothesis, this profile only resulted in balanced motivational behaviour and a (mild) tendency for controlled motivation to have personal importance (identified regulation) as a relevant regulatory process (Ryan & Deci, 2017, p. 193). From the perspective of SDT, the experience of basic psychological needs being thwarted can lead to psychological reactance (Brehm, 1966), malfunctioning, and ill-being (Vansteenkiste & Ryan, 2013). However, in this profile, the quality of motivation was not strongly controlled. Furthermore, the learner did not construct personal meanings that reflected their bond with the teacher. Accordingly, the Finnish learner did not connect external pressure with their teacher or need the teacher's help in order to feel effective or supported in terms of their internalization in mathematics class.

Certain mediators seem to exist within the Finnish social environment that guarantee the experience of self-determination in the mathematics classroom. In particular, for Profile 3–FIN, the hypothesized adapted model of social regulatory mechanism (cf. Figure 6) thus did not provide a holistic explanation of this profile's social experiences when these converge with the individual experiences explained through the perspective of SDT. Next to this notable result (Profile 3–FIN), each country's relevance systems, in particular their cross-cultural comparisons, provided strong evidence for the existence of these latent mediators affecting the learner's social experiences in the classroom. To entirely understand the individual's social regulations in mathematics class, these additional mediating factors need to be included in the adapted model. In the following, using a comparison of the profiles from Germany and Finland, I point out why the mediating role of the teacher and the learner's cultural setting are essential mediating factors that complement the adapted model of the social regulatory mechanism of the individual relevance system.

7.4.2 Comparing the Individual Relevance Systems in Germany and Finland

Conspicuous features of German and Finnish profiles emerged, highlighting similarities, but also differences across countries. The contrasting of results led to an enrichment of the predicted adapted model, that is, the social regulatory mechanism of the individual relevance system.

Certain dimensions of personal meaning that found empirical support across countries (cf. 5.1) emerged consistently in analogous profile orientations in Germany and Finland. For that reason, they were used to specify profiles' learning orientations in general. As indicated earlier, German and Finnish learners identify with four similar mathematics-learning orientations: emotional-social integration (characterized by the dimension: emotional well-being in class); enjoyment in mathematics learning (characteristic dimensions: all except duty); self-improvement (characteristic dimension: freedom of action), and external pressure (representative dimension: duty). Although the students are from different cultural settings, they share certain commonalities. German and Finnish ninth graders, approximately aged 14 to 16 (OECD, 2020a, 2020b), belong to the age group of middle adolescence (ages 14 to 17). Accordingly, their similar ages and commonalities between the cultures of these Western European countries might clarify why they constructed similar relevance systems when learning mathematics.

In both countries, the profiles of emotional-social integration were the most commonly occurring relevance systems among the students. According to their orientations, these students focus more on social rather than on individual processes. Through the non-construction of EXPERIENCE OF AUTONOMY, I expected them to experience controlled motivation. However, when interpreting the profiles from SDT's lens, I found the opposite to be the case. In line with SDT, the student is undergoing a process of becoming a self-determined individual, and for this reason it is necessary for them to have an emotionally warm relationship with their teacher (Wentzel, 2012) who lets them feel effective in mathematics class. During middle adolescence, learners are in the period of emotional, physical, and sexual development associated with puberty, but are also going through neurological changes (Byrnes, 2001; Keating, 2004) with consequences that can extend well into adulthood (Wigfield et al., 2006). In this spirit, to develop an internalized motivation they need a strong bond with their teacher (Haerens et al., 2015; Niemiec & Ryan, 2009; Vansteenkiste et al., 2018). Although the basic psychological need of relatedness is less vital for intrinsic motivation (Deci & Ryan, 2000), this particular need assists the individual to move in the direction of autonomy satisfaction (Ryan & Deci, 2017). As indicated earlier (cf. 2.1.2), Assor et al. (2002) stressed that the importance of the learner's striving for autonomy exists regardless of their development stage. The existence of these specific profiles underlines how critical the innate need to become a self-determined human being is (Ryan & Deci, 2017), a need that parallels the other critical individual changes associated with the development that takes place in middle adolescence (Steinberg, 2005; Wigfield et al., 2006). As is known, adolescence is affected by culture and thus varies over different cultural contexts (Wigfield et al., 2006). However, both German and Finnish learners value this emotional support greatly, and this emotional-social bond to the teacher exists in both cultures.

However, the Finnish profile showed self-determined motivational behaviour, whereas the profile in Germany scored a more balanced SDI. Even though both profiles are developing their internalization, for German students this process associates with duty in the sense of striving to perform well in order to please the teacher (focused on seeking acceptance from the teacher). The Finnish student connects the emotional-warmth relation with an additional focus on their professional qualification (focused on their own professional qualifications and on seeking acceptance from the teacher). As indicated earlier (cf. 6.2) due to the timing of the transition from lower to upper secondary education (academic or vocational track; OECD, 2020a), for Finnish ninth graders, good marks matter. In this connection, the Mann–Whitney U test pointed out that Finnish students perceived a higher relevance of vocational precondition and showed greater identified regulation than German learners (cf. 5.3). It becomes apparent that here, besides working for one's own qualifications, *specific support from the teacher* seems to result in a higher identified regulation, and this in turn seems to preponderate in Finnish students' self-determined SDI.

The second largest relevance system across countries is orientated towards enjoyment in mathematics learning; in this system, the social regulatory mechanism in the mathematics class appears to stimulate and fulfil all basic psychological needs through the realization of all three specific personal meanings. The complementary insights (through SDT and social constructivism) show that the learner socially experiences individual as well as social processes that result in a high degree of self-determination. Surprisingly, this specific learning orientation, enjoyment in mathematics learning, exists across cultures, without a culture-specific focus. Accordingly, the period of middle adolescence as a cultural phenomenon (Wigfield et al., 2006) shows almost exactly the same features, when all basic psychological needs are satisfied. Even though both groups construct almost identical orientation pattern in profiles, Finnish students scored higher relevance in the active practice of mathematics, self-perfection, marks, and vocational precondition (cf. 5.3), which associated with self-determined types of motivation within the correlation analyses (cf. 5.1). This could be why Finnish learners scored (again) a greater self-determination and seemed to be more satisfied in the context of their classroom setting than the German students, who additionally experienced internal pressure (introjected regulation). As indicated earlier, when learners are introjected, they may engage in this conduct because they are experiencing anxiety (Ryan & Deci, 2000a) guilt, or shame (Bartholomew et al., 2018), or because they are seeking self-worth to avoid a feeling of failure (Ryan & Deci, 2017). Ryan and Deci (2017, p. 186) suggested that this behavioural pattern is particularly common in highly pressured disciplines where a high level of competition and comparison between peers exists. The given differences in nuance between countries become more pronounced when contrasting the German and Finnish profiles that are oriented towards self-improvement and external pressure.

For German learners, studying mathematics for self-improvement means satisfying their own competence requirements (focus on mastery), but for Finnish students it means strengthening their personal development (focus on the individual). Across countries, these profiles reflected the highest degree of self-determination. Even though both relevance systems perceive EXPERIENCE OF AUTONOMY (in Germany there is only a tendency for construction) and COMPETENCE, Finnish students constructed personal meanings that relate to intrinsic types of motivation (purism of mathematics, active practice of mathematics, and self-perfection) or perceived more relevance in them (cognitive challenge) than German learners (cf. 5.1). When compared, the Finnish profile again scored a greater SDI than the German relevance system. However, in Germany, the orientation towards self-improvement is the third largest profile, while in Finland it is the smallest.

The most surprising difference over groups concerned the mathematics-learning orientations towards external pressure. Whereas the German learner connects their feeling of obligation and the performance requirements with their teacher (focusing on adaption to teacher's performance requirements), the Finnish student does not personalize the subject mathematics as connected with their teacher (focusing on adaption to the educational system). Quite the contrary; rather, they see the teacher as supporting them and probably relate mathematics to their Finnish educational system. Based on the learners' transactions with the social environment, personal meanings that reflect a collective relevance in mathematics, and the non-construction of EXPERIENCE OF AUTONOMY, I predicted these profiles would direct towards controlled motivational behaviour. Whereas in Germany, learners scored a higher relevance in EXPERIENCE OF COMPETENCE (cf. 5.3), which led to a less controlled quality of motivation, in Finland students only showed a tendency towards controlled motivation without constructing any specific personal meanings. As indicated above, interpretation of the German profile from a social constructivist perspective is in theoretical consensus with the lens of SDT, but the adapted model could not entirely explain SDT's perspective for Finnish Profile 3. Although Finnish learners constructed none of the specific personal meanings, the exclusive realization of duty in their learning orientation resulted in behaviour in mathematics class that was not explicitly controlled. In this example, it explicitly turned out that the adapted model (cf. Figure 6) could not provide convincing explanations regarding the learner's motivational outcome. In a nutshell, the adapted cycle fails to explain the support for self-determination in the Finnish classroom.

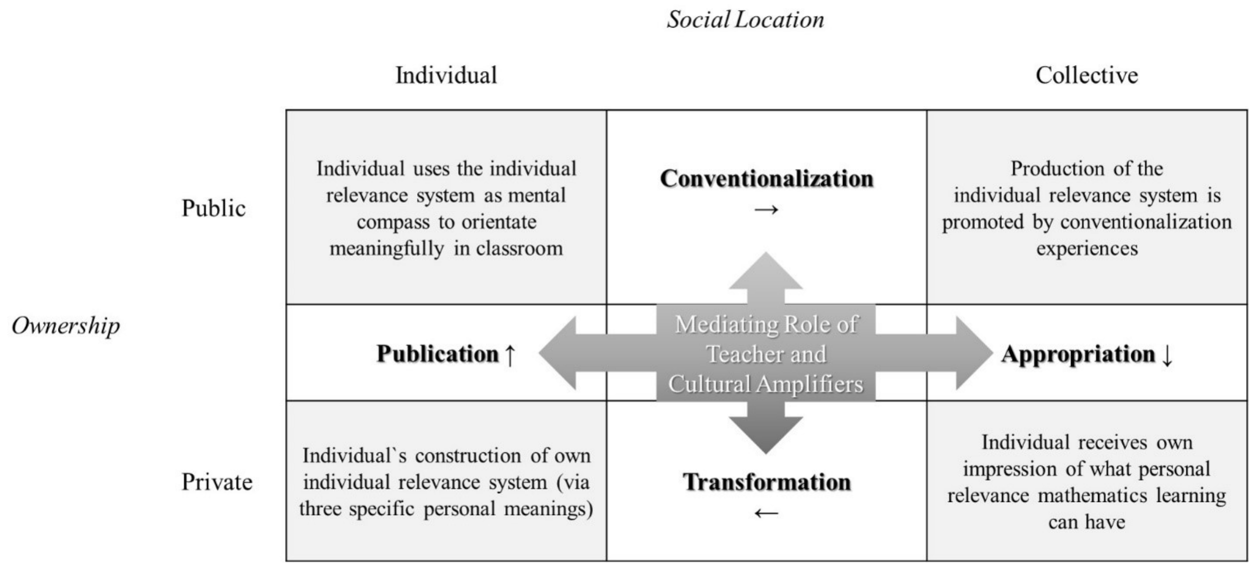
To conclude, contrasting the results from Germany and Finland by referring to learners' cultural setting in the classroom implied that the role of teacher is interpreted differently in German and Finnish schools. German students' relevance systems basically focus on performance-related learning orientations, whereas Finnish relevance systems in mathematics emphasize learning orientations that are inclined towards the individual learner. The results further show that in the German educational system, the teacher is seen as the personification of the subject mathematics and tends to emphasize the importance of academic performance—possibly mediated by the so-called *performance standards* within the German mathematics curriculum (German expression: *Leistungsstandards*; Blum, 2006, p. 15; Heymann, 2005). In particular, the consistent appearance of EXPERIENCE OF COMPETENCE in all profiles in Germany highlights the importance of academic performance in the mathematics classroom. The Finnish students do not associate the external pressure that is placed on them with the teacher; rather, they see the teacher as being on their side. In Finland, external pressure in mathematics seems to relate more to the educational system.

