Titel/Title: The Cube Rule in a Mixed Electoral System: Disproportionality in German Bundestag Elections

Autor*innen/Author(s): Philip Manow

Veröffentlichungsversion/Published version: Postprint

Zeitschriftenartikel/Journal article

Empfohlene Zitierung/Recommended citation:
Philip Manow (2011) The Cube Rule in a Mixed Electoral System: Disproportionality in German Bundestag Elections, West European Politics, 34(4), 773-794.

Verfügbar unter/Available at:
(wenn vorhanden, bitte den DOI angeben/please provide the DOI if available)
The original article is available at: https://doi.org/10.1080/01402382.2011.572391

Zusätzliche Informationen/Additional information:
The author can be contacted at: manow@uni-bremen.de

The Cube Rule in a Mixed Electoral System: Disproportionality in German Bundestag Elections

PHILIP MANOW

In the German mixed electoral system the PR tier is generally perceived as fully compensating for any disproportionality in the vote–seat translation generated by the plurality tier. However, as this article shows, the PR tier can itself increase disproportionality. In a mixed electoral system, small parties enter (hopeless) district races with the hope of boosting their PR vote share. But with a high number of district parties, parties may win districts at levels way below the usual 50 per cent vote share threshold. Looking at all 16 Bundestag elections from 1953 to 2009, the article identifies the effective number of district parties as a very strong predictor for the disproportional translation of votes into seats in the plurality tier of Germany's mixed electoral system. The article points to consequences for the internal composition of parliamentary parties, for parties' nomination strategies and for the occurrence of so-called overhang mandates.

Introduction

That plurality voting systems translate votes into seats in a nonlinear (disproportional) way, in which the ratio of seat shares of two parties roughly corresponds to the ratio of their vote shares cubed, has long been known and was first described for the British first-past-the-post electoral system (Kendall and Stuart 1950, 1952).[1] This finding is referred to as the Cube Law or Cube Rule in the literature (Taagepera and Shugart 1989; Grofman 1983; Gudgin and Taylor 1979; King and Browning 1987). We also find this pattern in the German mixed electoral system when we focus solely on its plurality tier, i.e. on the connection between candidate votes and district mandates (see Figure 1 and Table 1). However, the literature on Germany's mixed electoral system has rarely addressed this nexus, presumably because its PR tier is perceived as ensuring a full neutralisation of the disproportional seat distribution caused by the

Correspondence Address: manow@zes.uni-bremen.de

plurality tier, so that it was thought unnecessary to pay attention to disproportionality.

Yet, as will be shown below, the list vote does not completely neutralise the disproportionality caused by the candidate vote; in fact, it can even exacerbate it. This happens particularly in cases where a large number of (small) parties strive to increase their list votes by putting up candidates in district races although they have no reasonable prospect of winning them (see Cox and Schoppa 2002; Ferrara et al. 2005). As the number of district parties rises, the translation of candidate votes into seats becomes more disproportional. Furthermore, if, due to a high number of district parties, vote shares between 30 and 40 per cent in the candidate vote are enough to win a large number of districts, then this also increases the probability of a discrepancy between list vote shares for any given party and the share of districts won by it – and thus the probability of so-called overhang mandates. In the German electoral system about half the seats in parliament are assigned to candidates who gain a relative majority of candidate votes in one of the – currently – 299 single-member electoral districts. The other half are distributed among the parties according to the relative vote share that the regional party lists receive – after subtracting the number of seats already won by district candidates. But if the number of district mandates (Direktmandate) is larger than the number of seats that would have been allocated to the party on the basis of its list votes, the party can 'keep' these mandates as additional, so-called overhang mandates.

