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Fragile Robots, Economic Growth and Convergence

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Abstract

Technological progress leads to the development of robots that are more error-prone and fragile than their predecessors. As a consequence, the utilization of the existing automation capital stock is associated with higher wear and tear, CPU overload or communication downtime and, as a consequence, an increase of depreciation costs. This in turn affect new investments in the future. Considering a growth model with physical and automation capital utilization, we argue that in a fully automated society, the utilized automation capital is a perfect substitute for labor, not the automation capital stock per se. We show that it is not necessarily the introduction of capital utilization by itself, but the relationship between the elasticities of utilization of automation and physical capital that plays a crucial role in slowing down the convergence speed in a model that reflects an automated society.

Keywords

Automation, Capital Utilization, Perpetual Economic Growth

JEL Classifications

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1 Introduction

Automation and its induced economic, social as well as ethical consequences have attracted ample attention over the last few years. Some economists argue that automation might contribute to the well-being of a society (among others, see Steigum (2011), Acemoglu and Restrepo (2018a), Prettner (2019)), while others are more sceptical and point to the challenges a society has to cope with. For instance, automation might be one decisive factor explaining the rise in overall income inequality since the 1980s that has been observed in many countries (Piketty and Saez (2003), Piketty (2014), Milanovic (2016)) where increasing wage-related inequality is one of the driving forces behind this observation (Lankisch et al. (2019)). Moreover, it is often argued that automation is the culprit that is responsible for the stagnation of low-skilled worker's wages (Arntz et al. (2016), Acemoglu and Restrepo (2018b), Prettner and Strulik (2020)) or that automation simply leads to economic stagnation in the very long-run (Sachs and Kotlikoff (2012), Sachs et al. (2015)).

Over the past decades, in particular the development of industrial robots has allowed to reduce the amount of labor input in several industries. Looking at recent developments in automation technologies such as 3D printing, self-driving cars and trucks, it might be expected that these technologies have the potential replacing jobs in related industries further. But even high-skilled job are at stakes: According to a recent PwC survey, over the next 10-20 years, jobs related to surgery and anaesthesiology are at risk being replaced by robots[1]. Hence, these recent developments in automation technologies might have the potential to automate many low as well as high-skilled jobs in the not too distant future so that even a fully automated society seems to be possible.

An implicit assumption of existing contributions dealing with automation in the economic growth context is that automation capital per se is a (perfect) substitute for labor. This, however, implies that automation capital is fully utilized in every instant of time which seems to be a rather heroic assumption at first sight. For instance, the most frequently used industrial robot is the so-called vertical articulated arm robot that has often 5 or 6 axes. Due do this specific design, these axes are subject to wear and tear if they are in use (Uhlmann et al. (2020)). Hence, we argue that the more frequently this type of automation capital is used the higher is e.g. its wear and tear, the changes of CPU overload or communication downtime. This conjecture is supported by Gotlieb et al. (2020) that point to the fact that the increased complexity of robots in general makes them more fragile and more error-prone than their predecessors. Thus, we argue that it is not the automation capital per se that is a (perfect) substitute for labor but its utilization. Therefore, we suggest an endogenous growth model that takes this important difference into account by assuming that the depreciation rate of automation capital is endogenously explained by its utilization.

One the other hand and because of the fact that the afore mentioned studies focus on the effects of automation on *long-run* productivity, it could be also argued that there seem to be no compelling

[1] See https://www.pwc.com/m1/en/media-centre/2018/articles/ai-robotics-considerations-for-healthcare-workforce.pdf. The article was accessed on 4th. March 2022.

arguments to assume that automation capital should not be fully employed in the long-run as well. On contrary, it seems more plausible that fluctuation of (automation) capital can be observed in the short-run (with fixed capital). In the long-run however, it is hard to imagine that automation capital should undergo such substantial changes that justifies the assumption that automation capital is idle. From this point of view, it is less astonishing that capital utilization has found wider application in the context of real business cycle literature contrasting with what happens in economic growth theory. However, Rumbos and Auernheimer (2001), Dalgaard (2003), Monteiro et al. (2013) and Chatterjee (2005) and Chatterjee and Morshed (2011) have clearly demonstrated that physical capital utilization is important for the dynamics of growth and convergence as well[2]. Now, if we take the results in this strand of literature seriously, the question arises: If physical capital utilization has long-run relevance, why should this not be the case for automation capital as well? At least, to the best of our knowledge, there are not straightforwardly and empirically founded reasons that prevents us from following this argument.

Per se, the concept of capital utilization as an optimal decision is not new but dates back to the early work of Keynes (1936) who established the notion "user cost" of capital. Keynes (1936), pp. 69-70 pointed out that *"user cost constitutes the link between the present and the future. For in deciding his scale of production an entrepreneur has to exercise a choice between using up his equipment now or preserving it to be used later on. [...]"*.

In our model, automation and physical capital utilization affects the intertemporal growth path of the economy via two channels. First, the growth rate does not only depend on the ratio of physical and automation capital but also on the flow of services derived from these capital stocks, where the latter are endogenously determined by the respective utilization choices made by the firms. Hence, the crucial point here is that the firms have an additional margin to adjust output. Second, the depreciation rate on both types of capital is endogenously determined by the degree of capital utilization. The argument is simple: The more intensively capital is used[3], the higher is the wear and tear that, in turn, increases the depreciation rate. Intuitively, this seems particular relevant when thinking about industrial robots, where friction is the primary cause behind more than 50% of faults in heavy industrial robots (Kermani et al. (2004)). Unattended friction can be therefore seen as a source of wear and tear. Instead, a constant depreciation rate would imply zero marginal cots of automation capital utilization, which, in turn implies that is always optimal to employ the entire stock of automation capital.

The present contribution is mostly related to our study are the contributions by Prettner (2019), Lankisch et a. (2019), Geiger et al. (2018) and Antony and Klarl (2020). In our model with physical as well as automation capital, the optimizing agents weight the marginal gains in profits from increasing of automation as well as from physical capital utilization with the marginal costs that arises

[2] Chatterjee (2005) provides an exhaustive overview regarding the empirical evidence on capital utilization and depreciation in the growth context.

[3] Think of the "workweek" of (automation) capital.

from accelerated depreciation of these two capital stocks. As labor together with automation capital complements physical capital and, hence, off-sets the diminishing returns of physical capital, for homogenous elasticities of depreciation with respect to physical capital and automation capital utilization, we find that our model is equivalent to a standard AK framework. In turn, for a sufficiently pronounced heterogeneity of these elasticities, our model shows transitional dynamics, where the transitional path of new capital investments and future output is mainly determined by the interplay of theses elasticities. In other words, to generate an empirically plausible speed of convergence, in our setting, it is not sufficient to simply introduce capital utilization per se. Instead, we show that the speed of convergence slows down to an empirically plausible number if the heterogeneity of capital utilization is sufficiently pronounced[4].

To sum up, this article proposes a simple model that shows that non-negligible, long-run productivity differences might occur from the introduction and empirically plausible variations in the automation and physical capital utilization rates. More specifically, we propose a dynamic general equilibrium model where production takes place with labor, physical as well as automation capital as input factors. Utilized automation capital is a perfect substitute for labor. There are several key results that we wish to stress at the outset. First, we find that the introduction of endogenous capital utilization decreases the speed of convergence[5]. Second, we show that the interplay of the potentially heterogenous elasticities of depreciation with respect to physical capital and automation capital utilization plays a crucial role of slowing down the speed of convergence. Third, we compare our model with the full capital utilization model (we call this the "standard" model). Depending on the interplay of the utilization elasticities, inter alia, we find that the full utilization model either overstates or understates the steady state equilibrium fraction of automation capital investments. For any empirically plausible combination of the utilization elasticities, the full utilization model overstates the output-capital ratio by about 40%-72% and the speed of convergence by about 11% and 98%.

This paper is organized as follows. Section 2 presents the analytical framework. Section 3 analyzes the macroeconomic equilibrium, whereas Section 4 derives the speed of convergence. Section 5 discusses the numerical analysis of our model, while Section 6 concludes.

2 Related Literature

Initiated by Steigum (2011) who has introduced and addressed the implications of automation capital in a standard neoclassical growth model in the spirit of Ramsey (1928), Cass (1965) and Koopmans (1965), the thorough analysis of automation has gained further attention since then. Steigum (2011)

[4] Alternatively, one could introduce heterogeneous capital adjustment costs together with homogenous capital utilization. However, from a qualitative point of view, this would only increase complexity without changing the overall results.