Interestingly, these theory-driven analyses (investigating learners' social experiences) not only underpin and strengthen the empirical results that merged from the quantitative analyses, which pointed out that the classroom settings in mathematics vary greatly across countries; also, the results according to the coordination analysis indicated that analogously to the theoretical considerations (cf. 2.4), different foci within the mathematics curricula exist, for example academic performance and mathematical performance standards (GER; Blum, 2006, p. 15; Heymann, 2005; KMK, 2005) versus the equal support of academic and personal development (Finnish National Board of Education, 2016, p. 402). Like the empirical results (cf. 5.2), these findings also confirm the PISA results in which Finnish learners showed a lower level of anxiety about their mathematics performance in comparison to German learners (OECD, 2015, p. 2).

The mediating role of the teacher and the cultural context of the mathematics curricula seem to clarify why Finnish students show consistently higher levels of self-determined regulated behaviour in the mathematics classroom than is seen in German profiles. Apparently, the teacher mediates in the circular processes of the social regulatory mechanism and thus the teacher assists the process during all stages of social regulation, that is, appropriation, transformation, publication, and processes of conventionalization. To provide a holistic explanation of the learner's social regulatory mechanism, these mediators—the teacher's mediation activities and cultural amplifiers that reshape the learner's social regulation—should be included within the adapted model (see Figure 26).

Figure 26

Revised Overview: Learner's Social Regulatory Mechanism of the Individual Relevance System—Affected by the Mediating Role of the Teacher and Cultural Amplifiers.



Note. The empirical results of this study support the existence of latent mediators affecting learners' social regulations in the classroom. To provide a holistic explanation, teacher's mediation activities and cultural amplifiers are included. Cycle adapted from Ernest (2010).

7.4.3 Conclusions on the Effects Found in German and Finnish Mathematics-Learning Orientations

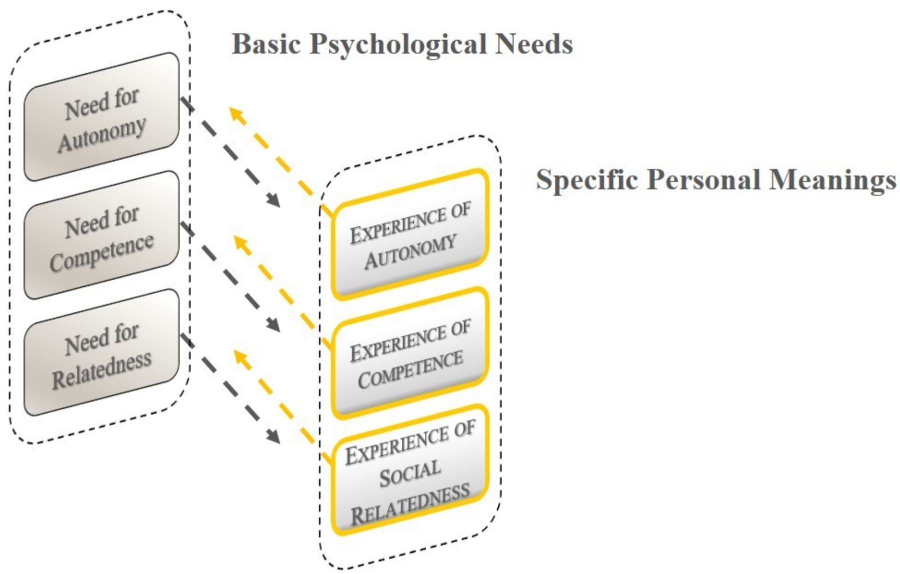
The coordination analysis came up with interesting insights that support the consideration of personal meaning as located at the boundary of SDT and social constructivism. From the perspective of social constructivism, the hypothesized adapted model, social regulatory mechanism of the individual relevance system, was not in conflict with the empirical profiles but did not provide an entire view of learners' motivational propensity. By means of this adapted model, the learner's whole relevance system can be understood as their experience of conventionalization processes, which thus emerges from the regulations of appropriation and transformation mediated by the teacher and is reshaped by cultural amplifiers.

Based on the empirical profiles across countries, it seems that the construction of the three specific personal meanings (EXPERIENCE OF AUTONOMY, EXPERIENCE OF COMPETENCE, and EXPERIENCE OF SOCIAL RELATEDNESS) in the social processes of appropriation and transformation increases the personal relevance of mathematics and regulates the publication of the learner's individual relevance system. In these regulations of appropriation and transformation, the three specific personal meanings play an essential role, as they seem to address the corresponding basic psychological needs.

The empirical evidence suggests that basic psychological needs can be stimulated by the construction (or fail to be stimulated in the case of non-construction) of the three specific personal meanings EXPERIENCE OF AUTONOMY, COMPETENCE, and SOCIAL RELATEDNESS through the learner's reaction to their social environment in the mathematics classroom. This in turn satisfies the respective basic psychological needs and regulates the learner's psychological well-being towards fulfilment (Deci & Ryan, 2000). The basic psychological needs, thus, wander into the regulations of appropriation and transformation and promote or restrict the process in view of their fulfilment. As a result of this (non-) satisfaction and in connection with significant others, in the processes of conventionalization, profiles are individually differentiated through strengths and weaknesses and emerge as patterns of personal meaning that are in line with the individual. Thus, the satisfaction of the respective basic needs seems to retroact to the learner's social space in class. Among the three specific personal meanings, the EXPERIENCE OF AUTONOMY has a crucial role as it seems to refer to the basic psychological need of autonomy, which is essential to experience deep-rooted personal importance and to put one's heart into learning situations in mathematics. In this way, the particular realization of EXPERIENCE OF AUTONOMY strengthens the relevance of both individual and mathematics-related personal meanings. If the learner's EXPERIENCE OF AUTONOMY is unrealized, they construct substitute personal meanings that weaken the relevance of both individual and mathematics-related personal meanings. On the basis of this *mutual relevance reaction* to the context of mathematics learning, the profiles of personal meaning can be explained not only by means of the adapted model (through the lens of social constructivism), but also through the theoretical lens of SDT that describes profiles' quality of motivation. In these profiles the three specific personal meanings show the same reaction, which are a result of the satisfaction of the respective basic psychological need, or the basic psychological needs mechanisms. As predicted, these three meanings, EXPERIENCE OF AUTONOMY, COMPETENCE, and SOCIAL RELATEDNESS, with their specific natures, seem to be more closely related to the basic psychological needs than to other personal meanings. The effects found in both independent data sets, explained by two different epistemic views, strongly support this assumption.

Figure 27

Inner Feedback Mechanism Between Basic Psychological Needs and Specific Personal Meanings.



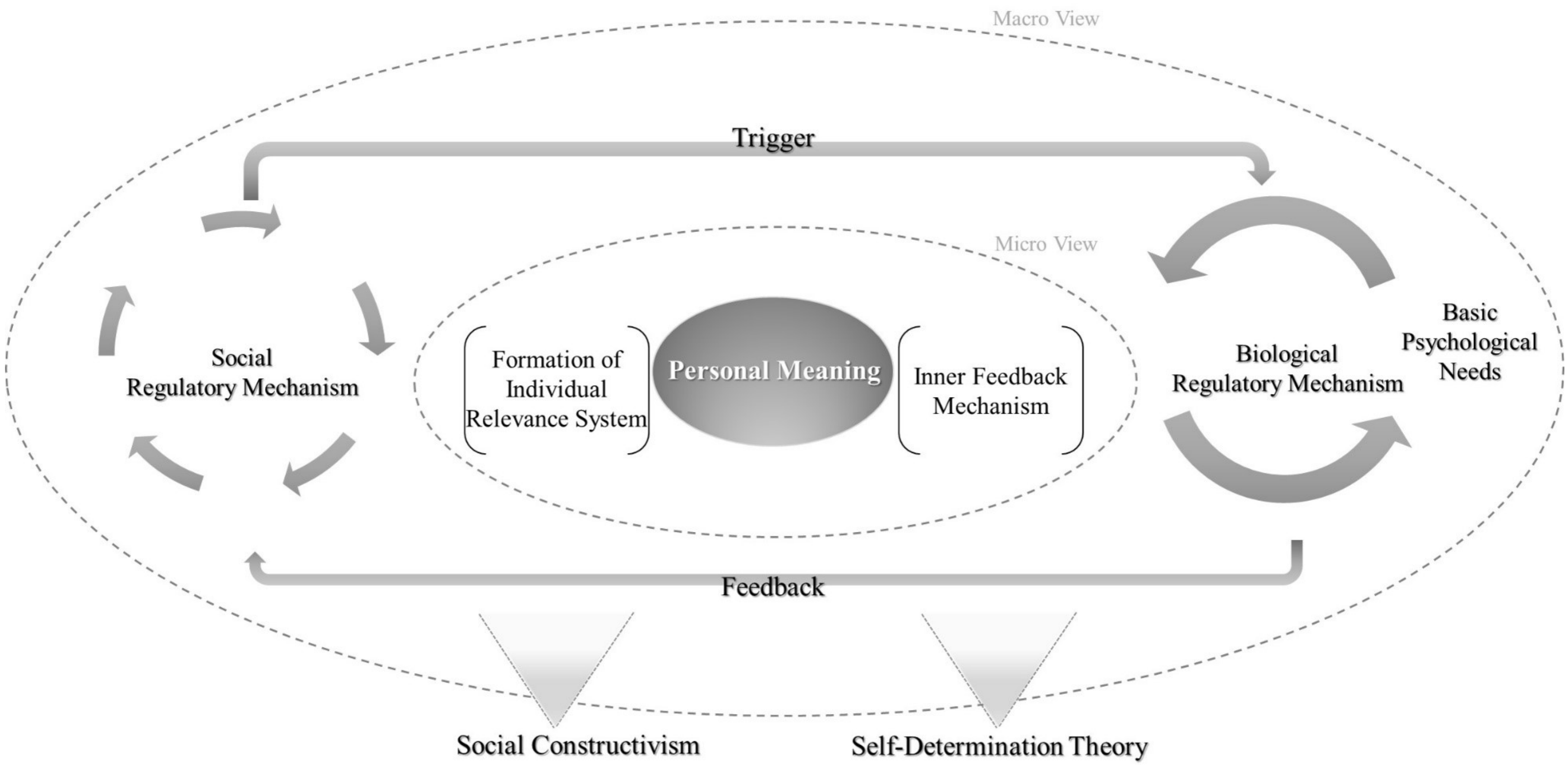
On a *micro level*, these effects indicate an inner feedback (see Figure 27) between the three specific personal meanings and the basic psychological needs that increases the learner's response regarding the personal relevance of learning mathematics. Not only does this particular inner feedback have an effect on the kind of relevance system the learner constructs through social interaction in the classroom setting, but it is also an essential component of the student's motivational behaviour - as the fulfilment of the innate basic psychological needs not only supports the individual's well-being, but also facilitates self-determined types of motivation (Deci & Ryan, 1993; Ryan & Deci, 2020).The published profiles in Germany and Finland highlight through both theoretical lenses the essential coherence between the three specific personal meanings and the basic psychological needs. Accordingly, a coordination between SDT and social constructivism provides a meaningful clarification of the content (or "what") of "why" the learner's motivational expression in mathematics lessons comes about.

7.4.4 Model of the Feedback Mechanism: The Genesis of Motivation in the Mathematics Classroom

The present research aimed to understand how the predicted biological regulatory mechanism (SDT) are coordinated with social regulatory mechanism (social constructivism) within the boundary object of personal meaning in light of the development of motivation in German and Finnish mathematics classrooms. As a response to this study's basic assumption, the effects found in the empirical evidence can be condensed into a hypothetical model that describes the development of the learner's quality of motivation in the mathematics classroom on two levels; namely, on a micro and on a macro level (see Figure 28).

Figure 28

Hypothetical Model of the Feedback Mechanism Between Social and Biological Regulatory Mechanisms to Explain the Genesis of Motivation in the Mathematics Classroom.



According to this coordination, from an epistemological view (*micro level*), the boundary object personal meaning mediates between the learner's outer and inner understanding. The link between SDT and personal meaning is the *inner feedback mechanism* (satisfaction of basic psychological needs) and the connection between social constructivism and personal meaning is the formation of the learner's *individual relevance system* (social experiences in the mathematics classroom). On the one hand, the learner's social regulatory mechanism regulates the construction of the individual relevance system. On the other hand, the biological regulatory mechanism regulates the learner's motivation. The learner's social regulatory mechanism seems to trigger their biological regulatory mechanism. This in turn seems to feedforward, on the basis of the inner feedback mechanism, a feedback (reaction) to their social regulatory mechanism. So, they both have a natural effect, and these two effects meet within the boundary object of personal meaning.