The increasing number of these overhang mandates in recent Bundestag elections[2] point to disproportionality in the plurality tier of Germany's mixed electoral system. This is presumably due to the increase in the effective number of (district) parties caused by the advent, first, of the Green party in the early 1980s, and then, later, by the establishment of the ex-communist PDS/Die Linke in the German party system after unification. This poses several questions about the disproportionality found in the plurality tier of the German mixed election system. How has it developed over time? What factors have influenced it? For this purpose I compare the ratio of candidate votes to the percentage of district mandates for the two major parties, Social and Christian Democrats (SPD and CDU/CSU) over time and at the state (Land) level. As will be shown, the number of district parties strongly affects the proportionality of the translation of candidate votes into district mandates. In demonstrating that the number of district parties does affect the proportionality of the plurality tier in Germany's electoral system and that its impact is strong and stable, the paper contributes to the literature on mixed electoral systems.

The article proceeds as follows: it first briefly examines the disproportional distribution of district mandates in the German electoral system. It then discusses the Cube Rule and suggest a modification when applying it to the German electoral system. This accommodates the possibility that, in a mixed electoral system, the effective number of district parties regularly

does not converge to two, i.e. that 'Duverger's Law' does not apply (Cox and Schoppa 2002). In fact, in Germany the average effective number of district parties is currently 3.7, not 2 (see below). The following section examines the ratio of candidate votes to district mandates on the national level over time. The article next provides a – longitudinal and cross-sectional

– overview of the development of disproportionality, for Bundestag elections since 1953 on the state level, on which the clearing between the list and candidate votes takes place. It then addresses the determinants of disproportionality – particularly the influence of the effective number of parties and of 'district size', meaning here the number of districts/seats per state. The next section briefly points to three consequences of increased disproportionality. First, the major parliamentary party in government is composed mainly of MPs who are directly elected in a district, whereas the main opposition party comprises mainly of MPs who were voted into parliament via the list. Second, parties need not only insure their district candidates against a potential vote loss and the ensuing disproportionate loss of district mandates, they also need to insure – in the case of vote gains – their list candidates against a disproportionate increase of district mandates, since these can render all positions on a party list unsafe. Using the instrument of the double candidacy, i.e. nominating candidates both in a district and on a list, is a tactic German parties have used to ensure their candidates against both eventualities. Finally, I show that the disproportionality of the plurality tier caused by the high number of district parties is a very strong predictor for the so-called overhang mandates which occur whenever a party wins more districts than seats would have been allocated to it according to its PR vote share. The article concludes with a summary of the argument and an outlook for future research.

Two Dimensions of Disproportionality

Figure 1 shows how the candidate vote shares for the CDU/CSU and SPD in the last 16 Bundestag elections[3] (on the x-axis) translate into the corresponding shares of district mandates awarded to both parties (on the y-axis). The result is the well-known non-linear pattern in which disproportionality increases – that is, in which there is a particularly high slope of the curve, near the 50 per cent mark for candidate votes (see Figure 1). The curve of the votes-to-seats ratio dissects the diagonal depicting the ratio of perfect proportionality at about 50 per cent. Close to this point, small vote gains lead to significantly overproportionate increases in the number of seats and small vote losses to significantly overproportionate decreases. Figure 1 also shows the graph of the Cube Rule if the exponent rho (ρ) is set as equal to 3.[4] The Cube Rule predicts the disproportionality pattern in the plurality tier of the German mixed voting system almost perfectly. If one estimates ρ according to the German data, it equals 3.17 with an explained variance of 0.986 (see Table 1).