[5] However, as a fully automated economy is something that is potentially realized in the future, it is worth to mention that we should be cautious by arguing that the speed of convergence in our model without full capital utilization converges towards empirically plausible values as empirically plausible values for the speed of convergence might be different as well in a singularity scenario compared to what conventional studies today tell. We are grateful to the referee to pointing us to this issue.

shows that automation is a potential source that explains the declining labor share observed for the United States[6]. Prettner (2018) introduces automation in a standard Solow (1956) setting and shows that the results obtained by Steigum (2011) also applies for an environment with an exogenous savings rate. Lankisch et al. (2019) generalize the setting of Prettner (2018) and analyze the evolution of wage inequality controlling for the skill-specific heterogeneity of workers. Antony and Klarl (2020) have introduced physical and human capital in a one-sector endogenous growth setting and focus on the irreversibility of investment decisions in physical and automation capital.

Reflecting the convergence debate in the growth literature, two issues are of central importance. The first is concerned with the question whether the speed of convergence[7] generated by theoretical models is in line with empirical evidence. The second issue is directly related to the question whether theoretical growth models can contribute to our understanding of observed cross-sectional differences in living standards and whether these growth models are able to identify channels through which these differences decrease or increase over time (i.e. see Maynou et al. 2022). Neglecting automation capital, Barro and Sala-i-Martin (1992, 2004), Mankiw et al. (1992), and Sala-i-Martin (1994,1996) estimate convergence rates in the range between 2%-3%, which are considerably smaller to what endogenous growth models suggests. For instance, the influential two-sector Lucas (1988) endogenous growth model implies converge rates between 7%-10%.

To the best of our knowledge, Dujava and Labaj (2019) are the first that have contributed empirically to the speed of convergence debate by focusing explicitly on automation capital. Using World Bank's World Development Indicators (WDI) data between 1993-2004 and by introducing robot capital into the Mankiw et al. (1992) setting, for a cross-section of countries, the authors find that the speed of convergence towards a steady-state is lower for countries when robots are used in the production process. This is an interesting results and motivates our contribution to the literature by making the following argument: If automation capital implies a further reduction of today's empirically plausible convergence rates, we have to make attempts to reconcile the discrepancy between theoretical models (that include automation capital) and empirical evidence if it turns out that these theoretical models produce implausibly large convergence rates.

One attempt of the theoretical growth literature to produce empirically plausibly convergence rates is to introduce multiple capital goods (see Eicher and Turnovsky (1999, a,b) and Turnovsky (2004)). A further way to dampen convergence rates can be achieved by introducing capital utilization. The idea behind capital utilization is simple: The more the stock of capital is utilized, the higher is its depreciation. We follow this way by introducing capital utilization in the Antony and Klarl (2020) setting and show that with this simple modification we are able to slow down the model's convergence speed.

In contrast to automation capital, there is a large (empirical) literature on physical capital utilization. Winston (1974) surveys the early microeconomic theory of capital utilization. An important

[6] For instance, see Karabarbounis and Neiman (2014) or Elsby et al. (2013).

[7] The speed of convergence is the rate at which the gap between a country's current and steady-state output per-capita is closed.

strand of early literature on capital utilization is motivated by the puzzling observation that the U.S. manufacturing firms' capital stocks are idle most of the times even in periods of economic prosperity (see Foss (1963)). Marris (1964) document a similar pattern for the U.K. In more recent contributions, Foss (1981a,b, 1995) shows that the work week of capital increased by 25 % between the years 1929-1976 followed by an increase of 8.1% over the time span 1976-1988. Taubman and Gottschalk (1971), Orr (1989), Shapiro (1986) and Beaulieu and Mattey (1998) find similar upward trends. By solving a dynamic cost-minimizing firm problem, Imbs (1999) computes quarterly capacity utilization by exploiting the deviation of the output-capital ratio from its long-run average. He documents a low correlation between the underlying U.S. capital stock and the growth in capital services with a value of 0.12. As also documented by a branch of the literature, empirical estimates of capital utilization varies also across time and industries. Chatterjee (2005) provides a nice summary of this evidence. Although sparse, there is also a literature that documents differences of capital utilization across countries. While Anxo et al. (1995) points to a large variation in utilization rates across European Community countries, Imbs (1999) documents downward trends for capital utilization in Australia, Canada, Germany, Japan and for the U.S. Recently, Comin et al. (2019) point to the fact that using hours per worker as a proxy for factor utilization is problematic for some European countries and instead suggests an alternative method that is based on survey data on capacity utilization.

3 The analytical framework

We consider a closed economy that is populated by a continuum of mass one of infinitely-lived, identical individuals. Individuals can invest in either physical capital such as machines, production halls etc. or in automation capital like 3D printers, industrial or health robots. Time t evolves continuously and population growth rate is assumed to be zero[8]. Our model contains three production factors: automation capital $P(t)$, physical capital $K(t)$ and labor $L(t)$. In contrast to Prettner (2018) and Antony and Klarl (2020), we do not assume that the stock of automation capital per se is a perfect substitute for labor but its utilization (e.g. assembly line workers in the car industry can be easily replaced by utilized industrial robots)[9].

At any instant of time each individual derives utility from her current consumption according to

$$\Omega = \int_0^\infty \exp[-\rho t]\ln(c(t))dt, \tag{1}$$

where $\rho > 0$ is the rate of time preference and $c(t) := \frac{C(t)}{L(t)}$ denotes per capita consumption. For the ease of exposition, we assume no population growth.

The individual's output, $Y(t)$, is represented by a Cobb-Douglas technology:

[8] Our general findings do not change qualitatively if we assume that the population growth rate is greater than zero.

[9] For the sake of simplicity, we do not distinguish between high-skilled and low-skilled workers because this would mainly complicate the exposition. Alternatively, one might follow Jones and Manuelli (1990) or Klarl (2016) and others and define $K(t)$ as a broad measure for capital that includes human capital as well.

$$Y(t) = F(L(t), P(t), K(t)) = [L(t) + \beta_P(t)P(t)]^{1-\alpha}[\beta_K(t)K(t)]^{\alpha},\ \alpha \in (0,1). \tag{2}$$

$\beta_i(t) \in (0,1]$ for $i = \{P, K\}$ represents the capital utilization rates for automation capital $P(t)$ and physical capital $K(t)$, respectively. Regarding (2), some comments are in order. First, (2) implies that utilized automation capital $\beta_P(t)P(t)$ is a perfect substitute for labor $L(t)$. For $\beta_i(t) = 1 \forall t$ for $i = \{P, K\}$, both types of capital are fully utilized which is the implicit standard assumption made in the growth literature. For this special case, we obtain the same production function as proposed by Prettner (2019) or Antony and Klarl (2020). Second, (2) is further motivated from the strict definition of automation provided by Merriam-Webster (2021). Even if we assume that substitution between human workers and robots is not perfect, the introduction of a constant elasticity of substitution production function with a sufficiently high elasticity of substitution between automation capital and labor does not change our results qualitatively (see Lankisch et al. (2019) or Steigum (2011)).

An immediate implication of the introduction of capital utilization for both types of capital (automation as well as physical) is that it makes no longer sense to assume constant depreciation rates for both types of capital. In other words, the rate of depreciation on physical as well as on automation capital is sensitive to the choice of capital utilization. Following Imbs (1999), Rumbos and Auernheimer (2001), Dalgaard (2003) or Chatterjee (2005, 2012), we introduce the following convex "depreciation-in-use" function that proposes a direct link between capital utilization and depreciation:

$$\delta_i(\beta(t)_i) = \frac{1}{\phi_i}\beta_i(t)^{\phi_i},\ \ \phi_i > 1,\ \ 0 \leq \delta_i(\beta_i(t)) \leq 1, \tag{3}$$

where $\frac{\partial \delta_i(\beta(t))}{\partial \beta_i(t))} > 0$ and $\frac{\partial^2 \delta_i(\beta_i(t))}{\partial \beta_i(t)^2} > 0$ for $i = \{P, K\}$.

Expression (3) implies that the rate of depreciation increases if capital is utilized more intensively. Some more comments regarding the specification of (3) are in order. First, $\phi_i = \frac{\partial \delta_i(\cdot)}{\partial \beta_i}\frac{\beta_i}{\delta(\cdot)}$ represents the elasticity of depreciation with respect to capital utilization. If ϕ_i increases, the depreciation rate reacts less sensitive with respect to capital utilization. Thus, the special case $\phi_i \to \infty^+$ for $i = \{P, K\}$ reflects a constant depreciation rate on capital which corresponds to the standard assumption made in the growth literature[10].