Social regulations act on the individual and the individual reacts with biological regulations to what is offered within the classroom's social environment. In this process, the profile of personal meaning arises through repetitive processes that take place in mathematics class, and which intensify and produce a reinforcing profile pattern; that is, an individual relevance system is produced by the increasing number of regulations between the inner and outer worlds.

One can say that the dominant role is that of the biological regulatory mechanism. However, analogous to the butterfly effect (Ghys, 2015; Lorenz, 1995, pp. 181–184), the subtle differences in how the profiles develop seem to be socially regulated (affected by the mediating role of the teacher and cultural amplifiers). To conclude, the social direction seems to *trigger* and the biological seems to give *feedback*, and this takes place through the processes of personal meaning. Accordingly, personal meaning can be interpreted twice ontologically and twice epistemologically, and therefore it is of central importance in light of the learner's quality of motivation in mathematics class.

From an ontological viewpoint (*macro view*): On the left-hand side of Figure 28 there is the social regulatory mechanism, which focuses on the individual and the individual-social environment ("conversation", Ernest, 1998). Accordingly, there are the individual and social levels accompanied by the role of culture and the teacher's mediating activities in the mathematics classroom. The biological regulation (right-hand side of Figure 28) includes the basic psychological needs. In line with SDT, if certain basic psychological needs are not satisfied, this will be visible in the well-being and health of the person. This aspect links the psychological level and the physiological level. Accordingly, there are the individual and biological levels. Through this linkage of the social and biological regulatory mechanisms, the social, psychological, and biological levels are addressed. So actually, all three levels, social, psychological, and biological levels, are interwoven when looking at the genesis of quality of motivation in mathematics class. Through this interdependence, we cannot see the social and biological regulatory mechanisms independently of each other.

The proposed feedback mechanism model condenses the overall effects found in this investigation's coordination analysis to explain the genesis of quality of motivation in the mathematics classroom. The described inner mechanisms within the hypothetical model were not directly observable, but their effects were visible in different individual relevance systems from Germany and Finland and could be explained by means of different theoretical lenses. These effects are not in conflict with the hypothesized model, but the model is not able to empirically illustrate the model's directionality, that is, the intensity of trigger or feedback effects.

8. Discussion

In this dissertation, I have examined the conceptual relationship between learners' individually constructed relevance systems (patterns of personal meaning) and the quality of their motivation in German and Finnish mathematics classrooms. Grounded on learners' empirical individual relevance systems, the specific factors within learners' inner biological regulations were examined through the lens of self-determination theory (SDT) to understand the corresponding motivational behaviour of each profile. By means of the lens of social constructivism, specific outer social regulations within the classroom context could be theoretically reflected within learners' inner biological regulations. Based on learners' individual experiences, the complementary perspective on their social experiences should help to increase our knowledge of teaching mathematics in the classroom in terms of possible external endeavour that addresses specific individual relevance systems which associate with autonomous motivation. At the same time, implications should be derived which aim to avoid the individual relevance systems that associate with controlled motivation.

In this chapter, I first present the main findings (see section 8.1) of this dissertation by summarizing the conceptual interconnections between learners' individual relevance systems (social regulatory mechanisms) and motivational behaviour (biological regulatory mechanisms), which theoretically explain the development of learners' motivation in the mathematics classroom. Secondly, to emphasize their generalizability and scope, I classify the main findings of this dissertation within the relevant theories of this work by relating them to existing knowledge; that is, within the landscape of mathematics-related affect research (see section 8.2) and SDTs research landscape (see section 8.3). Thirdly, I discuss the applied theoretical and the methodological approaches from a retrospective viewpoint (see section 8.4) to point out open-ended questions and suggest, from a prospective viewpoint, implications for future research, teaching mathematics in the classroom, and educational policy (see section 8.5). The final section of this chapter highlights the concluding remarks of this dissertation (see section 8.6).

8.1 Main Findings

The main findings within the empirical evidence of this study indicate that between the individual relevance system and motivation, a unique conceptual relationship exists that has an important role in students' mathematics learning in the classroom. In the following, I summarize the main findings of this dissertation: Learners' individual relevance systems found in Germany (GER) and Finland (FIN) can be differentiated into four kinds of mathematics-learning orientations in view of

1. Emotional-social integration—learners' starting internalization (either balanced (GER) or autonomous (FIN) motivation);
2. Enjoyment in mathematics learning—learners experience well-being and integration (autonomous motivation);
3. Self-improvement—learners feel freedom of action (autonomous motivation); and

4. External pressure—learners adapt to obligatory requirements (either balanced (FIN) or less controlled (GER) motivation).

- From the perspective of SDT, personal meaning is an effect of a basic psychological needs mechanisms, namely a biological regulatory mechanism.

- From the perspective of social constructivism, personal meaning results from interpersonal experiences in the classroom, namely a social regulatory mechanism.

- The biological and social regulatory mechanisms are coordinated within personal meaning, and the motivational construct of personal meaning links both regulations and can be interpreted as a boundary object through both theoretical lenses.

- Indicators within the empirical evidence support the finding that basic psychological needs can be stimulated by the construction (or fail to be stimulated in the event of non-construction) of the three specific personal meanings EXPERIENCE OF AUTONOMY, COMPETENCE and SOCIAL RELATEDNESS through the learner's reaction to their social environment in the mathematics classroom, termed as an inner feedback mechanism.
  1. Through this inner feedback mechanism, the biological regulatory mechanism regulates the learner's self-determination and well-being in the classroom (for autonomous to controlled motivation).
  2. Through this inner feedback mechanism, in the classroom setting the social regulatory mechanism regulates the direction of the learner's kind of individual relevance that is produced in line with the biological regulation (geared towards or away from the relevance in individual and mathematics-related personal meanings).
  3. The mediating role of the teacher and cultural amplifiers reshape the learner's social regulatory mechanism and this in turn explains why similar individual relevance systems result in different biological regulations across countries.

- Accordingly, basic psychological needs are the factors within personal meaning that have a strengthening or weakening effect on biological and social regulatory mechanisms.

- The effects (not directly observed, but not in conflict with the hypothesized model) found within the coordination of biological and social regulatory mechanisms can be modelled as a hypothetical feedback mechanism to explain the genesis of motivation in the mathematics classroom.
  1. The social regulatory mechanism triggers the biological regulatory mechanism and that in turn seems to feedforward to the social regulatory mechanism.
  2. Through this conceptual interrelation, one cannot regard the biological and social regulatory mechanisms as independent of each other.
  3. Within this conceptual interrelation, the dominant role is determined by the biological regulatory mechanism, but how the individual relevance system develops in terms of its subtle differences from other similar systems is socially regulated.

- To conclude, the boundary object personal meaning can be interpreted twice, ontologically (biological and social nature) as well as epistemologically (motivational behaviour or social interaction), and as a result of its multimodal ontological nature it consequently has major significance for the genesis of motivation in the mathematics classroom.

8.2 Complementing References to the Field of Mathematics-Related Affect

8.2.1 Networking Theories to Counter the Terminological Ambiguity

As indicated earlier (cf. 4.1.1), in qualitative empirical case studies the tradition of linking theoretical lenses is particularly widespread, but it is rarely realized in quantitative empirical investigations (Prediger et al., 2008). Connections between theories can take place at the level of principles, methodologies, and of questions, or by combining these elements (Radford, 2008a).

In this study, which belongs to the quantitative field, I considered the connections between theories on the level of principles (Radford, 2008b). The networking of theories approach sharpened the view of both theories' alignment (SDT and social constructivism) by means of coordinating the complementary analyses, that is, of the development of the individual relevance system and of the way this learning orientation is processed as motivational behaviour in the mathematics classroom. The applied theoretical knowledge (conceptualized through the principles of the theories) helped to view motivation from different angles by dissecting focus-relevant questions with respect to the role of cultural imprints. In particular, the consideration of the German and Finnish data helped to contrast and refine the relational understanding between learners' individual relevance systems and quality of motivation. This decision challenged the principles of both theories, SDT and social constructivism, in terms of their descriptive and explanatory power (Schoenfeld, 2002). This challenge led to working out (on a theoretical level) the mediating factors affecting the social regulations of the learners.

Although the results have certain limitations (see section 8.4), the above-mentioned decisions about the way this study was conducted, given the challenges associated with the need to take into account two independent data sets from different cultures and to target the selection of both theories, led to findings that supported the explanatory power (Schoenfeld, 2002) of the hypothetical model to a certain degree: The joint results (R as the fourth element; Radford, 2012) of SDT and social constructivism, obtained from this dissertation's networking practice, influenced and complemented each other at the same time. The results show that a one-sided view, either from the perspective of SDT or of social constructivism, of learners' motivation would have led to limited knowledge. Generated on the basis of a sequential-dependent methodology (M), both theoretical lenses (P) gave results that shed light on complementary insights answering common questions (Q) in view of learners' mathematics-related motivational behaviour in the classroom: Substitute personal meanings or less mathematics-/individual-related meanings can be avoided by realization of the three specific personal meanings and teachers' mediation activities reshaped by cultural amplifiers. This in turn strengthens the learners' individual and mathematics-related learning orientation (*what*), and that explains *why* a self-determined motivational expression in mathematics lessons comes about. Bringing both lenses together by means of coordination, the thus-built conceptual framework creates a locally integrated conceptual tool (R; Prediger et al., 2008, p. 166) that works together on the same research problem by jointly complementing the resulting picture, thereby improving their new concept's (hypothetical model of a feedback mechanism, Figure 28) *explanatory power* as well as both theories' *descriptive power* (Schoenfeld, 2002). Accordingly, one can see that SDT and social constructivism are nicely aligned when used to investigate the role of personal meaning in learners' motivation.

This networking-of-theories approach also seems to be a helpful strategy with which to tackle the problem of terminological ambiguity in the definitions of motivational constructs (Hannula, 2012) in a sustainable way. Even though the coordination analyses, which follow the tradition of linking theoretical lenses, lead to a feedback mechanism that is hypothetical in nature, this contribution, creating a dialogue between constructs, establishes a fruitful research practice to counter the terminological ambiguity in mathematics-related affect research (Hannula, 2012; Suriakumaran et al., 2020) by elaborating the connections between the affective constructs. In fact, the present research approach helped to increase our knowledge as regards the theoretical conceptualization of personal meaning and to enrich our understanding concerning the interrelation between personal meaning and quality of motivation. However, although we have found empirical evidence for learners' patterns of personal meaning in two different cultural contexts (Vollstedt, 2011b; Vorhölter, 2009), all kinds of personal meaning could not be assessed within one measurement model (cf. 6.1). The conducted correlation analyses support an empirical link between the socially experienced personal meanings and the biologically perceived quality of motivation. Building on the insights gained so far, we could focus within future research on specific social conditions in the classrooms (e.g., the teacher's mediating role and the foci within mathematics curricula; see 8.5 for more), which seem to affect our learners' biological regulatory mechanisms. However, future research needs to find a solution that can methodologically estimate all factors (personal meanings) within one comprehensive model in order to move to the next level; for example, this might involve investigating the causal effects between personal meaning and motivation.

In summary, this research approach has not only contributed to addressing the problem of terminological ambiguity; also, the present investigation's application of networking theories intends to encourage the practice of "networking" with other fields from mathematics education to broaden scholars' horizons with regard to fruitful research strategies that could advance our research within the field of mathematics-related affect.

8.2.2 References to Meta-Theoretical Knowledge

Within the frame of this quantitative study, relevant indicators referring to the meta-theoretical knowledge (cf. 3.2.3) of this present study's research interest emerged in the evidence; this meta-theoretical knowledge is discussed in the following.

Relation to Learners' Interest in Mathematics. As indicated earlier (cf. 2.2.2), the construction of personal meanings is particularly influenced by various factors from learners' personal characteristics. Some of the factors may also come to the fore as consequences of a socially experienced individual relevance system (Vollstedt, 2011b; Vorhölter, 2009). Although various concepts were considered as influencing factors during the coding process (see Vollstedt, 2011b), the three basic psychological needs, beliefs, and interest became relevant concepts (in the sense of Grounded Theory) for the construction of the learner's personal meaning (Vollstedt, 2011b; Vorhölter, 2009).