FIGURE 1
CANDIDATE VOTE SHARE AND SHARE OF DISTRICT MANDATES, CDU/CSU AND SPD, 1953–2009 AT THE NATIONAL LEVEL

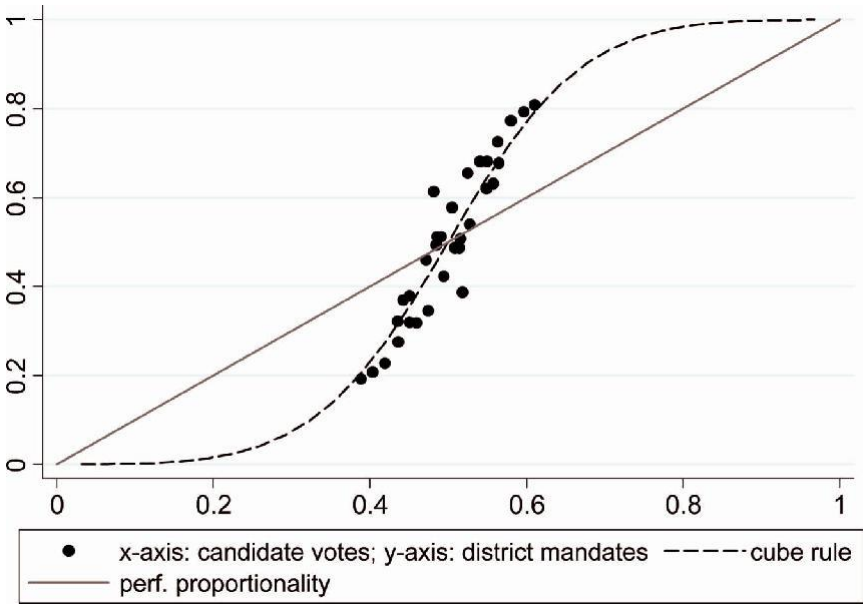


TABLE 1
DISTRICT MANDATE SHARES FOR CDU/CSU AND SPD IN THE BUNDESTAG ELECTIONS 1953–2009, DEPENDENT ON THEIR SHARE OF THE CANDIDATE VOTE; ESTIMATION OF THE PARAMETERS ρ

r (ρ)	3.175 (0.000)***
Observations	32
R^2	0.986

Note: p-value in parenthesis, *significant to the 10% level; **significant to the 5% level; ***significant to the 1% level.

In the literature on the German electoral system, this disproportionate distribution pattern within the plurality tier is rarely debated. As frequent references to 'personalised proportional representation' indicate (see § 1 para. 1 Federal Election Law), the view prevails that the element of plurality in the German electoral system is of secondary importance and at most introduces an element of personalisation in a voting system otherwise shaped by proportional representation (Schreiber 1994). However, this viewpoint disregards the fact that the disproportionality 'produced' by the candidate vote is not completely neutralised by the list vote. On the contrary, as we shall see, disproportionality can even be exacerbated. Therefore, it is worth examining in depth the ratio between candidate votes and district mandates.

It is apparent from Figure 1 that the degree of disproportionality varies from election to election. Disproportionality is dependent on at least two factors: the 'district size' and the effective number of district parties.

District size refers to the size of the 'secondary district' (Cox 1997: 49), i.e. to the number of constituencies and thus district mandates in each German state (Land), because it is at the state level that the clearing between list-vote shares and number of districts won by a party takes place. Here, disproportionality in the translation of votes into seats can be due to a simple 'arithmetic effect'. This might be demonstrated by a simple numerical example: in a (single-member) district, a party can receive 0 or 100 per cent of the seats; in a total of two districts, 0, 50, or 100 per cent; in three constituencies, 0, 33, 66, or 100 per cent, etc. In other words, the more constituencies there are, the more closely the share of seats won may approximate the share of votes received – whereby the maximal number of seats always defines the upper limit of proportionality (Taagepera and Shugart 1989). Therefore, it should make a difference for the proportionality of seat distribution whether the vote/seat translation takes place on the basis of two districts (as in Bremen 2002 and 2005) or of 73 districts (as in North Rhine-Westphalia in the Bundestag elections between 1965 and 1976). Also, with smaller secondary districts, deviations from a normal distribution of voters become more likely (Gudgin and Taylor 1979). This applies not only to a comparison of states of different size, but also to a longitudinal comparison of parliamentary elections: when we compare Bundestag elections over time, it may make a difference whether – as after German unification – the number of states with comparably few constituencies increased.