Physical and automation capital are the only investment and savings vehicles in the economy. With variable capital utilization, the dynamics of the two capital stocks are governed by

$$\dot{P}(t) = I_P(t) - \delta_P(\beta_P(t))P(t), \tag{4}$$
$$\dot{K}(t) = I_K(t) - \delta_K(\beta_K(t))K(t). \tag{5}$$

Moreover, we assume that a fraction $\tau(t) \in [0,1]$ of the economy's total investments $I(t) =$

[10] It is worth to note that the "depreciation-in-use" function is a commonly used specification in the context of business cycle analysis. See Greenwood et al. (1988), Finn (1995), and Burnside and Eichenbaum (1996).

$I_K(t) + I_P(t)$ is used for automation capital investment $I_P(t)$. We will show below that $\tau(t)$ exhibits transitional dynamics if and only if the heterogeneity between the elasticities of depreciation with respect to physical and automation capital utilization is sufficiently pronounced.

Finally, the economy's resource constraint reads as:

$$Y(t) = F(L(t), P(t), K(t)) = C(t) + I_K(t) + I_P(t), \tag{6}$$

where $I_K(t)$ and $I_P(t)$ stands for gross investment in physical and automation capital, respectively. Finally, neither $K(t)$ nor $P(t)$ can be negative: $0 \leq K(t), 0 \leq P(t)$. In the following, we exclude corner solutions with $P(t) = 0$, where output is solely produced using labor and physical capital[11]. Next, we derive the macroeconomic equilibrium.

4 Macroeconomic equilibrium

The social planner maximizes the intertemporal utility of households (1) subject to the two constraints represented by equations (4) and (5) and subject to the economy-wide resource constraint given by equation (6).

Using per capita notation $x := \frac{X}{L}$ for $X = \{P, K, I, Y, C\}$[12], the present-value Hamiltonian reads as

$$\begin{aligned}\mathscr{H}(c, \tau, k, p, \lambda_1, \lambda_2) := \ln(c)\exp[-\rho t] + \lambda_1[(1 + p\beta_P(t))^{(1-\alpha)}(k\beta_K(t))^{\alpha} - c - \tau i - \delta_K(\beta_K(t))k] \\ + \lambda_2[\tau i - \delta_P(\beta_P(t))p],\end{aligned} \tag{7}$$

where λ_1 and λ_2 are shadow prices associated with $\dot{k}$ and $\dot{p}$, respectively.

Remark. *If human labor and automation capital are perfect substitutes and the implicit automation condition* $P(t) > \frac{Y(t)K(t)(1-\alpha)}{Y(t)\alpha + K(t)(\delta_P(t) - \delta_K(t))} - \frac{L(t)}{\beta_P(t)}$ *is met, the build-up of both stocks of capital do not rise the marginal product of human labor*[13]*. Hence, asymptotically, the production function (2) exhibits constant returns to scale in the two capital stocks and the economy exhibits sustained endogenous growth as it will be shown below. In other words, if the automation condition is met, the production function (6) changes to*[14]*:*

$$Y(t) = F(0, P(t), K(t)) = [\beta_P(t)P(t)]^{1-\alpha}[\beta_K(t)K(t)]^{\alpha},\ \alpha \in (0, 1). \tag{8}$$

[11] In fact it can be shown that if the implicit non-automation condition $P(t) \leq \frac{Y(t)K(t)(1-\alpha)}{Y(t)\alpha + K(t)(\delta_P(t) - \delta_K(t))} - \frac{L(t)}{\beta_P(t)}$ is met, output is produced with $Y(t) = [L(t)]^{1-\alpha}[\beta_K(t)K(t)]^{\alpha}$. Note further that due to endogenous depreciation, there is no closed-form solution for the non-automation condition.

[12] Where appropriate, throughout the rest of the paper, we omit time indices for clarity.

[13] Thus, the scarcity of human labor does not lead to transitional growth which would be the case if the automation condition is not met.

[14] This asymptotic view of full automation is in line with Berg et al. (2018), Prettner and Bloom (2020), p. 94, Antony and Klarl (2019), Lankisch et al. (2019) or Prettner (2019) all deriving a asymptotic balanced growth path for this singularity scenario of full automation.

In line with Berg et al. (2018), Lankisch et al. (2019) or Prettner (2019) and others, in the following, we focus on this so-called singularity scenario, i.e. an economy that is fully automated. In this regard, it should be noted that this singularity scenario is a realistic description of what should be expected in the future. For instance, if we assume a broad definition of the physical capital stock $K(t)$ that also includes human capital, in this singularity scenario, output is produced solely with human capital (i.e. high-skilled labor) and robot capital [15].

The first-order conditions for an interior optimum read as

$$\mathscr{H}(\cdot)_c = 0 \Leftrightarrow c^{-1}\exp[-\rho t] = \lambda_1, \tag{9}$$

$$\mathscr{H}(\cdot)_\tau = 0 \Leftrightarrow \lambda_1 = \lambda_2, \tag{10}$$

$$\mathscr{H}(\cdot)_{\beta_K} = 0 \Leftrightarrow \beta_K = \left[\alpha\frac{y}{k}\right]^{\frac{1}{\phi_K}}, \tag{11}$$

$$\mathscr{H}(\cdot)_{\beta_P} = 0 \Leftrightarrow \beta_P = \left[(1-\alpha)\frac{y}{p}\right]^{\frac{1}{\phi_P}}, \tag{12}$$

$$-\mathscr{H}(\cdot)_k = -\left[\frac{\partial y}{\partial k} - \delta_K(\beta_K)\right]\lambda_1 = \dot{\lambda_1}, \tag{13}$$

$$-\mathscr{H}(\cdot)_p = -\left[\frac{\partial y}{\partial p}\right]\lambda_1 + \delta_P(\beta_P)\lambda_2 = \dot{\lambda_2}, \tag{14}$$

together with the transversality conditions

$$\lim_{t\to\infty}(\lambda_1 k) = \lim_{t\to\infty}(\lambda_2 p) = 0. \tag{15}$$

Using conditions (9)-(14), the growth rate of per capita consumption reads as

$$\gamma_c := \frac{\dot{c}}{c} = \alpha\frac{y}{k}\left(\frac{\phi_K - 1}{\phi_K}\right) - \rho. \tag{16}$$

From (16), we see that an under-utilized physical capital stock leads to an immediate reduction of the growth rate of consumption as $\left(\frac{\phi_K-1}{\phi_K}\right) < 1$. Intuitively, if ϕ_K increases together with a decreasing depreciation rate leads to an increase of the marginal and average product of capital that in turn increases the steady-state rate of utilization and the investments. This also tends to increase the speed of convergence as will be shown in this article.

Using (10), we will show below that the ratio of the two capital stocks is constant. In turn, this

[15] This in turn highlights the importance of human capital investments in a society confronted with automation (see Prettner and Strulik (2020)).

implies that diminishing returns do not apply. Using expressions (11) and (12), we can rewrite the production in its intensive form:

$$y = \left[\xi^{\frac{(1-\alpha)(\phi_P - 1)}{\phi_P}} (1-\alpha)^{\frac{1-\alpha}{\phi_P}} \alpha^{\frac{\alpha}{\phi_K}} \right]^{\frac{\phi_P \phi_K}{\alpha(\phi_K - \phi_P) - \phi_K + \phi_P \phi_K}} k, \tag{17}$$

with $\xi \equiv \frac{p}{k}$. Thus, from (17) we see that our model exhibits an AK-structure.

Lemma 1. *To ensure a positive consumption-output and consumption-capital ratio, we impose the mild restriction $\phi_P(\phi_K - \alpha) > \phi_K(1-\alpha)$.*

This mild restriction is similar to the heterogenous capital utilization model of public and private capital introduced by Chatterjee and Morshed (2011).