In this connection, the identified profiles of personal meaning in Germany and Finland indicate connections to the affective domain-related concept, namely *interest as a person–object relationship* (Krapp, 1992, 2002). Learners who are oriented towards "enjoyment" (Profile 2–GER and Profile 2–FIN) and "self-improvement with a focus on the individual" (Profile 4–FIN) seem to be interested in mathematics as a scientific discipline. In all these three profiles, learners specifically perceived the two specific personal meanings EXPERIENCE OF AUTONOMY and EXPERIENCE OF COMPETENCE and constructed the personal meaning "purism of mathematics". Referring to the inner feedback mechanism (cf. Figure 27), the learning situations' conditions thus seem to satisfy the basic psychological needs for autonomy and competence that in turn nurture the development and maintenance of intrinsic motivation. In these three cases (Profile 2–GER, Profile 2–FIN as well as Profile 4–FIN), in line with Bikner-Ahsbahs (2005, 2014), these results underline that the perception of *meaningful* mathematics lessons and active *involvement* (Mitchell, 1993) in activities (here through the EXPERIENCE OF AUTONOMY) can encourage their (situational) interest.

By contrast, in Profiles 1 in Germany and in Finland, learners are involved in the subject specifically with regard to social integration with the teacher and work *for* the teacher; they seem not to be interested in the subject of mathematics as a scientific discipline. However, as these students are on their way to becoming self-determined (starting internalization), the learning orientation towards emotional-social integration has a *fertile soil* in which learners' situational interest can be supported by offering them opportunities over time to benefit from EXPERIENCE OF AUTONOMY. Accordingly, the mathematics-learning orientations detected offer pointers for how lessons should be designed (see 8.5.2 for more) in order to promote interest and intrinsic motivation, conveyed through personal meaning.

Learners' Gender Performance Characteristics. Next to the references to related affective concepts, implicit knowledge about further characteristics of personal meaning emerged, namely gender and performance. Whereas the Profiles 1 (GER and FIN) of "emotional-social integration" did not display a noticeable difference in gender, they showed differences with respect to learners' performance. In Finland this profile is mostly preferred by high-achieving students, and in Germany most average students belong to this profile. Moreover, in both countries the learning orientation of "enjoyment" was mostly perceived by high-achieving students (in Germany it was also preferred by many average students). However, in Germany the proportion of male students is higher in this profile, while in Finland it was the other way around; that is, more female students engaged in learning through this orientation. Similar results were exhibited in profiles of "self-improvement"; in both countries more male students engaged in mathematics for self-improvement than female learners. As a profile this shows connection to the affective concept of interest; these results are in line with findings from the literature that state that male students are more interested in mathematics than girls are (Eccles et al., 1983; Watt, 2004). In both countries, this learning orientation was perceived mostly in high-achieving students, whereas in Germany this profile is also experienced by average students. And finally, many high achievers and some average learners (in the case of Germany, some high achievers and many average students) engage in mathematics due to "external pressure". Whereas in Finland this orientation is mostly found in boys, in Germany there was no noticeable difference in gender.

Altogether, there is no strong trend regarding gender performance properties and profile orientations (which were also not tested in terms of their statistical significance), but it also becomes clear that in this context, cultural differences exist that could be clarified by having a closer look at the different classroom settings. If this is done, I recommend that these characteristics be discussed through a "fresh perspective" in future with languages that respects the individual's gender identity (American Psychological Association, 2020) and referring to all individuals as human beings first, i.e., avoiding the more common, dichotomous way of talking about gender.

State- or Trait-Like Characteristics. The discussion on the *state*- or *trait*-like properties of an affective concept has been an ongoing discourse within mathematics-related affect research (Hannula, 2012). Briefly, traits refer to relatively stable personality traits, but states of affectivity can vary over time or situationally (Kelava & Schermelleh-Engel, 2012). Referring to the state- or trait-like characteristics of personal meaning, published individual relevance systems represent an outcome after a series of experiences from the adapted cycle (cf. Figure 26). Accordingly, the realization of the three specific personal meanings that strengthen or weaken the individual- and mathematics-related personal meanings is based on multiple cycles in which the learner experiences their social interaction in the classroom. So far, there has been no clarification of the trait- or state-like properties of personal meaning. However, the conceptual relation between personal meaning and motivation implies some interesting clues with respect to this issue.

Although individual relevance systems are based on a series of experiences, the different kinds of learning orientations (emotional-social integration, enjoyment in mathematics learning, self-improvement, and external pressure) and the encompassed personal meanings can be tentatively related to state or trait. As indicated earlier (cf. 2.1.2), autonomous regulatory styles such as identified, integrated, and intrinsic regulation associate with persistence, whereas controlled regulatory styles do not. Furthermore, externally regulated students comply with expectations as long as the contingency lasts (Ryan & Deci, 2017). That is, in the absence of the external expectations in the classroom that constrain the conduct of students, those students would not exhibit consistently "good behaviour" in the long term and would show little engagement in their learning tasks, eventually ceasing to participate in them altogether (Ryan & Deci, 2017). Following this background knowledge, the thus-constructed kinds/dimensions of personal meaning and the profiles seem to have a more trait-like character (see Hannula, 2012) when they refer more to the subject of mathematics, the individual, and autonomous motivation (cf. 5.1 and 5.3), rather than to kinds/dimensions of personal meaning and the individual relevance systems that are associated with controlled motivation. The latter might have more state-like characteristics. Anyway, these relations provide limited cues; proper methodological procedures are required in order to dig deeper into this issue and to find solid answers.

8.2.3 The Body as an Essential Element of Mathematical Thinking

This dissertation's main findings can be condensed into a model which theoretically explains the genesis of motivation in the mathematics classroom as a function of learners' individual relevance systems. As indicated earlier (cf. 7.4.4), from an ontological viewpoint, this conceptual interrelation between personal meaning and quality of motivation can be theorized through the embodied, psychological, and social perspectives. Thus, all three perspectives seem to be interwoven when theorizing the genesis of motivation in the mathematics classroom. Besides the conceptual links between both constructs' central elements (cf. 2.3), the potential to theorize the hypothetical feedback mechanism from three different levels might be a further reason why a strong conceptual interrelation between personal meaning and motivation exists.

Hence, the knowledge in this model is not only underpinned by Hannula's (2012) meta-cube, which proposes to theorize affective concepts like quality of motivation or personal meaning through physiological, psychological, and social lenses. In line with Vygotsky (1962, 1987), this feedback mechanism also highlights why the emphasis on the monist perspective on mind and body, in which it is stated that affect "is never external to intellect" is important (Roth & Walshaw, 2019, p. 112; see also Hannula, 2012). Because, in accordance with Hickey (1997), this study's findings emphasize that motivation as an affective construct is not separable from cognitive activities when learning.

These important connections point out that the "corpus" of mathematics learning can neither be separated from body-related functions nor restricted to cognitive processes as an "unemotional activity" (Drodge & Reid, 2000, p. 249). In fact, this specific discussion on the study's findings goes beyond frontiers where mathematics learning or mathematical activities are considered as *embodied*, and as such I adhere to the research that considers and conceptualizes mathematical thinking as based on an interdependence of embodied, psychological, and social phenomena (de Freitas & Sinclair, 2014; Ferrari & Ferrara, 2020; Hannula, 2012; Roth & Walshaw, 2019).

8.3 Complementing References to SDT's Research Landscape

At the beginning of this thesis (cf. 1), it was stated that students can also be involved positively with mathematical activities through a learning environment that supports basic psychological needs and that highlights the personal relevance of mathematics for the student (more individually valued than socially, Ryan & Deci, 2017, 2020; Vansteenkiste et al., 2018). The knowledge gathered from the present study research can now clarify on a conceptual level why learners' personal relevance relates in particular to autonomous motivation when it refers more to the individual. It also shows why the consideration of all kinds of personal relevance (regardless of their relation to more or less internalized forms of motivation) was important to helping us holistically understand the conceptual interrelations between personal relevance and motivation in the mathematics classroom.

As indicated earlier, the learner's personal meaning is an effect of a basic psychological needs mechanisms; namely, a biological regulatory mechanism. Furthermore, the basic psychological needs are the factors within personal meaning which have a strengthening or weakening effect on biological and social regulatory mechanisms. Accordingly, personal meanings with respect to the relevance in individual and mathematics-related personal meanings are in line with the learner's satisfaction of basic psychological needs, which in turn associate with autonomous forms of motivation. This aspect clarifies why the kinds of personal relevance that are close to the learner as an individual are associated with autonomous (extrinsic) regulatory styles (Vansteenkiste et al., 2018).

Moreover, Vansteenkiste et al. (2018) stated that the notion of personal relevance can only associate with autonomous forms of motivation. From the present knowledge, a differentiated consideration would take into account that learners' personal relevance can associate either with autonomous or controlled forms in line with the fulfilment of basic needs, and this depends on whether personal meanings are aligned with or diverge from the individual's personal relevance of mathematics (in an academic context). Based on these insights, the consideration of personal meanings which are less close to the individual or to mathematics was important to studying the complex associations of personal meaning with the various behavioural outcomes of self-determined and controlled types of motivation. In particular, the use of the composite score SDI (Levesque et al., 2004; Ryan & Deci, 2017, p. 195, 2020) was helpful to reflect learners' perceived relative autonomy in addition to the examination of the regulatory styles.

Altogether, these results from Germany and Finland underpin that basic psychological needs are not only universal (Vansteenkiste et al., 2020), but also significant factors in learners' *need for meaning* (Vinner, 2007) and thus are an essential constituent of their mathematics education. Moreover, Ryan and Deci (2017) discussed the notion of *meaning* on a more general level (e.g., meaning in life), and considered meaning to be a resulting effect of the satisfaction of basic psychological needs. Although both concepts, meaning and the learner's personal relevance in terms of studying mathematics, are not identical, the results of this dissertation indicate that Ryan and Deci's suggestion can be applied to the learner's mathematics-related personal meaning.

As indicated earlier, in comparison to other forms of autonomous motivation (intrinsic or integrated regulation), teachers' external endeavours can address identified regulation by encouraging students to understand (e.g.,) the personal relevance of mathematics learning for them. For this reason, it was a matter of particular interest with which individual relevance systems learners associate identified regulation. In Finland all profiles associated with particularly identified regulation, whereas in Germany all profiles apart from Profile 4, "external pressure," displayed identified regulation. Although Profile 3 in Finland was also oriented towards "external pressure", they still identified with the importance (Deci & Ryan, 2000) of mathematics learning. As mentioned earlier, Haerens et al. (2016) advocated that teachers should refrain from using classroom tactics of exerting pressure and control when students' conduct does not conform to their expectations, as this can cause frustration of their basic psychological needs. It seems that the degree to which the learner felt obligation towards the teacher and adapted themselves to the teacher's performance requirements (such as in Profile 4 in GER) influences the learner's (less) controlled behaviour.

Referring to the dimensions of personal meaning, in both cultural contexts the dimension of professional qualification and application in life associated highly positively with identified regulation. Accordingly, particularly when learners are not interested in the subject, highlighting the personal relevance learners attach to participating in the social environment and valuing mathematics knowledge as part of their identity, as well as understanding why mathematics will be important for their future profession—or further education—helps learners to actively internalize extrinsic content as part of their identities. To conclude, a focus on the students' relevance, or rationales that are student-centred and support the experience of autonomy, as well as autonomy-supportive teaching styles that foster a positive teacher–student relationship, contribute to students' positive learning processes and well-being.

While studies within SDT have identified motivational profiles in order to explore configurations of behavioural regulations in mathematics education (e.g., Wang et al., 2017), in this dissertation I have focused both on learners' individual mental compasses, which provide them with their orientations towards studying mathematics, and on the respective motivational behaviour they exhibit in the classroom. Considering learners' inner and outer worlds helped to move the analysis to the next level and focus on the interconnection between the learners' individual relevance systems and their behavioural regulations.

8.4 Retrospection

With regard to the general research interest in the genesis of motivation in the mathematics classroom, the given discussion has emphasized that several factors support the evidence gained about the hypothetical model of a feedback mechanism in the social and biological regulatory mechanisms: the effects can be understood through the joint lens that emerged by means of coordinating SDT and social constructivism, and the evidence behind the hypothetical model could be replicated, validated, and refined on the basis of two independent data sets, namely those from Germany and Finland. Furthermore, we were able to increase our knowledge regarding personal meaning's conceptualization and in this relation the power and range of SDT's basic psychological needs. The evidence of this dissertation fits into the renewed debate about the role of the body in learning and meta-theoretical links could be found between the identified empirical profiles and existing knowledge. Despite this support for the evidence of this present research, there exist several limitations that lead to certain open-ended questions.