With regard to the number of parties,[5] it is not inevitable but probable that disproportionality grows as the number of participating parties increases. When only two parties are competing, a party needs at least 51 per cent of the vote in order to win 100 per cent of the seats in a (single-member) constituency; with three parties it needs at least 34 per cent; with four parties at least 26 per cent, etc. Consequently, a large number of parties, particularly a strong 'third party' (Behnke 2007), can lead to a constellation in which a party with a relatively low percentage of candidate votes still wins many districts. There is then also a higher probability that the percentage of list votes and the percentage of district mandates won in a state will diverge, leading to overhang mandates (Behnke 2003a, 2003b). These overhang mandates are criticised because they appear to base parliamentary majorities on a moment of arbitrariness in the conversion of votes into seats. When the percentages of list votes and of district mandates differ, it is probably also due to the increased opportunities for vote splitting when a higher number of parties contest districts (see Gschwend 2003). When many smaller parties compete in a constituency, more voters may decide not to 'waste' their

candidate vote and to cast it for a promising local candidate (Bawn 1999).

In the end, this indicates that the PR tier in the voting system, which was actually designed to balance out the disproportionality linked to the plurality tier, can – on the contrary – also enhance its disproportionate effect. This points to the debate on 'contamination' between the different tiers of a mixed electoral system (Herron and Nishikawa 2001; Ferrara et al. 2005; Cox and Schoppa 2002). In this recent literature, one central indicator for contamination is usually considered to be the number of parties competing in a district. In a mixed electoral system, this number does not follow Duverger's Law – that is, it does not converge into two – but remains higher because smaller parties send constituency candidates into what are actually hopeless races in the hope that they can positively influence their percentage of the list vote in the district (Hainmüller and Kern 2008; Manow 2011). This is a classic case of contamination, which is defined in the literature as existing

> whenever at the micro-level, the behaviour of a voter, a party, a candidate or legislator in one tier of the election is demonstrably affected by the institutional rules employed in the other tier. At the aggregate level, contamination is observed when a particular outcome produced in one tier (like the number of parties) is affected by the institutional features of the other tier. (Ferrara et al. 2005: 9)

If, however, these contamination effects give parties a systematic incentive to compete in districts they have no hope of winning; if, at the same time, the number of competing parties influences the degree of disproportionality in the plurality tier; and if this eventually increases the probability that the parties' percentages of district mandates also deviate considerably from their share of the list vote, then we are dealing here with a contamination effect that has not been noticed and studied previously. Previous work focused on the effects that strategic party action within the plurality tier had on results within the proportional representation tier – namely, more list votes due to (even hopeless) party candidatures for a district mandate in constituency races. Here, however, we are confronted with an effect in which party strategies geared toward the PR tier have an impact on the results within the plurality tier – namely, an increased disproportionality caused by a higher effective number of parties competing in a constituency.

This has methodological consequences for the context considered here. The Cube Rule was formulated based on a strictly (British) single-member district voting system and therefore assumes implicitly the existence of a two-party system. It is precisely this development that is blocked in the German election system, which combines elements of plurality voting and proportional representation. The high effective number of constituency parties must therefore be accounted for in the formula to calculate

disproportionality. The applied Cube Rule formula (see above, note 4) needs to be modified. I supplement it here with the additional parameter β, which makes it possible that the estimated function – unlike the 'classic' Cube Rule – predicts a seat share far above 50 per cent even before a vote share reaches 50 per cent. The modified formula, on which the following estimations and figures are based, therefore is:

$$Seatshare = \left\{1 + e\left[-\rho \ln\left(\frac{Voteshare + \beta}{1 - Voteshare - \beta}\right)\right]\right\}^{-1} \qquad (2)$$

Now the formula contains two parameters that measure, on one hand, the slope of the curve (ρ) and, on the other, its shift to the left (β). Figure 2 shows, by way of illustration, the course of the curve of the function for different values of ρ (3, 5, 10 and 20) and β (0, 0.1, 0.15, 0.2).