In the following, we define $\mu \equiv \frac{(1-\alpha)(\phi_P - 1)\phi_K}{\alpha(\phi_K - \phi_P) - \phi_K + \phi_P \phi_K} \in (0,1)$[16]. μ and (1-μ) represent the reduced form output elasticities of the *utilized* automation and *utilized* physical capital stock, respectively. These former elasticities depend on the output elasticity of physical capital, α, the output elasticity of automation capital, $1-\alpha$, and the elasticities of depreciation with respect to physical as well as with respect to automation capital utilization, ϕ_K and ϕ_P, respectively. In the limit, and for the special case of homogeneous elasticities of depreciation with respect to capital utilization ($\phi_K = \phi_P = \phi$), μ approaches $1-\alpha$ and $1-\mu$ approaches α. Denoting further $\chi \equiv \frac{c}{k}$, it is straightforward to express the equilibrium dynamics of the centralized economy in terms of the redefined variables ξ and χ. Together with β_K and β_P, the dynamics of the model can be expressed as:

$$\frac{\dot{\chi}}{\chi} = (\alpha + \tau - 1)\xi^{\mu}\Gamma + (1-\tau)\chi - \rho, \tag{18}$$

$$\frac{\dot{\xi}}{\xi} = (\xi^{\mu}\Gamma - \chi)(\tau\xi^{-1} - 1 + \tau) - \xi^{\mu}\Gamma\left(\xi^{-1}\frac{1-\alpha}{\phi_P} - \frac{\alpha}{\phi_K}\right), \tag{19}$$

$$\frac{\dot{\beta}_K}{\beta_K} = \left(\frac{\phi_P - 1}{\phi_K \phi_P - 1}\right)\frac{\dot{\xi}}{\xi}, \tag{20}$$

$$\frac{\dot{\beta}_P}{\beta_P} = -\left(\frac{\phi_K - 1}{\phi_K \phi_P - 1}\right)\frac{\dot{\xi}}{\xi}, \tag{21}$$

with $0 < \Gamma \equiv \left[(1-\alpha)^{\frac{1-\alpha}{\phi_P}} \alpha^{\frac{\alpha}{\phi_K}}\right]^{\frac{\phi_P \phi_K}{\alpha(\phi_K - \phi_P) - \phi_K + \phi_P \phi_K}} < 1.$

Condition (10) says that, in equilibrium, the compensation of physical capital owners has to be equal to the compensation of automation capital owners. Using expression (11) and (12) in (3), we obtain $\delta_K(\cdot) = \frac{\alpha}{\phi_K}\frac{y}{k}$ together with $\delta_P(\cdot) = \frac{1-\alpha}{\phi_P}\frac{y}{p}$. Thus, as long as output per capita grows with the same rate as automation and physical capital per capita, both depreciation rates are constant. The latter can be shown as follows: Inserting the last two expressions for $\delta_K(\cdot)$ as well as for $\delta_K(\cdot)$ into expression (13) and (14), and further exploiting the fact that $\lambda_1 = \lambda_2$ (see (10)), we find that

[16] $\mu < 1$ as $\phi_K > 1$. $\mu > 0 \Leftrightarrow \phi_P(\phi_K - \alpha) > \phi_K(1-\alpha)$ as imposed by Lemma 1.

$$\left(\frac{p}{k}\right)^* \equiv \xi^* = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\phi_P - 1}{\phi_P}\right)\left(\frac{\phi_K}{\phi_K - 1}\right). \tag{22}$$

Expression (22) shows that automation capital rises with physical capital: If k goes up, the return of automation capital rises relatively to physical capital so that automation capital is rising as well. Finally, using (22) in (17) delivers the result that in the steady state, both depreciation rates are constant.

Next, we want to pin down the steady state fraction of physical capital investment, τ^*. To obtain this measure, first, impose the steady state condition $\dot{\xi} = 0$ on (19) as well as $\dot{\chi} = 0$ on (18). Solving the latter delivers

$$\chi = \frac{(1-\alpha-\tau)\xi^{\frac{(1-\alpha)(\phi_P-1)\phi_K}{\alpha(\phi_K-\phi_P)-\phi_K+\phi_P\phi_K}}\Gamma + \rho}{1-\tau}. \tag{23}$$

Second, inserting expression (23) and (22) in (19), and solving for τ[17] yields:

$$\tau^* = \frac{(1-\alpha)\left[\alpha\Gamma\left(\phi_k - 1\right)\phi_p - \rho\,\xi^{*\frac{(\alpha-1)\phi_K\left(\phi_P-1\right)}{\phi_K(\alpha+\phi_P-1)-\alpha\phi_P}}\phi_K\left(\phi_p - 1\right)\right]}{\alpha\Gamma\left(\phi_K - 1\right)\phi_P - \rho\,\xi^{*\frac{(\alpha-1)\phi_K\left(\phi_P-1\right)}{\phi_K(\alpha+\phi_P-1)-\alpha\phi_P}}\left[\phi_K\left(\phi_P - 1\right) + \alpha(\phi_K - \phi_P)\right]} \in [0,1]. \tag{24}$$

Note that for the case of homogeneous elasticities of depreciation with respect to capital utilization ($\phi_P = \phi_K = \phi$), expression (24) reduces to

$$\tau^* = 1 - \alpha. \tag{25}$$

In this case, the optimal fraction of automation capital investments, τ, is equal to the output elasticity of automation capital, $1-\alpha$. For the case of heterogenous elasticities of depreciation with respect to capital utilization, we observe that τ^* is also affected by the time preference parameter, ρ, while this is not the case for the homogeneous case. From (24), it is straightforward to see that only for the case that $\xi(t) \neq \xi^*$ and hence $\tau \neq \tau^*$, there is transitional dynamics. In turn, for the case of homogeneous elasticities of depreciation with respect to capital utilization ($\phi_P = \phi_K = \phi$), or for the case that both (heterogeneous) elasticities go to infinity, from (22), we find that in these cases transitional dynamics is absent.

Using the steady state conditions, it is straightforward to show that

Proposition 1. *The unique long-run growth rate of the economy is*

$$\gamma = \frac{\dot{y}}{y} = \frac{\dot{i}}{i} = \frac{\dot{p}}{p} = \frac{\dot{k}}{k} = \frac{\dot{c}}{c} := \alpha\left(\frac{y}{k}\right)^*\left(\frac{\phi_K - 1}{\phi_K}\right) - \rho. \tag{26}$$

[17] It can be shown that $\gamma_c > 0$ directly implies that $\tau \in (0,1)$.

Proof. The transversality conditions holds for $\rho > 0$. As we will show below, we find that $\chi := \frac{c}{k}$ is constant. Thus, consumption grows with the same rate as physical capital. Moreover, (22) shows that physical capital grows with the same rate as automation capital. Using (22) in (17), we find that final output grows with the same rate as physical capital. (24) is constant as long as (22) is constant as well. Finally, from the resource constraint (6), we observe that investment in both types of capital grows at the constant rate as consumption.

A further important remark is that capital utilization affects the savings rate as well. In the steady state, we note that Proposition 1 implies a gross savings rate $s = 1-\frac{c}{y} = \frac{\rho\Gamma^{-1}\xi^{*\mu}[\alpha(\phi_K-\phi_P)+\phi_K(\phi_P-1)]}{\alpha(\phi_K-1)\phi_P}$. For the special case of homogenous elasticities of depreciation with respect to capital utilization, i.e. $\phi_K = \phi_P = \phi$, the savings rate reduces to $s = (1-\alpha)^{-(1-\alpha)}\alpha^{-\alpha}\rho\left[(1-\alpha)^{\frac{1-\alpha}{\phi}}\alpha^{\frac{\alpha}{\phi}}\right]^{\frac{-\phi}{\phi-1}}$, while it tends to $s = (1-\alpha)^{-(1-\alpha)}\alpha^{-\alpha}\rho$ for $\phi \to \infty^+$. The latter expression of the savings rate corresponds to the "full utilization" assumption made in the growth literature[18].

Proposition 2. *In the steady state, the optimal rates of capital utilization for physical as well as for automation capital are constant and read as*

$$\beta_K^* \equiv \beta_K(\xi^*) = \left[\alpha\left(\frac{y}{k}\right)^*\right]^{\frac{1}{\phi_K}}, \tag{27}$$

$$\beta_P^* \equiv \beta_P(\xi^*) = \left[(1-\alpha)\left(\frac{y}{k}\right)^*\xi^{-1*}\right]^{\frac{1}{\phi_P}}, \tag{28}$$

with

$$\left(\frac{y}{k}\right)^* = \left[\xi^{*\frac{(1-\alpha)(\phi_P-1)}{\phi_P}}(1-\alpha)^{\frac{1-\alpha}{\phi_P}}\alpha^{\frac{\alpha}{\phi_K}}\right]^{\frac{\phi_P\phi_K}{\alpha(\phi_K-\phi_P)-\phi_K+\phi_P\phi_K}}. \tag{29}$$

Proof. The proof of the proposition follows immediately from using (11) and (12) together with (17).

Proposition 3. *In the steady state, the capital depreciation rates for physical as well as for automation capital are constant and read as*

$$\delta_K(\beta_K^*) \equiv \delta_K[\beta_K(\xi)] = \frac{1}{\phi_K}(\beta_K^*)^{\phi_K}, \tag{30}$$

$$\delta_P(\beta_P^*) \equiv \delta_P[\beta_P(\xi)] = \frac{1}{\phi_P}(\beta_P^*)^{\phi_P}. \tag{31}$$

Proof. The proof follows directly from Proposition 2.