8.4.1 Limitations of the Theoretical Approach

Reflection on Mathematics Curricula. With regard to the two mathematics curricula, there is no uniform curriculum in Germany. Since the German data set comes from various federal states (from west, north, and south), I looked at the overall cultural setting of many different school systems that are shaped by the KMK[40] and their educational standards (KMK, 2005) or, as Blum (2006) and Heymann (2005) refer to them, their "performance standards."

In contrast to the federal system in Germany, there is an overall mathematics curriculum in Finland. Regarding the Finnish curriculum, in which self-determination is explicitly promoted (e.g., in particular Profile 4, "self-improvement with a focus on the individual"), one can see that this mathematics curriculum has different effects compared to that of Germany, where such systematic promotion across all federal states simply does not exist—except for the standardization of the educational standards focusing on mathematics-related

[40] German: Kultusministerkonferenz–KMK

competencies. In Germany, these competence-related standards have to be implemented by everyone; that is, in all the respective federal states with their different educational systems and traditions. It is interesting that a uniform mathematics curriculum has the effect of fostering internalization, for example, students' readiness to work independently. In contrast, in the German federal system, which has a non-uniform mathematics curriculum which takes diversity of ability into account, this effect is relatively small.

What these differences effectively mean is that if the German context does not have these cultural *amplifiers or mediators*, we basically cannot expect to find similar profiles to those that are produced in Finland, because that would not work in a different cultural context. In terms of the mediators, the empirical evidence implied that learners' social regulations are affected by the cultural setting and mediating role of the teacher. Although classroom observations were not part of this research, based on learners' individual and interpersonal experiences it is thought that certain types of teacher behaviour in the classroom might support the construction of learners' individual relevance systems.

Reflection on the Teacher's Mediating Role. In particular, by discussing the teacher–learner interpersonal relation, the "model of sign appropriation and use" by Ernest (2010) describes how the zone of proximal development (ZPD, Vygotsky, 1978) may embrace the overall cycle that explains how an individual learns concept development though signs; that is, the ways in which learning and knowledge come to be realised (Ernest, 2010, p. 45). Ernest further stated that signs or texts are a kind of cultural representation (although there are others), and they appear in/are relevant to all four quadrants of the model. Since culture encompasses signs and texts, culture affects learning (Ernest, 2010).

Although the process of conventionalization and learners' social experiences in general in the various classrooms were not observed, application of the adapted model implied, in line with Ernest, that the cultural context affects learners' social experiences. Moreover, Ernest stated that the theoretical approach of social constructivism places emphasis (among other factors) on learner–teacher interactions (Ernest, 2010). The mediating role of the teacher also becomes a central component within the adapted model as it seems that the teacher triggers the profiles through specific behaviours. That is, within the different individual relevance systems across countries, the learners' points of view imply that they experience the various mediation activities of the teacher within the context of social interaction. Again, I have not observed the behaviour of teachers here, but using the data of specific profiles, we could predict which teacher behaviour would then support that profile, giving a spectrum of possible ways in which a teacher can act, and thus assumptions could be made about the ways in which a teacher supports different profiles in their actions and behaviour.

In this spirit, I briefly review teachers' mediation activities in terms of their effect on the process of internalization and benefits for mathematics learning. Generally, we can say that the existence of the eight profiles across countries implies that a teacher has more than one mediation activity and that mediations are not carried out by teachers in exactly the same way. They have to mediate very differently when students develop different profiles (e.g., within an inclusive mathematics classroom). The empirical profiles show implicit knowledge regarding teachers' mediation activities, which may take the form of an "emotional-social presence," caring for emotional-social integration in the class (Profile 1–GER, Profile 2–GER, Profile 1–FIN, and Profile 2–FIN); a "potential developer" with a versatile presence in class (Profiles 2–

GER and Profiles 2–FIN); a "self-improvement supporter", who, as the name suggests, supports self-improvement (Profiles 2–GER, Profile 3–GER, Profiles 2–FIN, and Profile 4–FIN); and finally, a "pressure supporter," who brings pressure to bear on students (Profile 4–GER). Almost all mediating roles were found across countries, apart from "pressure supporter", which only emerged in Profile 4–GER.

Taking into account the profiles' sample sizes, in both countries the profiles of emotional-social integration (Profile 1–GER and Profile 1–FIN) and enjoyment (Profile 2–GER and Profile 2–FIN) are preferred most. In light of middle adolescence's preference for an emotionally warm teacher–student relationship, the teacher supports learners' internalization in the mathematics class through an emotional-social presence. Since these profiles are the most preferred ones in all the groups, this specific mediation activity as an emotional-social presence supports learners during middle adolescence on the way to becoming self-determined human beings and thus has importance for their mathematics-learning processes as well as their personal development.

A prototype for acting in many ways was implied by profiles that were marked by "enjoyment in mathematics learning," these profiles emphasize how teachers serve many different relevance constructions in individual as well as social processes. Again, these profiles exist independently from cultural settings in both countries (having no culture-specific focus). Moreover, this learning orientation experienced all three specific personal meanings, and it is assumed that the social setting of these profiles fulfils all three basic psychological needs; this is because these learners identified with fun as well as relevance, and this directed them towards self-determined behaviour in the mathematics class. Accordingly, this profile, above all the others, seems excellent for teaching, and the activities of the teacher as a potential developer have a powerful effect in supporting learners' learning processes and autonomous motivation. Accordingly, this means that the teacher has to organize mediation in a variety of ways, as not all students are the same.

In conclusion, a teacher as a potential developer can support, from the learner's point of view, self-determined learning in mathematics if they act as a potential developer serving the roles of emotional-social presence and self-improvement supporter, but, importantly, *not* the role of pressure supporter. Of course, further research is necessary to underpin these limited insights. To summarize, the insights into learners' classroom experiences were not empirical and thus somewhat incomplete. Therefore, no profound knowledge can be gained about learners' experiences from their mathematics curricula and the teachers' mediating activities. Although implicit knowledge in this connection exists, we would have to observe these conventionalization processes, which affect the learners' development processes, from the perspective of the specific profiles in question.

8.4.2 Limitations of the Methodological Approach

To investigate the given ontological characteristics of the empirical phenomenon personal meaning, I carried out research activities following a sequential-dependent (Schoonenboom-Johnson, 2017) research design. Not only did the inclusion of these practices enriched the investigation by adding different methodological perspectives, but their integration also allowed a more detailed insight into the research topic and an increase in knowledge in terms of the *problematique*, work on the conceptual understanding between personal meaning and motivation in mathematics.

The mono-methodological quantitative research approach as well as the coordinating research approach carried out in this dissertation produced evidence supporting the linkage between the social and the individual (SDT and social constructivism) as well as the existence of cultural differences, potentially affected by the mediating role of the teacher and the educational system in both cultural settings. Based on the quantitative approach, support of the dimensionality of personal meaning initially highlighted an implicit theoretical link between socially experienced personal meaning and the regulatory styles (SDT). The empirical stability of the predicted dimensions of personal meaning not only supported studying the psychometric properties in a meaningful way; their emergence within the empirical profiles also helped to characterize students' mathematics-learning orientations and to work out the spectrum of possible forms of action, based on profile levels, of how a teacher can act to support different profiles in their relevance construction and behaviour.

The correlation analyses across countries not only explored the predicted connection between these meta-factors—orientation towards the regulatory styles and to the relatedness of mathematics—that is, a positive linear connection between (kinds/dimensions of) personal meaning and quality of motivation; they also showed that Finnish students associate their personal meanings more strongly with self-determined types of motivation than German learners do. Analogous theoretical consistency emerged by contrasting the latent mean structure in which the student's constructed personal relevance was in line with the country's motivational peculiarity. Furthermore, these valid group comparisons undergirded and somewhat explained the correlational structures detected across groups: The correlation pattern in Germany showed most associations with controlled regulatory styles and resulted similarly in a latent mean structure in which German students identified less with individual and mathematics-related personal relevance than Finnish learners did. The sequential-dependent methodology also added to the value of certain empirical results and provided an enhanced insight into their role through the strategy of coordinating. That is, regarding the role of the three specific personal meanings within learners' social interactions, there was no clear evidence within the empirical work that clarified the roles of these three specific meanings. The coordination analysis helped to point out their roles for learners' individual relevance systems and to understand the differences in learners' biological regulations by theoretically describing their social experiences in the classroom. However, the quantitative statistical analyses provided the empirical basis for understanding these results in more depth within the model of feedback mechanism built by coordinating the theories.

Accordingly, the additional value of integrating these research activities, which are motivated by ontological assumptions (Buchholtz, 2021), can be underlined through the findings within the hypothetical model of a feedback mechanism. Although the networking approach compensated for the weaknesses of the mono-methodological quantitative approach, the strategy of coordinating presented the author with its own challenges when it came to conduct a theory-driven interpretation procedure of numerical data. In this spirit, the adjusted terminologies "identify with," "not identify with," and "enacting potential" (originating within qualitative research paradigm; Bohnsack, 1989) not only helped to validate the computed profiles in terms of theoretical consistency, that is, judged by the lens of social constructivism;

they also provided a secure framework in which to carefully conduct the coordinating analysis of standardized data (qualifying the quantitative data for each evaluation step that was carried out), and to benefit from the knowledge gained in light of this dissertation's specific networking activity and of motivation-relevant articulation of both theories (Prediger et al., 2008).

Limitations of Investigation. In terms of the methodological issues that led to the theoretical limitations of this dissertation, despite the usefulness of the two independent data sets, it is important to note that this study could not consider each country equally. Whereas the reconstructed personal meanings concern the kinds of personal relevance German (and Hong Kongese) students assign to mathematics learning, Finnish learners' personal meanings that specifically address the Finnish cultural context are not included. Because of this limitation it was not possible to decode the "what" (certain personal meanings) that support Finnish learners' autonomy experiences in Profile 3–FIN "External pressure focusing on adaptation to the educational system." Although the students did not construct EXPERIENCE OF AUTONOMY, COMPETENCE, and SOCIAL RELATEDNESS within their social context, the motivational orientation only showed tendency for controlled motivation to be associated with having identified regulation. Accordingly, the students seem to construct personal meaning(s) that reflect a personal importance (identified regulation) in mathematics learning. So, which personal relevance associates with identified motivation in this profile? In view of supporting autonomously motivated learning, it would be interesting to explore what kind of relevance Finnish culture offers to foster students' basic needs satisfaction which is not included within the German classroom (e.g., Profile 4–GER) considered here.

Further, this study's profiles cannot easily claim validity for the whole country. For this reason, the results and evidence gained here cannot be generalized to the whole country; that is, the generalizability and representativeness (a relatively small sample size is given) are very limited. Moreover, the hypothetical model of a feedback mechanism (cf. 7.4.4) is only based on theoretical knowledge, albeit evidence-based. To understand to what extent the social and the biological regulatory mechanisms interact in respect of the genesis of learners' motivation, further investigation is necessary. Accordingly, no causal effects can be derived.

Concerning the evidence found in Germany and Finland, the results did not include any classroom observations about the processes of conventionalization. For that reason, the social cultural context was only reconstructed on the basis of the individual patterns of personal meaning that learners constructed. Additional empirical observations could have helped explore interpersonal conversations between teachers and students and teachers' mediation activities (in connection with teachers' need experiences) in depth and also to investigate the question of what specific learning situations support students' personal experiences when constructing the specific personal meanings of EXPERIENCE OF AUTONOMY, COMPETENCE, AND SOCIAL RELATEDNESS.

In summary, in this study's context, connecting theories in quantitative studies is helpful not only to solve practical problems in research analysis but also to broaden one's mind with respect to implicit knowledge behind applied theoretical approaches and pointing out the methodical benefits and technical obstacles for future research.

8.5 Prospects

8.5.1 Implications for Future Research

In order to increase the understanding of autonomy-supportive teaching in connection with students' individual relevance systems, the focus in subsequent research should be on integrating qualitative approaches. As already pointed out by Ryan and Deci (2020), more qualitative studies are necessary to obtain a detailed and full picture of learners' everyday classroom experiences.