In the following sections, I apply such a modified Cube Rule formula in order to estimate ρ and β for Bundestag elections over time and as a basis of comparison among the German states.[6] In turn, these estimated parameters serve as dependent variables to study the effect of district size and effective number of parties on the degree of disproportionality within the plurality tier in Germany's mixed election system. It is suspected that district size primarily influences the slope of the curve (Taagepera and Shugart 1989: chapter 15), while the effective number of parties primarily influences their shift to the left.

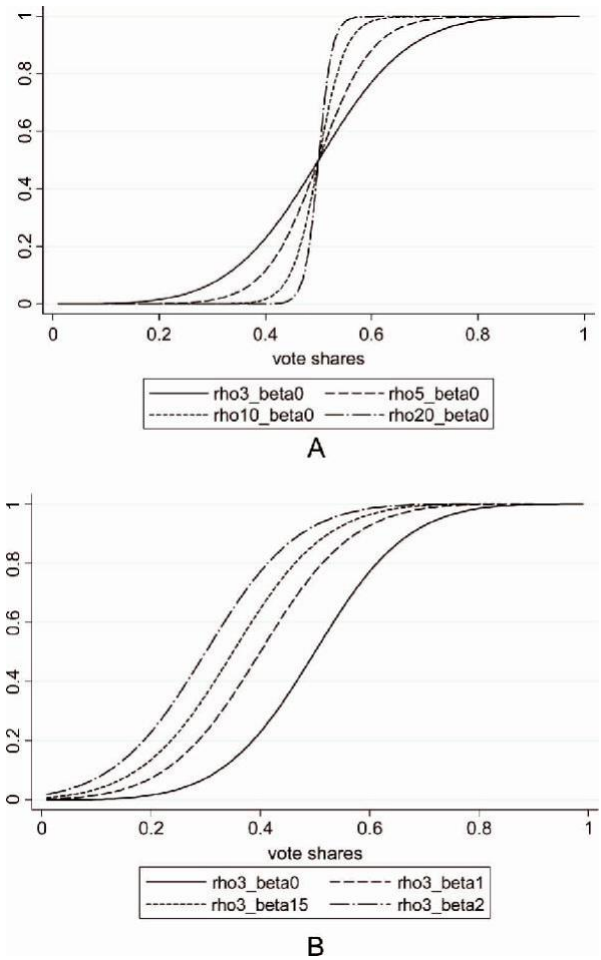
Candidate Votes and District Mandates: A Study of Bundestag Elections and the German States

Disproportionality as an Indicator of Electoral Success: The National Level

When we consider the plurality tier at the national level for each of the two major parties that have been able to win the vast majority of constituencies – the Christian (CDU/CSU) and Social Democrats (SPD) – several correlations become immediately noticeable (see Figure 3).

Figure 3 confirms the known finding that the SPD suffers from a structural disadvantage with regard to candidate votes and district mandates. In the majority of the Bundestag elections, the CDU/CSU wins substantially more constituencies than the SPD. We can conjecture that the primary reason for this is linked to electoral geography (Rodden 2010; Gudgin and Taylor 1979). While the SPD wins several constituencies (Germany's industrial centres and heavily populated areas) with a large majority, it also loses many (mainly rural) constituencies by a much narrower margin. From the standpoint of the party, the number of 'wasted votes' is therefore higher for the SPD than for the CDU/CSU. This is a