Proposition 4. The steady state consumption-capital ratio, $\left(\frac{c}{k}\right)^*$, reads as

$$\chi^* = \frac{\rho\left[\alpha(\phi_K-\phi_K)+\phi_K(\phi_P-1)\right]}{\alpha(\phi_K-1)\phi_P} > 0, \tag{32}$$

as, under the mild assumption imposed by Lemma 1, $\alpha(\phi_K-\phi_P)+\phi_K(\phi_P-1) > 0$. Note that

[18] If population grows with a positive rate, $\frac{\dot{N}}{N} \equiv n > 0$, n would negatively affect the savings rate as well.

for $\phi_K = \phi_P$, this reduces to $\chi = \frac{\rho}{\alpha} > 0$ or to $\chi = \frac{\rho - n}{\alpha} > 0$ if we would allow a positive growth rate of population, $\frac{\dot{N}}{N} \equiv n > 0$[19].

Proof. The proof follows directly using (32).

The next section shows that (ξ^*, χ^*) is a unique steady state equilibrium which is (locally) a saddle point.

5 Transitional dynamics

In this section, we derive a measure for the speed of convergence for our model. To achieve this, we first linearize the model around its steady-state derived in section 4. The linearized dynamics of system (18)-(19), which is the core dynamics of the model, can be compactly summarized as

$$\dot{\Xi} = \Lambda(\Xi - \Xi^*), \tag{33}$$

with $\dot{\Xi}' = [\dot{\xi}, \dot{\chi}]$, $\Xi' = [\xi, \chi]$ and $\Xi^{*'} = [\xi^*, \chi^*]$. Λ is a (2×2) coefficient matrix of the linearized system.

In order to guarantee a positive growth rate for consumption, we have to restrict the parameter space of the rate of time preference, ρ.

Lemma 2. *From (16) we see that for $\rho \in (0, \rho_0)$, with $\rho_0 \equiv \frac{\alpha \xi^{*\mu} \Gamma(\phi_K - 1)}{\phi_K}$, the steady state growth rate of consumption is positive.*

As shown by Ortigueira and Santos (1997), the convergence speed can be defined as the absolute value of the stable root of the matrix Λ of the linearized system (18)-(19). As shown with Proposition 5, the determinant of the linearized system (18)-(19) is negative while the trace is positive. That in turn implies that there is one positive and one negative eigenvalue, whereas the absolute value of the latter corresponds to the asymptotic speed of convergence. Proposition 5 and 6 summarize these findings in more formal detail.

Proposition 5. *The equilibrium (ξ^*, χ^*) is an unique saddle-point as the determinant of the matrix Λ of the linearized system (18)-(19) is negative and reads as*

$$\mathscr{D}(\rho) = (1 - \alpha)\left(\phi_p - 1\right) \xi^{-\frac{\alpha(\phi_K - 1)\phi_P}{\phi_K(\alpha + \phi_P - 1) - \alpha\phi_P} - 1} \frac{A(\rho)}{B(\rho)} < 0, \text{ for } \rho \in (0, \rho_0), \tag{34}$$

while the trace $\mathscr{T}(\rho) > 0$.

For the special case of homogeneous elasticities of depreciation with respect to capital utilization ($\phi_K = \phi_P = \phi$), expression (34) simplifies and reduces to $\mathscr{D}(\rho) = -\rho(\rho_0 - \rho) < 0$.

Proof. See Appendix.

Proposition 6. *Proposition 5 implies that system (18)-(19) has one negative and one positive*

[19] In the latter case, the tranversality conditions holds for $\rho > n$.

Eigenvalue. The absolute value of the negative eigenvalue, say $|\tilde{\zeta}|$, which by definition is the asymptotic speed of convergence to the steady state equilibrium, is given by

$$|\tilde{\zeta}| = |\tilde{\zeta}(\mu, 1-\mu, \alpha, 1-\alpha, \phi_K, \phi_P, \rho)|. \tag{35}$$

Proposition 6 shows that the convergence rate $|\tilde{\zeta}|$ does not only depend on production and preference parameters of the considered economy, but also on the output elasticities of the utilized stocks of physical as well as of automation capital, μ and $1-\mu$ and, therefore, on the elasticity of depreciation with respect to capital utilization of both types of capital, ϕ_P and ϕ_K[20]. For an empirically plausible parameter space, it can be shown that the convergence rate is an increasing function of ϕ_K and ϕ_P for $\rho \in (0, \rho_0)$[21]. The crucial point here is that the "less than full" utilization of automation and physical capital by the agent may help to reduce the discrepancy between the theoretical and empirically observed speed of convergence. Based on numerical experiments, in the next section, we will show that the empirically consistent assumption of "less than full" utilization of (physical) capital will prevent the model from overstating the speed of convergence, the balanced growth rate as well as the steady state equilibrium of the output-capital ratio. The latter observation is a typical (and well known) drawback of most one-sector models of economic growth settings when confronting the model with the data (see Chatterjee (2005) or Monteiro et al (2013)).

6 Idle (automation) capital and convergence: A numerical exemplification

We now proceed to a numerical exemplification of endogenous capital utilization for the above outlined one-sector growth model with heterogenous stocks of capital. The following exercises are particularly designed to gauge the sensitivity of the long-run and transitional dynamics behavior of our model regarding the parameter uncertainty for the elasticity with respect to automation capital utilization, ϕ_P. We will see that for some calibrations, even small changes of ϕ_P lead to relatively large long-run changes of the consumption-output ratio, the output-capital ratio, the long-run growth rate as well as for the speed convergence. Depending on the calibrated value for ϕ_P, we also observe different dynamic adjustment patterns during transition for the consumption-output ratio and the output-capital ratio.

We start with a parameter calibration that is in line with empirical estimates. The rate of time preference, ρ, is set at 0.04. The output elasticity of capital, α, is set to 0.35 which is consistent with the estimates of the capital share in U.S. GDP. More problematic is the assignment of numerical values for the elasticities of depreciation with respect to physical as well as automation capital utilization, ϕ_P and ϕ_K, respectively. Reflecting the literature, we find few studies (mainly from the RBC

[20] If the agent's instantaneous utility is assumed to be iso-elastic, $u(c) = \frac{c^{1-\theta}-1}{1-\theta}$ with $\theta \neq 1$, the intertemporal elasticity of substitution, $\frac{1}{\theta}$ would affect (35) as well. However, this alternative specification would not change our main results qualitatively.

[21] The numerical simulation study executed below focuses on this point as well.

literature) attempting to estimate the elasticity parameter ϕ_K. For example, Burnside and Eichenbaum (1996) estimate $\phi_K = 1.56$ for U.S. manufacturing. Also for U.S. manufacturing, Finn (1995) estimate $\phi_K = 1.44$. Dalgaard (2003) with a growth focus, estimates $\phi_K = 1.7$ for Denmark. Basu and Kimball (1997) emphasis that $\phi_K = 2$ is the upper bound of the 95 percent confidence interval for the parameter estimate of ϕ_K. Due to that rare empirical studies, Chatterjee (2005)restricts the parameter space for ϕ_K to $\phi_K \in (1.4, 2)$, while Chatterjee (2012) calibrates $\phi_K = 2$. To the best of our knowledge, there is no parameter estimates for ϕ_P available. Due to that reason, for our experiments, we allow ϕ_P to vary between the same interval as does ϕ_K. Further, we have to check whether the chosen values for ϕ_P and ϕ_K generate values for ρ_0 (see Proposition 5) smaller than the calibrated ρ. For $\phi_K = \phi_P = \phi = 1.4$, the steady state is unstable, so that we restrict the values for ϕ_i $i = \{K, P\}$ to vary between 1.5 and 2.Table 1 summarizes the parameters for the benchmark model.

Paramter	ρ	α	ϕ_K	ϕ_P
Value	0.04	0.35	1.7	1.7

Table 1: Benchmark Parameters

Next, we calibrate the growth model using above derived equilibrium solutions. Moreover, we compute the implied speed of convergence and other relevant equilibrium quantities for a benchmark model, where we assume homogenous but finite elasticities of depreciation with respect to capital utilization, $\phi_K = \phi_P = \phi$. The results of this benchmark model is then compared for variations in the parameter space of ϕ_K and ϕ_P. Moreover, we also pay attention to the dynamic adjustment process in response to a shock of the parameter space of ϕ_P. It should be mentioned that the primary focus of this simulation study is to understand the implications of heterogenous capital utilization on the speed of convergence and equilibrium values relative to a model with full utilization. Hence, the numerical illustration executed below should not be primarily viewed as a calibration for a particular economy or group of economies, although the speed of convergence as well the long-run growth rate lies within their empirically observed ranges for OECD countries. In this context it should be mentioned that we have normalized the level of technology to one in order to avoid that long-run growth potential due to automation is confounded by a second engine of growth, such as technological progress (see (2)).