Supporting this demand, also with regard to the findings of the present dissertation, evidence from qualitative research would help to increase and develop our understanding of the interplay between learners' individual relevance systems and quality of motivation in multiple ways: based on the linkage which was emphasized by the hypothetical feedback mechanism, future research should adopt theoretical perspectives which theorize the learner's experiences on three levels: social, psychological, and embodied (Hannula, 2012). In this connection, empirical observations within the classroom can shed light on learners' everyday experiences, specifically in order to understand how the identified four profiles' social experiences—grounded on teacher–student and student–student interpersonal activities—affect their construction of the specific personal meanings (EXPERIENCE OF AUTONOMY, COMPETENCE, and SOCIAL RELATEDNESS). These insights might additionally highlight in which situations the experiences fluctuate on an intra-individual level and what decisive factors direct the profile formation. Alongside these observations, to get a full picture, teachers' mediation activities and especially their basic-psychological-needs experiences should also be observed as a counterpart to learners' experiences within educational reality. Ryan and Deci (2020) stated that the satisfaction of teachers' basic psychological needs has an impact on their students' learning processes. These insights can be used to explore to what extent teachers' needs fulfilment/frustration affects their mediation activities and in turn learners' construction of their individual relevance systems. The knowledge gained could subsequently help to design interventions (as suggested by Vansteenkiste et al., 2018), not only in order for learners to experience autonomy support in the mathematics classroom but also to allow teachers to satisfy their own basic psychological needs within their teaching activities.

Furthermore, on the basis of this qualitative research evidence, the intensity between social ("intensity of trigger") and biological regulatory mechanisms ("intensity of feedback") could subsequently be quantitatively assessed, whereby the networking approach could be applied to a qualitative and quantitative database in order to refine the coordinated conceptual framework that explains the development of motivation in the mathematics classroom. Hence, future research needs not only to care about students' autonomous motivations, but also teachers' well-being in mathematics education in order to sustainably support learners' autonomous motivation in the mathematics classroom.

8.5.2 Implications for Teaching and Educational Policy

Autonomously Oriented Mathematics Learning in the Classroom. What do these profiles mean for teaching mathematics? Based on the evidence gained, certain perspectives can be used to support learners' autonomously motivated learning in the classroom. Of the 17 personal meanings, the three specific personal meanings (EXPERIENCE OF AUTONOMY, COMPETENCE, and SOCIAL RELATEDNESS) proved their power consistently in all profiles regardless of culture. Accordingly, teachers could create learning situations and impulses that support the experience of those three particular personal meanings. Thereby, even though it cannot be guaranteed that each learner (as an individual snowflake) can become interested in mathematics (Bikner-Ahsbahs, 2005; Ryan & Deci, 2017), there exists good potential for them to experience autonomous motivation and to personally value their mathematics learning.

As pointed out, Profile 2, "Enjoyment in mathematics learning", exists regardless of culture. Here, the teaching style was heterogeneous: as potential developers, teachers acted in a versatile manner: they supported emotional-social integration and self-improvement, importantly, free from pressure. Significantly, as emphasized by several scholars within SDT (Haerens et al., 2016; Reeve, 2016; Ryan & Deci, 2017), the exclusion of teachers' controlling behaviour also had here a positive impact on learners' experiences in the classroom that led them to construct a mental compass in terms of enjoyment and psychological well-being in that classroom, independent of the cultural context. That is, the teacher should perform and arrange various mediation activities in a number of ways in order to suit the individual profiles, which are different due to the biological development process considered here. Because all profiles across the countries displayed identified motivation apart from Profile 4 in Germany (where the teacher is connected with external pressure), the mediating role of "Pressure supporter" should be avoided to support learners' experiences of self-determination.

Other interesting evidence referred to the most valued profile in both countries, namely Profiles 1 in Germany and Finland, "Emotional–social integration". Irrespective of social cultural contexts and the foci within mathematics curricula, students wished for a warm bond with a mathematics teacher who mediated "emotional-social presence". SDT stated that feedback on learners' performance (Ryan & Deci, 2020) can have a meaning of that is of functional importance to students. However, these profiles showed that teachers' emotional-social presence is not limited to providing students with meaning and providing them with learning orientation in mathematics. This specific teacher–student interrelation can also be used to accompany the progressive intensification of students' internalization processes (EXPERIENCE OF COMPETENCE AND SOCIAL RELATEDNESS), particularly in the early stages of those processes.

Foci within Mathematics Curricula. Besides the differences between German and Finnish mathematics curricula, in both countries the identified overall learning orientations were similar, just having differences in nuance. Nonetheless, the social cultural influences were visible in respect of autonomy support in the mathematics classroom. An "excessive emphasis on grades, performance goals, (…)" (Ryan & Deci, 2020, p. 10) and also controlling teaching behaviour on the part of teachers (e.g., the mediating role of the teacher as "pressure supporter") were prominent within the profiles from Germany, but not within the Finnish mathematics-learning orientations. All profiles in Finland display identified motivation, whereas in Germany, Profile 4, oriented towards "external pressure," did not display identified motivation.

The comparison between the two countries did not intend to judge whether one is better than the other. Each country has to face individual issues within its educational system. Furthermore, heterogeneity in Germany (population ca. 83.1 million in 2020; Statista, 2021) is much greater than within Finland (population ca. 5.5 million in 2020; Statista, 2021), and accordingly the strategies and instructions within the Finnish educational system cannot be simply applied to German classrooms by ignoring the individual differences between them.

Each learner is connected to their cultural setting and therefore they should be treated as such. If the German context does not have these cultural amplifiers or mediators, we basically cannot expect that profiles will arise like those that are produced in Finland, because that does not work in a different cultural setting.

Instead of "copying" the Finnish instructions, we could learn from each other and "refresh" our view. The comparison intended to improve our perspective by questioning our own picture of the culture of mathematics education. If one investigates the school system where one has grown up, experience of something foreign helps one to question procedures that were hitherto accepted as natural. In this spirit, the reinterpretation of the German results made by contrasting them with the Finnish profiles showed that a differentiated perspective on the German mathematics curricula is necessary to support autonomous mathematics learning. Similar to the individual educational school systems of the 16 federal states, consideration of each learner's individual affective factors during their development is just as important. Although German policies have added further objectives (e.g., digitalization; KMK, 2019, p. 307) over recent years, there is no explicit objective concerning learners' personal development (like the transversal competencies in Finnish basic education; Finnish National Board of Education, 2016). Mentioning this topic explicitly within the mathematics curriculum may induce educators to take learners' personal development just as seriously as they do the "performance standards" (Blum, 2006; Heymann, 2005) for mathematics lessons.

8.6 Conclusion

The findings of this dissertation can be condensed into two central concluding remarks: Firstly, the evidence of the present investigation points out that besides the support of learners' basic psychological needs, the interpersonal relationship between teachers and students is very important. Like snowflakes, each learner is unique and thus they bring their individual motivations into the classroom to study mathematics. What is more, all learners are in the process of becoming self-determined individuals, and therefore they need an emotionally warm relationship with a teacher (Wentzel, 2009, 2012), who enables them to feel effective in the mathematics classroom. Because of this, experiencing an emotionally warm bond with the teacher or when the teacher mediates very differently in the classroom can support students to engage in mathematics learning for their own sake. This in turn simultaneously promotes learners' autonomous motivation.

In this regard, although teachers face difficulties in their everyday classroom routine, if they deliberately refrain from controlling teaching techniques (Reeve, 2009) this can help encourage learners to build a positive relationship with their teacher, which kicks off the internalization process and eventually leads to their attaining self-determination. Eschewing controlling educational styles does not entail a reduction in the structure and guidance provided, however. Indeed, many students find structure useful as a means of countering anxiety and uncertainty (Haerens et al., 2016; Mouratidis et al., 2013).

As these findings show, the interpersonal relationship between the teacher and the student is a decisive factor in students' learning processes as well as in their personal development.

Although this aspect seems to be a helpful strategy in supporting the learner's autonomy experience as well as their mathematics-learning process, it creates a critical concern when access to mathematics education in school is limited (e.g., due to the COVID-19 pandemic crisis of 2020/2021) and it is not yet clear how educators can counterbalance the negative impacts of this on learners' cognitive development (Zacharopoulos et al., 2021).

This is because, according to the second concluding remark, mathematics learning is an interdependence of embodied, psychological, and social phenomena (de Freitas & Sinclair, 2014; Ferrari & Ferrara, 2020; Hannula, 2012; Roth & Walshaw, 2019). Therefore, the affective quality of the teacher–student relationship has a huge impact on learners' autonomously oriented motivation, emotional well-being (Wentzel, 2009), and the thus-resulting cognitive development (Hannula, 2012; Vygotsky, 1987). The present investigation's results emphasize (further) that besides a positive bond between teacher and student, we have to consider the individual's body and mind—or affect and intellect—through a monist perspective instead of a dualist approach (Hannula, 2012; Roth & Walshaw, 2019; Vygotsky, 1987). An intertwined view while teaching (and within our research) may holistically promote students' mathematics-learning processes, in order to support them on their individual journeys towards actualizing themselves as fully self-determined individuals.

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Appendix A
Original Survey Instrument – English Version

Dear 9th Graders,

Thank you for agreeing to take part in this important survey[41]. Within the scope of our project, we would like to find out what you think about mathematics. It is about your attitude towards mathematics and mathematics class respectively. Please fill out the following questionnaire.

There are no right or wrong answers and you do not have to be an expert in mathematics. It is important for us that you complete the questionnaire carefully and honestly. It would be a pity if your thoughts get lost.

The questionnaire is completely anonymous. Therefore, it is not apparent who has completed which questionnaire. Hence, do not write your name on the questionnaire.

General

What is your gender?

Female	☐
Male	☐
Other	☐
Don't want to answer	☐

What was your math grade in the last academic year? ☐

Notes:

- Please answer the following questions quick and in sequence.
- Read the individual questions carefully and tick that answer which describes you most applicable.

Please tick only ***one*** box belonging to each statement.	☐	☐	☒	☐
If you want to ***correct*** your answer than *colour the wrong box completely* and tick the right one!	☐	■	☐	☒

Let's start! ☺

[41] Please contact the author for the German or the Finnish version of the survey.

Appendix A
Original Survey Instrument – English Version

Part: 1

Appendix A
Original Survey Instrument – English Version

- Mathematics at school

	Statement	strongly disagree	rather disagree	rather agree	strongly agree
Emo3	If I do not like my math teacher, I also don't like the math lessons.	☐	☐	☐	☐
Aus1	It is important to me that there are recovery phases in mathematics lesson in which I can lean back.	☐	☐	☐	☐
Kom3	I am proud of myself, when I realize what I have learned in the last years in math lessons.	☐	☐	☐	☐
Emo2i	I don't care who teaches us, as contents matter and not the teacher.	☐	☐	☐	☐
Pos4	I intensively deal with the mathematical contents so that I can help others in lessons.	☐	☐	☐	☐
Emo4	I pay more attention in math lessons if I like the teacher.	☐	☐	☐	☐
Kom7	I am proud of myself, when I can present my solution in front of my class.	☐	☐	☐	☐
Klas1.2/ St81Q01	It annoys me if no one listens to what the teacher says.	☐	☐	☐	☐
Soz4	It is important to me that there is a positive classroom climate during math lessons.	☐	☐	☐	☐
Amo1	In math lessons, I am just physically present but I don't think.	☐	☐	☐	☐
Aut7	It is important to me to explore new mathematical contents independently.	☐	☐	☐	☐
Klas2.3	It is important to me that the central theme in mathematics lesson is recognizable clearly.	☐	☐	☐	☐
Amo2	If the teacher doesn't recognize I deal with other things.	☐	☐	☐	☐
Lzk7	I work carefully on my math homework in order to pass quizzes and tests.	☐	☐	☐	☐

Appendix A
Original Survey Instrument – English Version

- Mathematics at school

	Statement	strongly disagree	rather disagree	rather agree	strongly agree
Aus5	I enjoy if e.g. historical knowledge about mathematics is mentioned in class.	☐	☐	☐	☐
Klas2.2/ St81Q02	It bothers me if the math lessons is noisy and chaotic.	☐	☐	☐	☐
Lzk6	I learn mathematical contents in order to be able to show my calculations at the blackboard.	☐	☐	☐	☐
Soz1	It is important to me that a good sense of community does exist in math lessons.	☐	☐	☐	☐
Klas3.2/ St81Q03	I don't like if my teacher has to wait a long time until it becomes quiet in the lesson.	☐	☐	☐	☐
Lzk2	Without any exams or tests in mathematics, I would be less engaged in subject matter.	☐	☐	☐	☐
Aus4	I like it when we can finally relax after a strenuous working phase in mathematics.	☐	☐	☐	☐
Klas4.2/ St81Q04	It is important to me that we can work well in math lessons.	☐	☐	☐	☐
Akt4i	As soon as I am aktively challenged, I lose interest/pleasure in math lessons.	☐	☐	☐	☐
Emo5	I prefer to go to the math lessons when I realize that the teacher is interested in me.	☐	☐	☐	☐
Klas5.2/ St81Q05	It annoys if the students start working only a long time after the lesson has already begun.	☐	☐	☐	☐
Lzk3	The examination are an important reason for me to deal with mathematics.	☐	☐	☐	☐
Klas1.3/ Steve1_1	It is important to me that the math lessons follow a logical order.	☐	☐	☐	☐

Appendix A

Original Survey Instrument – English Version

- Mathematics at school

	Statement	strongly disagree	rather disagree	rather agree	strongly agree
Lzk1	I deal with mathematics in order to pass exams and tests.	☐	☐	☐	☐
Aus3	It is important to me to sometimes play games in math lessons.	☐	☐	☐	☐
Klas3.3	It is important to me that the contents in the math lesson build on each other stringently.	☐	☐	☐	☐
Akt13	When I am aktively challenged in lessons, I have the feeling to understand the math contents easily.	☐	☐	☐	☐
Amo3	I do not care about the math lessons.	☐	☐	☐	☐
Lzk8	I prepare intensely before a math examination in order to do well.	☐	☐	☐	☐
Klas4.3	It is important to me that also things are discussed, which are a bit off topic.	☐	☐	☐	☐
Her5	I am unmotivated when the tasks in the math lessons are too simple.	☐	☐	☐	☐
Akt11	I understand more in math lessons due to phases of active practice.	☐	☐	☐	☐
EfU1	It is important to me that my teacher supports me in learning mathematics.	☐	☐	☐	☐
Klas5.3	It is important to me that the math lesson is not interrupted for too long by interposed questions.	☐	☐	☐	☐

Appendix A
Original Survey Instrument – English Version

- Learning and understanding mathematics

	Statement	strongly disagree	rather disagree	rather agree	strongly agree
Pur6	Due to the logical structure of mathematics, it is easy for me to understand the contents.	☐	☐	☐	☐
Akt10	Working actively on tasks helps me understand the contents.	☐	☐	☐	☐
Sel4	It is particularly important for me that I really understand the ideas between the mathematical relationships.	☐	☐	☐	☐
Lzk4	I calculate exceptionally many tasks in order to have a good feeling before exams.	☐	☐	☐	☐
EfU2	Without any support from my teacher, it is more difficult for me to understand mathematics.	☐	☐	☐	☐
Soz8	It is important to me that we work on new contents together.	☐	☐	☐	☐
Sel3	It is important to me to perceive my learning progress.	☐	☐	☐	☐
Lzk5	Before exams I work on more tasks than those given to us so that I know that I can do the topic.	☐	☐	☐	☐
EfU3	It is important to me that my teacher explains the contents well to me.	☐	☐	☐	☐
Akt9	When I deal actively with the contents It is easier for me learn mathematics.	☐	☐	☐	☐
Sel1	I work on many mathematical tasks in order to become better and faster.	☐	☐	☐	☐
EfU4	My teacher's support is important for my learning process in mathematics.	☐	☐	☐	☐

Appendix A
Original Survey Instrument – English Version

➢ Mathematics and I

	Statement	strongly disagree	rather disagree	rather agree	strongly agree
Kom1	I have a positive feeling when I succeed in mathematics.	☐	☐	☐	☐
Aut2	I like to search for own approaches to solve mathematical tasks.	☐	☐	☐	☐
Pur1	In its purity, mathematics is uniquely beautiful to me.	☐	☐	☐	☐
Zen1	I deal with mathematics in order to be proud of my marks.	☐	☐	☐	☐
Sel2	I deal with mathematics in order to improve my logical thinking.	☐	☐	☐	☐
Akt1	I enjoy calculating.	☐	☐	☐	☐
Her7	I get bored when the mathematical contents are too simple.	☐	☐	☐	☐
Kom2	I deal with mathematics because my learning success makes me feel good.	☐	☐	☐	☐
Pur2	The structure of mathematics fascinates me.	☐	☐	☐	☐
Sel5	I deal with mathematics in order to increase my mathematical self-confidence.	☐	☐	☐	☐
Zen3	I make an effort in mathematics in order to be pleased with good marks.	☐	☐	☐	☐
Pfl2	I deal with mathematics mainly because it is a compulsory subject.	☐	☐	☐	☐
Pur3	The beauty of mathematics appears to me in simple results for difficult problems.	☐	☐	☐	☐
Akt2	I am happy when I can do mathematical tasks.	☐	☐	☐	☐

Appendix A

Original Survey Instrument – English Version

- Mathematics and I

	Statement	strongly disagree	rather disagree	rather agree	strongly agree
Pfl5	Dealing with mathematics is nothing more than an obligation.	☐	☐	☐	☐
Pur4	I am overjoyed by the uniqueness of the mathematical technical language.	☐	☐	☐	☐
Sel8	It is important to me to become more confident in mathematics (e.g. by practicing).	☐	☐	☐	☐
Pur7	I am excited about mathematics because mathematical statements are either right or wrong.	☐	☐	☐	☐
Zen4	I deal with mathematics as good marks strenghten my self-confidence.	☐	☐	☐	☐
Akt12	I look for additional tasks myself and work on them, in order to better keep up in math lessons.	☐	☐	☐	☐
Sel6	Mathematics is important to me because I can develop myself in a number of ways.	☐	☐	☐	☐
Pfl3	I only deal intensively with mathematics if I get a reward (e.g. money,…).	☐	☐	☐	☐
Akt3	I like to deal actively with mathematics.	☐	☐	☐	☐
Her2	If I cannot solve tricky tasks immediately, I think about it until I got it.	☐	☐	☐	☐
Kom6	I deal with mathematics because then I realize what I am capable of.	☐	☐	☐	☐
Her1	I work on additional mathematical tasks in order to challenge myself.	☐	☐	☐	☐
Aut8	When dealing with mathematics it is important to me to recognize that I can manage tasks without any help.	☐	☐	☐	☐

Appendix A
Original Survey Instrument – English Version

- Mathematics and I

	Statement	strongly disagree	rather disagree	rather agree	strongly agree
Kom4	I am proud of myself when I've solved a mathematical task which was difficult for me.	☐	☐	☐	☐
Her3	It is important to me that I feel challenged from mathematics.	☐	☐	☐	☐
All6	Mathematics is important to me because thinking mathematically has many advantages.	☐	☐	☐	☐
Akt5	I like to deal with varied tasks.	☐	☐	☐	☐
Her4	I have great pleasure with mathematics if I succeed in solving complex problems.	☐	☐	☐	☐
Pur5	Mathematics is beautiful to me as it is unique in its formalism.	☐	☐	☐	☐
Zen5	Bad marks in mathematics frustrate me.	☐	☐	☐	☐
Aut1	It is important to me to independently come to the solution for new types of tasks.	☐	☐	☐	☐
Aut5	It is important to me to organize the time for working on mathematical tasks on my own.	☐	☐	☐	☐
Brf5i	I will not need school-mathematics for my professional life.	☐	☐	☐	☐
Aut3	Working autonomously is important to me to when it comes to mathematics.	☐	☐	☐	☐
Pfl1	I mainly deal with mathematics because I have to.	☐	☐	☐	☐
Aut6	I like when I may choose math exercises on my own.	☐	☐	☐	☐
Sel7	Mathematical competitions motivate me to deal more with contents.	☐	☐	☐	☐
Pfl6	To deal with mathematics is an obligation to me.	☐	☐	☐	☐

Appendix A
Original Survey Instrument – English Version

- Mathematics in everyday life

	Statement	strongly disagree	rather disagree	rather agree	strongly agree
All3	I deal with mathematics so that I do not lack important knowledge later on.	☐	☐	☐	☐
Rea3	I prefer mathematical tasks which take up situations from real life.	☐	☐	☐	☐
Brf4	I learn mathematical contents because I have to apply them during my studies / vocational training.	☐	☐	☐	☐
All5	Learning mathematics is important to me because in doing so you learn for life.	☐	☐	☐	☐
Rea2	I understand mathematics easier if tasks refer to examples from my life.	☐	☐	☐	☐
Brf2	I deal with mathematics because I will have a larger selection of professions later on.	☐	☐	☐	☐
All7	I deal with mathematics because logical thinking pertains to general education.	☐	☐	☐	☐
Brf1	I deal with mathematics as I need it for my desired profession.	☐	☐	☐	☐
Rea1	It is important to me that the contents from math lessons refer to everyday life.	☐	☐	☐	☐
All4	I think that learning mathematics is important because it is of major importance to society.	☐	☐	☐	☐
Rea4	Tasks referring to everyday life make me occupy myself more deeply with the mathematical content.	☐	☐	☐	☐
All2	Learning mathematics is meaningful/sensible/reasonable because responsible citizens should have mathematical skills.	☐	☐	☐	☐

Appendix A
Original Survey Instrument – English Version

➢ Mathematics in everyday life

	Statement	strongly disagree	rather disagree	rather agree	strongly agree
Rea5	Learning mathematics is important to me because I can apply it to everyday life.	☐	☐	☐	☐
All8	I learn mathematics as otherwise a participation in life is only possible to a limited extent.	☐	☐	☐	☐
Rea6	I think that learning mathematics is important because it is of great importance for other sciences.	☐	☐	☐	☐
All1	I deal with mathematics because it pertains to general education.	☐	☐	☐	☐
Rea8	Learning mathematics is important to me in order to understand processes found in everyday life.	☐	☐	☐	☐
Brf3	I try to be good at mathematics because thereby I will have better chances on a profession later on.	☐	☐	☐	☐
Rea7	Doing mathematics is important to me because it occurs everywhere in life.	☐	☐	☐	☐

➢ Other people and I

	Statement	strongly disagree	rather disagree	rather agree	strongly agree
Pos1	It is important to me that I am given credit for my achievement in mathematics from people who are important for me.	☐	☐	☐	☐
EfU5	It is important to me that my teacher communicates my progress in mathematics to me.	☐	☐	☐	☐
Kom5	I feel good when I can help others in math lessons.	☐	☐	☐	☐
Aus2	I think It is good to have group or partner work in math lessons because then I can also talk about other things than mathematics.	☐	☐	☐	☐

Appendix A
Original Survey Instrument – English Version

- Other people and I

	Statement	strongly disagree	rather disagree	rather agree	strongly agree
Her6	In difficult tasks I try to be faster than the others.	☐	☐	☐	☐
Zen2	It is embarrassing when I have worse marks in mathematics than the others.	☐	☐	☐	☐
Pos2	It is important to me that others get a positive impression of me through my mathematics achievement.	☐	☐	☐	☐
Soz2	It is important to me that I can collaborate with others in math lessons.	☐	☐	☐	☐
Pfl4	I deal with mathematics because it is expected from me.	☐	☐	☐	☐
Pos3	I deal with mathematics in order to be a role model for others.	☐	☐	☐	☐
Soz5	If I can, I am happy to help the others with their problems in mathematics.	☐	☐	☐	☐
Pos5	If my teacher has a poor impression of me in mathematics I make special effort in order to change that.	☐	☐	☐	☐
Emo1	It is important to me that I have a good relationship to my teacher in math lessons.	☐	☐	☐	☐
Soz3	It is important to me that preferably everybody should understand the contents in math lessons.	☐	☐	☐	☐
Aut4	In math lessons it is important to me to choose with whom to collaborate with.	☐	☐	☐	☐
Emo6	It is important to me that I feel valued by my teacher.	☐	☐	☐	☐
Pfl7	I learn mathematics because my family wants me to.	☐	☐	☐	☐
Pos7	I deal with mathematics in order to get appreciation for my achievement.	☐	☐	☐	☐

Appendix A

Original Survey Instrument – English Version

- Other people and I

	Statement	strogly disagree	rather diagree	rather agree	strongly agree
Soz6	I prefer to do mathematics when I collaborate with others in a group.	☐	☐	☐	☐
Pos8	It is important to me that my family can be proud of my mathematics achievement.	☐	☐	☐	☐
Soz7	I like group work because one can help each other.	☐	☐	☐	☐
EfU6	It is important to me that my teacher is interested in my progress in math lessons.	☐	☐	☐	☐
Pos6i	I do not care what others think about my mathematics performance.	☐	☐	☐	☐