6.1 Steady state results

Table 2 and Figure 1 presents the implied speed of convergence measures for the capital utilization model. Figure 1 delivers the massage that the speed of convergence is increasing for *any* combination of ϕ_i for $i = \{K, P\}$ if ϕ_i increases for $i = \{K, P\}$. This result extends Chatterjee (2005), who found within one-sector models of economic growth (with one type of capital) that the speed of convergence increases with ϕ. More precisely, Chatterjee (2005)'s finding is equivalent to our model with two types of capital but homogenous and finite elasticities of depreciation with respect to capital utilization as can be directly seen from inspecting the values on the principal diagonal of Table 2.

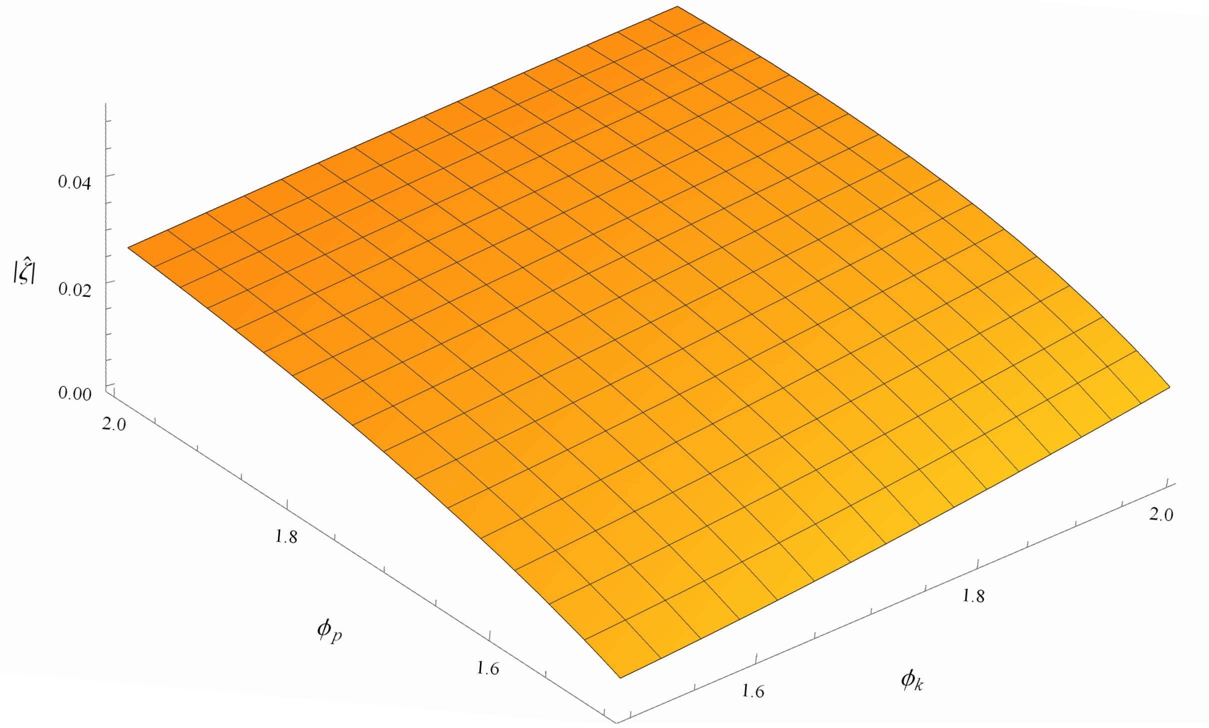

Figure 1: Speed of convergence $|\hat{\zeta}|$ in the capital utilization model over the feasible region $[1.5, 2] \times [1.5, 2]$.

Moreover, while the standard model considerably overstates the speed of convergence (see Table 3 below), the capital utilization model generates speeds of convergence ranging from 0.4% to 5.2%. Hence, for a reasonable choice of ϕ_K and ϕ_P, the capital utilization model is able to produce convergence speeds that fall well within the empirically observed range of 2-3 percent. This is particularly true for our benchmark model that produces a speed of convergence of 2.44% for $\phi_K = \phi_P = 1.7$. For heterogenous elasticities of depreciation with respect to capital utilization, in some cases, the model produces speeds of convergence below 2.44%.

	$\phi_P = 1.5$	$\phi_P = 1.6$	$\phi_P = 1.7$	$\phi_P = 2$
$\phi_K = 1.5$	0.0041	0.0113	0.0166	0.0270
$\phi_K = 1.6$	0.0059	0.0143	0.0204	0.0322
$\phi_K = 1.7$	0.0083	0.0177	0.0244	0.0373
$\phi_K = 2$	0.0175	0.0289	0.0370	0.0522

Table 2: Speed of convergence $|\hat{\zeta}|$ in the capital utilization model for selected values for ϕ_K and ϕ_P.

Now, we proceed by comparing the capital utilization model with a "standard" one-sector, heterogenous capital model commonly used in growth literature. For this "standard" model, we assume, first, full capital utilization for both types of capital, i.e. $\beta_i = 1$ for $i = \{K, P\}$, and, second, identical, and constant depreciation rates on both types of capital. Table 3 in Appendix 8.2 shows the relative differences in computed equilibrium values between the capital utilization model and a "standard" model. The numbers which are presented in Table 3 show the relative differences between two equilibrium values, i.e. $\frac{x^{\texttt{standard}}-x}{x^{\texttt{standard}}}$, where $x^{\texttt{standard}}$ is the equilibrium value of the "standard" model and x is the corresponding value of the capital utilization setting.

For finite elasticities of depreciation with respect to capital utilization, Table 3 highlights the fact that the "standard" model overstates output-capital ratio by about 40% and 72%, the speed of convergence by about 11% and 98%, and, the growth rate by about 51% and 98%. For *homogenous*, but finite elasticities of depreciation with respect to capital utilization ($\phi_K = \phi_P = \phi$), we also observe that the fraction of automation investment, τ, is identical between the capital utilization and the standard model, while the fraction tend to converge in the limit. We also observe that the "standard" model massively understates the consumption-output ratio between 28% ($\phi_K \to \infty^+, \phi_P = 2$) and 265% for $\phi_K = \phi_P = 1.5$. This also reflects how sensitive some of the equilibrium values react with respect to changes of the calibrated elasticities of depreciation with respect to capital utilization. An exemption is the fraction of automation capital investments, τ, that is hardly affected by changes of ϕ_P and ϕ_K. The "standard" model either slightly overstates (by 3% for low ϕ_K and $\phi_P \to \infty^+$) or understates τ relative to the capital utilization model by 5% for low ϕ_P and $\phi_K \to \infty^+$.

6.2 Increase of the elasticity of depreciation with respect to automation capital utilization, ϕ_P

In the preceding sections, we have shown that the speed of convergence is the larger, the smaller is the elasticity of depreciation with respect to automation and/or physical capital utilization. The purpose of this section is to directly compare the dynamic response of the endogenous core variables of our model for two scenarios: The first considers a small and unexpected increase of the elasticity of depreciation with respect to automation capital utilization, ϕ_P from $\phi_P = 1.7$ to $\phi_P = 2$, while the second reflects a shock that unexpectedly increases the elasticity of depreciation with respect to automation capital utilization from $\phi_P = 1.7$ to $\phi_P = 20$. For the latter case, the new steady-state (with $\phi_P = 20$) reflects an economy where automation capital is relatively more employed, while for the first case automation capital is still relatively less employed even after the shock has been realized. Hence, we conjecture that the dynamic adjustment takes a longer time for an economy that is hit by a smaller rather than by a larger elasticity of depreciation shock. Hence, the following analysis shows that not only the long-run behavior of the model is different between the two scenarios, but also the short- and medium-run dynamic adjustment towards the new long-run equilibrium differs. This conjecture is graphically supported with Figure (2). The two regimes start with identical equilibrium allocations. The purple (orange) color shows the dynamic adjustment in a response to the small (large) shock. As the depreciation rate is endogenous, a shock on ϕ_P affects capital accumulation via two channels: the marginal product of capital and the depreciation rate. From the analysis above, we saw that with endogenous capital utilization, it is not optimal to fully utilize capital. Hence, shocks (induced via ϕ_p) have a lower impact on the marginal product of capital compared to a setting where capital is fully utilized. This finding is supported by our numerical simulation carried out in this section.

a) ***Steady state behavior***

An increase of ϕ_P implies a less sensitive reaction of the depreciation rate with respect to automation capital utilization. Thus, the marginal costs of capital utilization decline. It is therefore more profitable to increase the amount of automation capital in the production process. Turning to our numerical exercise, in the long-run, we observe that an increase of ϕ_P increases the utilization rate of automation capital and, in turn, the corresponding depreciation rate. Also the fraction devoted to automation capital investments, τ, decreases as the increase of automation capital utilization allocates more resources towards consumption and physical capital investment. The latter finding is due to the fact that an increase of ϕ_P raises the productivity of physical capital and its rate of utilization as well. Consequently, the increase of utilization and accumulation of both types of capital raises the equilibrium growth rate. Although, we observe a drop of τ, in the new steady state, 68% of investments still go into automation capital. Hence, the investments in automation capital more than compensates the increased depreciation of automation capital due to its more intensively employment in the production process. On the other hand, the relatively small increase of physical capital investment cannot compensate the relatively large increase of physical capital depreciation induced by its utilization. Consequently, this leads to an increase of the equilibrium automation-physical capital ratio and the equilibrium consumption-capital ratio as reflected by Figure (2).

b) ***Transitional Dynamics***

However, as also clearly reflected with Figure (2), the transitional dynamics across the two regimes are different. The increase of ϕ_P implies an increase in the expected long-run productivity of automation capital that is the larger the more pronounced is the increase of ϕ_P. However, as the stock of automation capital cannot be adjusted instantaneously, the household increases the rate of its utilization, β_P. For the large shock size regime, the new long-run β_P closely mirrors a full automation capital utilization model. We also see that for both shock sizes, β_P overshoots its higher long-run equilibrium. In turn, the higher expected long-run stock of automation capital raises the expected long-run productivity of physical capital as well. This causes a jump of its utilization rate, β_K, but, importantly, this jump is less pronounced compared to the jump of ϕ_P in order to maintain the equality in their respective returns (see (22)). Thereafter, as more automation capital is accumulated, its average product falls and β_P converges from above to its new steady state equilibrium. In turn, as the average product of physical capital rises, β_K increases from below to its new steady state equilibrium.

Further, the higher long-run productivity of physical capital (due to automation capital accumulation and higher utilization) causes an immediate increase of the consumption-capital ratio, χ (cf. Proposition 4), accompanied by a pronounced drop of the fraction of investment devoted to automation capital investments, τ. Thereafter, as the average product of automation capital excels the average product of physical capital, the investment in automation capital recovers. Thus, τ increases from below to its new steady state equilibrium.

This corresponds with the dynamic adjustment of the consumption-capital ratio, χ. After the shock, automation capital is utilized more heavily than physical capital, which implies that the depreciation rate of automation capital is far above the depreciation rate of physical capital. This, in turn raises the productivity of automation capital more than the productivity of physical capital. Thus, investments in automation capital increase while initially overshooting investments in physical capital decline. This also implies that, first, the initially increased consumption-capital ratio increases further but at a diminishing rate as capital and consumption converge to the unique steady state growth rate (26); second, we observe that the automation-physical capital ratio monotonically increases but at a diminishing rate towards its new steady state equilibrium.

c) ***Speed of Convergence***
Our experiment shows that the adjustment speed towards the new steady state increases with ϕ_p. Stated in other words, the speed of convergence increases with the size of the elasticity of depreciation with respect to automation capital utilization, ϕ_P. Reflecting the large shock regime, inter alia, we hardly see a dynamic adjustment phase for the automation capital utilization rate, β_p. This can be analytically verified by looking at (20): For large but finite ϕ_P, the growth rate of β_P is close to zero. A similar behavior can be observed for the utilization rate of physical capital if ϕ_K is large but different from ϕ_P. As an example, let us consider an economy that initially utilizes a large but finite fraction of its capital stocks at different rates. Now, if ϕ_P increases, $\frac{\beta_P}{\beta_K}$ goes to one and hence, the automation-physical capital ratio hardly changes (see (22)) and, further, the adjustment to the new steady state equilibrium is relatively fast for the model's variables[22]. To sum up, for sufficiently low ϕ_i, $i = \{P, K\}$, the model implies a considerable reduction of the convergence rates compared to a setting where the depreciation rates of both types of capital are exogenous.

To summarize, the introduction of (automation) capital utilization as an optimal choice in an one-sector, two capital model of endogenous growth offers a simple but plausible way to reduce the speed of convergence provided the elasticities of depreciation with respect to physical and automation capital utilization are sufficiently heterogenous. Together with the findings made by Dujava and Labaj (2019) who estimate a reduction of today's empirically plausible speeds of convergence for countries

[22] The results are available upon request from the authors.

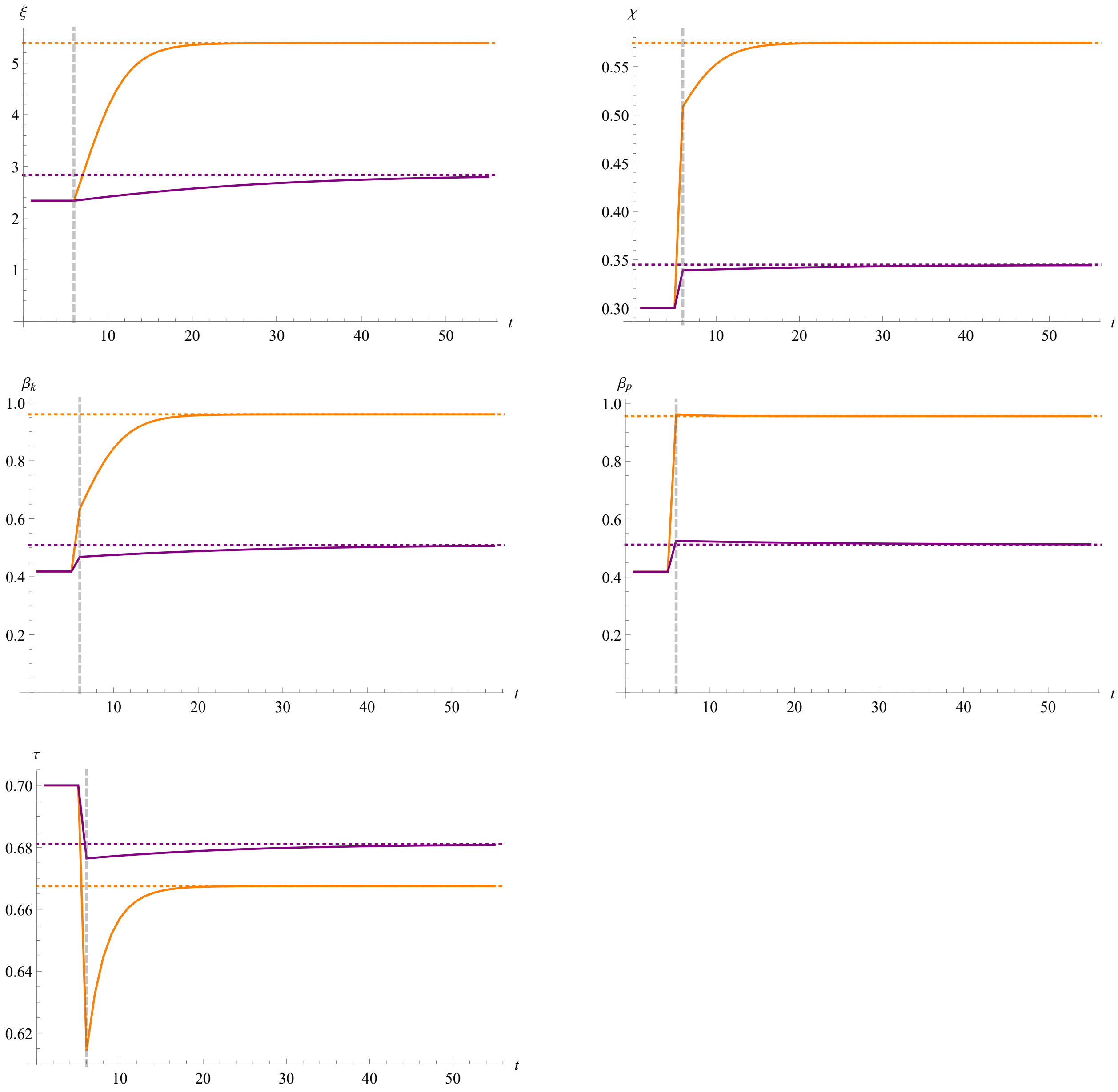

Figure 2: Transitional dynamics after an unanticipated ϕ_P shock in $t = 6$.