Part: 2

Appendix A
Original Survey Instrument – English Version

➢ Learning mathematics from my point of view

	I work and learn in the subject mathematics…	**strongly disagree**	**rather disagree**	**rather agree**	**strongly agree**
OIntrinsisch1	… because it is fun.	☐	☐	☐	☐
OIdentifiziert1	… because thereby I will have more opportunities to choose profession.	☐	☐	☐	☐
OIntrojiziert1	… because I want the teacher to think I am a good student.	☐	☐	☐	☐
OExternal1	… because otherwise I will get into trouble at home.	☐	☐	☐	☐
OIntrinsisch2	… because I like to deal with mathematics.	☐	☐	☐	☐
OIdentifiziert2	… because the things I learn here will be useful later on.	☐	☐	☐	☐
OIntrojiziert2	… because I like to be better than my classmates.	☐	☐	☐	☐

Appendix A
Original Survey Instrument – English Version

- Learning mathematics from my point of

	I work and learn in the subject mathematics…	strongly disagree	rather disagree	rather agree	strongly agree
OExternal2	… because otherwise I will get in trouble with my teacher.	☐	☐	☐	☐
OIntrinsisch3	… because I like to solve mathematical tasks.	☐	☐	☐	☐
OIdentifiziert3	… in order to be able to do a further education (e.g. school, vocational training or studies).	☐	☐	☐	☐
OIntrojiziert3	… because I would feel guilty.	☐	☐	☐	☐
OExternal3	… because my parents request that from me.	☐	☐	☐	☐
OIntrinsisch4	… because I like to think about the things in the subject mathematics.	☐	☐	☐	☐
OIdentifiziert4	… because thereby I can get a better profession.	☐	☐	☐	☐
OIntrojiziert4	… because I want the other students to think I am smart.	☐	☐	☐	☐
OIntrinsisch5	… because I enjoy to deal with mathematics.	☐	☐	☐	☐
OIntrojiziert5	… because I like to receive praise.	☐	☐	☐	☐
OExternal4	… because I have to.	☐	☐	☐	☐

Thank you very much for your participation! ☺

Appendix B.

Table—*Confirmatory Factor Analysis of 17–Factor Model of Personal Meaning in German and Finnish Students*.

Model	χ^2	*df*	CFI	TLI	RMSEA	SRMR
17 Factors (GER)	3140.704	2074	.848	.834	.043	.061
17 Factors (FIN)	3243.324	2074	.857	.843	.047	.062

Note. CFI =comparative fit index; TLI = Tucker–Lewis index; RMSEA = root mean square error of approximation; SRMR = standardized root mean square residual.

Appendix C1.

Table—*Intercorrelations of 17 Personal Meanings in Finish Students.*

Personal meaning	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1 Active practice	-															
2 Classroom management	.50															
3 Cognitive challenge	.87	.39														
4 Duty	-.70	*-.27*	-.59													
5 Emotional-affective relation to the teacher	.28	.52	*.31*	(-.04)												
6 Examination	.48	.43	.36	-.25	.46											
7 Experience of Autonomy	.81	.47	.82	-.49	.33	.41										
8 Experience of competence	.70	.67	.72	*-.34*	.70	.64	.72									
9 Experience of relatedness	.37	.73	.42	(-.09)	.78	.43	.37	.79								
10 General education	.76	.43	.80	-.57	.42	.49	.68	.68	.49							
11 Marks	.47	.45	.44	*-.22*	.56	.56	.47	.73	.61	.51						
12 Positive image	.46	.32	.48	*-.19*	.55	.48	.54	.69	.58	.54	.70					
13 Purism of mathematics	.90	.36	.87	-.62	.29	.42	.70	.56	.33	.78	.40	.45				
14 Reference to reality	.14	.35	.21	(-.01)	.40	.29	(.10)	.38	.50	.30	.16	.31	*.24*			
15 Self-perfection	.89	.48	.80	-.61	.44	.64	.77	.74	.45	.92	.56	.60	.88	.37		
16 Support by the teacher	(.16)	.48	.18	(.00)	.91	.47	.21	.63	.66	.33	.50	.59	.20*	.44	.39	
17 Vocational precondition	.58	.34	.54	-.50	.40	.45	.50	.60	*.36*	.80	.49	.58	.51	*.25*	.68	.35

Notes. Correlations in parentheses were not statistically significant, correlations printed in italics were significant at $p < .01$, all other correlations reported were significant at $p < .001$

Appendix C2.
Table—*Intercorrelations of 17 Personal Meanings in German Students.*

Personal meaning	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1 Active practice																
2 Classroom management	.34															
3 Cognitive challenge	.84	.32														
4 Duty	-.80	-.25	-.59													
5 Emotional-affective relation to the teacher	(.05)	.33	(.14)	(.17)												
6 Examination	(.21)	.25	(.13)	(-.05)	(.19)											
7 Experience of Autonomy	.73	.49	.88	-.51	.25	.34										
8 Experience of competence	.30	.41	.25	(-.13)	.60	.51	.55									
9 Experience of relatedness	.24*	.57	(.20)	(-.14)	.52	.34	.48	.74								
10 General education	.71	.37	.73	-.51	.28	.31	.73	.40	.37							
11 Marks	.20*	.37	(.14)	(-.03)	.38	.58	.39	.58	.42	.40						
12 Positive image	.26	.41	(.18)	(-.07)	.42	.55	.42	.70	.40	.30	.75					
13 Purism of mathematics	.85	.30	.80	-.66	.21	.29	.75	.37	.28	.72	.26	.35				
14 Reference to reality	(.14)	.30	.26	(.09)	.41	(.14)	.28	.34	.41	.27	(.09)	.25	.20			
15 Self-perfection	.77	.43	.70	(-.52)	.26	.53	.78	.48	.35	.83	.53	.53	.83	(.19)		
16 Support by the teacher	(-.02)	.38	(-.10)	.29	.77	.35	(.22)	.68	.50	(.17)	.54	.59	(.00)	.32	.22	
17 Vocational precondition	.40	.23*	.49	-.29	.16*	.21	.45	.28	.27	.73	.40	.29	.43	.23	.53	(.13)

Notes. Correlations in parentheses were not statistically significant, correlations printed in italics were significant at $p < .01$, all other correlations reported were significant at $p < .001$

Appendix D1.

Table—*Factor Loadings of Personal Meaning Items in Finish Students.*

Item	Act	Cla	Cog	Dut	Emo	Exa	Au	Com	Soc	Gen	Mark	Pos	Puri	Rea	Self	Supp	Voc
Akt1	.791																
Akt2	.664																
Akt3	.715																
Klas1		.776															
Klas2		.680															
Klas3		.676															
Klas4		.600															
Klas5		.589															
Her2			.580														
Her3			.828														
Her5			.469														
Pfl1				.892													
Pfl2				.863													
Pfl5				.764													
Pfl6				.620													
Emo1					.733												
Emo4					.433												
Emo5					.532												
Emo6					.850												
Lzk4						.859											
Lzk5						.797											
Lzk8						.673											
Aut1							.678										
Aut3							.709										
Aut7							.547										
Aut8							.585										
Kom1								.635									

APPENDIX D1–APPENDIX D4

Kom4	.717							
Kom5	.589							
Kom7	.608							
Soz1		.692						
Soz3		.504						
Soz4		.601						
Soz5		.624						
All2			.659					
All4			.685					
All6			.856					
All7			.727					
Zen1				.853				
Zen3				.756				
Zen4				.772				
Pos1					.645			
Pos2					.690			
Pos7					.781			
Pos8					.641			
Pur1						.788		
Pur2						.839		
Pur3						.739		
Pur4						.796		
Pur5						.828		
Pur6						.575		
Pur7						.693		
Rea1							.659	
Rea2							.721	
Rea3							.606	
Rea4							.834	
Sel1								.625
Sel2								.768

Sel5	.737		
Sel6	.745		
EfU4		.636	
EfU5		.809	
EfU6		.746	
Brf1			.708
Brf2			.767
Brf3			.767
Brf4			.580
Brf5u			.492

Notes. Act=Active practice; Cla=Classroom management; Cog=Cognitive challenge; Dut=Duty; Emo=Emotional-affective relation to the teacher; Exa=Examination; Au= Experience of autonomy; Co= Experience of competence; So= Experience of social relatedness; Gen=General education; Mark= Marks; Pos=Positive image; Puri=Purism of mathematics; Rea= Reference to reality; Sel=Self-perfection; Supp=Support by the teacher; Voc= Vocational precondition; Am = Amotivation; Ext = External; Intro = Introjected; Iden = Identified; Intr = Intrinsic. Item abbreviations and wording are displayed in Appendix A.

Appendix D2.
Table—*Factor Loadings of Personal Meaning Items in German Students.*

Item	Act	Cla	Cog	Dut	Emo	Exa	Au	Com	Soc	Gen	Mark	Pos	Puri	Rea	Self	Supp	Voc
Akt1	.740																
Akt2	.820																
Akt3	.775																
Klas1		.447															
Klas2		.748															
Klas3		.805															
Klas4		.551															
Klas5		.642															
Her2			.615														
Her3			.782														
Her5			.501														
Pfl1				.794													
Pfl2				.731													
Pfl5				.801													
Pfl6				.487													
Emo1					.618												
Emo4					.493												
Emo5					.627												
Emo6					.692												
Lzk4						.881											
Lzk5						.665											
Lzk8						.662											
Aut1							.713										
Aut3							.578										
Aut7							.547										
Aut8							.599										
Kom1								.576									

Kom4	.563							
Kom5	.677							
Kom7	.600							
Soz1		.688						
Soz3		.389						
Soz4		.592						
Soz5		.515						
All2			.600					
All4			.648					
All6			.793					
All7			.681					
Zen1				.824				
Zen3				.809				
Zen4				.782				
Pos1					.626			
Pos2					.630			
Pos7					.625			
Pos8					.627			
Pur1						.768		
Pur2						.774		
Pur3						.647		
Pur4						.728		
Pur5						.734		
Pur6						.574		
Pur7						.644		
Rea1							.730	
Rea2							.739	
Rea3							.736	
Rea4							.705	
Sel1								.483
Sel2								.603

Sel5	.637		
Sel6	.742		
EfU4		.565	
EfU5		.723	
EfU6		.705	
Brf1			.746
Brf2			.672
Brf3			.685
Brf4			.722
Brf5u			.549

Notes. Act=Active practice; Cla=Classroom management; Cog=Cognitive challenge; Dut=Duty; Emo=Emotional-affective relation to the teacher; Exa=Examination; Au= Experience of autonomy; Co= Experience of competence; So= Experience of social relatedness; Gen=General education; Mark= Marks; Pos=Positive image; Puri=Purism of mathematics; Rea= Reference to reality; Sel=Self-perfection; Supp=Support by the teacher; Voc= Vocational precondition; Am = Amotivation; Ext = External; Intro = Introjected; Iden = Identified; Intr = Intrinsic. Item abbreviations and wording are displayed in Appendix A.

Appendix D3.
Table—*Factor Loadings of Regulatory Style Items in Finnish Students.*

		Regulation			
Item	Amotivation	External	Introjected	Identified	Intrinsic
Amo1	.79				
Amo2	.59				
Amo3	.54				
Ext1		.81			
Ext2		.47			
Ext3		.62			
Intro2			.73		
Intro4			.93		
Intro5			.70		
Ident1				.85	
Ident3				.78	
Ident4				.87	
Intrin1					.88
Intrin2					.91
Intrin3					.91
Intrin4					.81
Intrin4					.87

Notes. Item abbreviations and wording are displayed in Appendix A.

Appendix D4.

Table—*Factor loadings of Regulatory Style Items in German Students.*

		Regulation			
Item	Amotivation	External	Introjected	Identified	Intrinsic
Amo1	.81				
Amo2	.68				
Amo3	.70				
Ext1		.79			
Ext2		.39			
Ext3		.91			
Intro2			.78		
Intro4			.91		
Intro5			.59		
Ident1				.90	
Ident3				.76	
Ident4				.87	
Intrin1					.87
Intrin2					.91
Intrin3					.93
Intrin4					.77
Intrin4					.87

Notes. Item abbreviations and wording are displayed in Appendix A.