Note: The impulse responses in orange color show the effect of a large, while the impulse responses in purple show the effect of a small shock. The dotted lines show the new steady state equilibrium after realizing the shock, whereas the grey dotted, vertical line visualizes the point in time of the shock.

when robots are employed in the production process, our model is thus able to produce convergence rates that are in line with empirical evidence.

7 Conclusion

This paper introduces a simple model that discusses the implications of idle automation and physical capital for economic growth and convergence. Under some relatively mild assumptions, we establish the existence of a unique balanced growth path equilibrium that is locally saddle-path stable. We also show that capital utilization can bring down the convergence speed provided that the elasticities of depreciation with respect to physical and automation capital utilization are sufficiently heterogenous. If not, or if heterogenous utilization elasticities tend to infinity, we show that the model (nearly) converges to a standard AK model, which implies that we observe little or no transitional dynamics. We argue that if automation capital implies a further reduction of today's empirically plausible convergence rates, the introduction of idle (automation) capital provides a useful way bringing down the speed of convergence. Moreover, by comparing our model with the full utilization setting within an extensive numerical exercise, we find that the "standard" model overstates the speed of convergence by about 11% and 98%, and, the growth rate by about 51% and 98%. Our model yields a number of predictions that can be investigated further empirically. For instance, data for robot stocks can be used to estimate the elasticity of depreciation with respect to automation capital utilization. As our numerical exercises show, a precise interval estimate of the elasticity of automation capital utilization is from crucial importance as, for some cases, even small changes of this elasticity lead to sizable long-run changes and short- as well as medium-run adjustment dynamics of the consumption-output and the output-capital ratio. There are several avenues for further research: First, it can be then tested whether the required pronounced heterogeneity between the elasticities of depreciation with respect to physical and automation capital utilization to bring down the speed of convergence is in line with the data. Second, it would be also straightforward to introduce idle (automation) capital in the Lankisch et al. (2019) framework in order to investigate the role capital utilization might play explaining the empirical evolution of the skill premium. Third, it would be interesting to include capital utilization into a Schumpeterian growth context in order to explore the effects of an automation subsidy policy versus an R&D subsidy on welfare as recently done by Chu et al. (2022).

8 Appendix

8.1 Proof of Proposition 5.

Note first that

$$\mathscr{D}(\rho) = \underbrace{(1-\alpha)(\phi_P - 1)\,\xi^{*-\frac{\alpha(\phi_K-1)\phi_P}{\alpha(\phi_K-\phi_P)-\phi_K+\phi_K\phi_P}-1}}_{>0}\frac{A(\rho)}{B(\rho)}, \tag{36}$$

with

$$\begin{aligned} A(\rho) \quad &\equiv \quad \left[\alpha^2\Gamma(\phi_K-1)+\rho^2\phi_K\xi^{*-\mu}\right]\left[\alpha\phi_P-\phi_K\left(\alpha+\phi_p-1\right)\right] \\ &+ \quad \alpha\Gamma\rho\,(\phi_K-1)\,\xi^{*-\mu}\left[\phi_K\,(\alpha+2\phi_P-1)-\alpha\phi_P\right] \end{aligned}$$

and

$$B(\rho) \equiv \left[\phi_K\,(\alpha+\phi_P-1)-\alpha\phi_P\right]\left\{\alpha\Gamma\,(\phi_K-1)\,\phi_P+\rho\,\xi^{*-\mu}\left[\alpha\phi_P-\phi_K\,(\alpha+\phi_P-1)\right]\right\}.$$

The transversality conditions (2) imply that $\rho > 0$. $A(\rho)$ is a quadratic equation in ρ with $A(0) < 0$ if $\mu > 0$ per assumption. Solving $A(\rho) = 0$, it has two solutions in ρ, $\rho_1 > 0$ and $\rho_2 > 0$. $A(\rho)$ takes a local maximum value at $A(\rho_{max}) > 0$. Thus $A(\rho)$ is first increasing and after attaining its maximum at $\rho = \rho_{max}$ it is decreasing for $\rho > \rho_{max}$ and takes only positive values for $\rho \in (\rho_1, \rho_2)$. Hence, $0 < \rho_1 < \rho_{max} < \rho_2$. Let us turn to $B(\rho)$ which is linear and strictly decreasing in ρ with $B(0) > 0$ if $\mu > 0$ per assumption. Assume that ρ_3 solves $B(\rho) = 0$. Finally, it can be shown that $\rho_0 \equiv \frac{\alpha\xi^{*\mu}\Gamma(\phi_K-1)}{\phi_K} < \rho_1 < \rho_3$, so that for $\rho \in (0, \rho_0)$, $A(\rho) < 0$, $B(\rho) > 0$ and $\gamma_c|_{\rho<\rho_0} > 0$. Thus $\mathscr{D}(\rho) < 0$ for $\rho \in (0, \rho_0)$. Hence, the dynamic system can be characterized with one stable and one unstable Eigenvalue for $\rho \in (0, \rho_0)$.

8.2 Comparison of the capital utilization model with a "standard model" of full capital utilization

$\phi_K = 1.5$	$\phi_P = 1.5$	$\phi_P = 1.6$	$\phi_P = 1.7$	$\phi_P = 2$	$\phi_P \to \infty$
Consumption-output ratio	-2.6506	-2.1499	-1.8077	-1.2289	-0.1040
Output-capital ratio	0.7260	0.6567	0.5893	0.4055	-1.0824
Automation investment	0	0.0128	0.0195	0.0271	0.0280
Growth rate	0.9838	0.9588	0.9345	0.8682	0.3311
Speed of convergence	0.9838	0.9563	0.9360	0.8961	0.7573
$\phi_K = 1.6$	$\phi_P = 1.5$	$\phi_P = 1.6$	$\phi_P = 1.7$	$\phi_P = 2$	$\phi_P \to \infty$
Consumption-output ratio	-2.3597	-1.9419	-1.6502	-1.1442	-0.1047
Output-capital ratio	0.7238	0.6600	0.5986	0.4326	-0.8850
Automation investment	-0.0144	0	0.0081	0.0187	0.0258
Growth rate	0.9706	0.9447	0.9197	0.8523	0.3173
Speed of convergence	0.9769	0.9447	0.9213	0.8762	0.7238
$\phi_K = 1.7$	$\phi_P = 1.5$	$\phi_P = 1.6$	$\phi_P = 1.7$	$\phi_P = 2$	$\phi_P \to \infty$
Consumption-output ratio	-2.1325	-1.7753	-1.5217	-1.0721	-0.1038
Output-capital ratio	0.7202	0.6606	0.6034	0.4501	-0.7464
Automation investment	-0.0239	-0.0089	0	0.0123	0.0238
Growth rate	0.9580	0.9314	0.9059	0.8376	0.3040
Speed of convergence	0.9679	0.9319	0.9059	0.8564	0.6930
$\phi_K = 2$	$\phi_P = 1.5$	$\phi_P = 1.6$	$\phi_P = 1.7$	$\phi_P = 2$	$\phi_P \to \infty$
Consumption-output ratio	-1.6797	-1.4322	-1.2496	-0.9106	-0.0967
Output-capital ratio	0.7076	0.6556	0.6064	0.4766	-0.5038
Automation investment	-0.0383	-0.0238	-0.0144	0	0.0193
Growth rate	0.9244	0.8963	0.8697	0.7994	0.2686
Speed of convergence	0.9325	0.8888	0.8577	0.7994	0.6143
$\phi_K \to \infty$	$\phi_P = 1.5$	$\phi_P = 1.6$	$\phi_P = 1.7$	$\phi_P = 2$	$\phi_P \to \infty$
Consumption-output ratio	-0.4157	-0.3837	-0.3550	-0.2875	0
Output-captial ratio	0.5996	0.5708	0.5441	0.4756	0
Automation investment	-0.0499	-0.0435	-0.0384	-0.0282	0
Growth rate	0.6495	0.6182	0.5893	0.5152	0
Speed of convergence	0.1362	0.1317	0.1263	0.1104	0

Table 3: Comparison of the capital utilization model with a "standard model" of full capital utilization.

Note: Numbers show relative differences in steady state equilibrium levels.

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