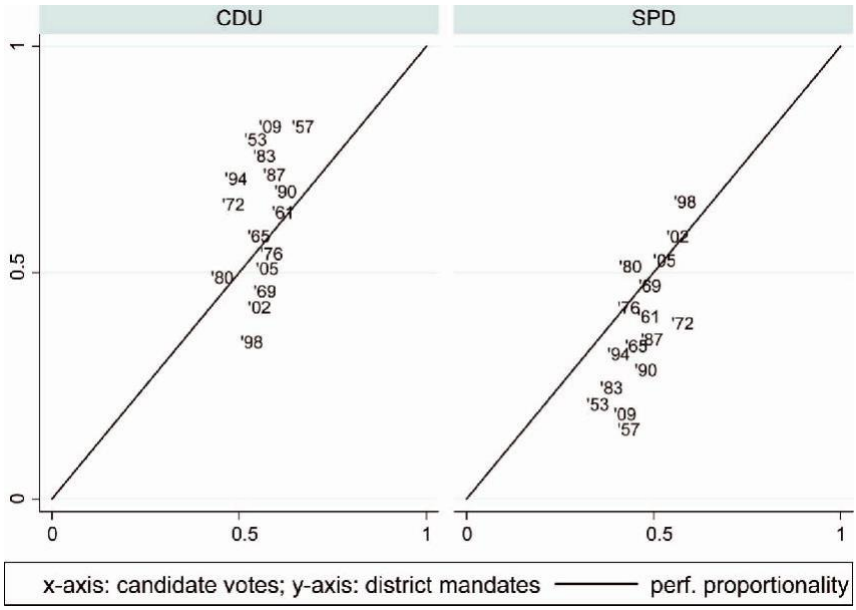
FIGURE 2
(A) DISPROPORTIONATE CONVERSION OF VOTES INTO SEATS, DEPENDENT ON VARIOUS VALUES FOR ρ (WHEN $\beta = 0$)
(B) DISPROPORTIONATE CONVERSION OF VOTES INTO SEATS, DEPENDENT ON VARIOUS VALUES FOR β (WHEN $\rho = 3$)



general pattern in plurality voting systems for parties with a regionally concentrated electorate.

As evident in Figure 3 is the connection between the general election success of a party and its overproportionate gain of district mandates. We could even, with the aid of Figure 3, reconstruct a history of the alternating electoral successes of the two major parties. In it, the first two Bundestag elections examined here – in 1953 and 1957 – mark the positive and negative extreme points for the CDU/CSU and the SPD, respectively. It is not particularly surprising that the extraordinary election of 1957, in which a

FIGURE 3
SHARES OF CANDIDATE VOTES AND SHARES OF THE TOTAL NUMBER OF DIRECT MANDATES IN THE BUNDESTAG ELECTION 1953–2009 FOR THE CDU AND SPD



German party won an absolute majority of votes and seats for the first and – so far – last time, was also the election in which the distribution of district mandates was out of proportion to the most extreme degree observed to date. Correspondingly, it was the Bundestag elections in which the SPD was particularly successful – namely, the elections of 1969, 1998, and 2002, which propelled the Social Democrats over the line of perfectly proportional representation with regard to district mandates.

Yet we find no proof of a secular trend toward ever greater proportionality between candidate votes and district mandates. This strengthens the presupposition that the degree of disproportionality for the plurality tier of the German electoral system is possibly influenced by the number of parties competing for votes. In the elections of 1953 and 1957 and once again in recent Bundestag elections (particularly those of 1998 and 2002), a relatively high degree of disproportionality is evident. In the interim period, however, the ratio between the percentage of candidate votes and district mandates lies closer to the ideal line of proportional representation. Whether this finding can actually be attributed to the relatively high (effective) number of parties in both the early elections and those held recently is the topic to be examined in the following section.

To do this, however, it is necessary to look at the ratio between the percentages of votes and mandates at the level of the German states, because an examination at the national level can paint a misleading picture (in which disproportional seat distribution in one state – for instance, a SPD stronghold – is possibly offset by disproportional seat distribution in another – for instance, a CDU stronghold). Yet it is at the state level, in the course of the so-called Unterverteilung or 'sub-distribution' (according to §6 of the Federal Election Law), where the actual adjustment occurs between the district mandates won and the seats won by a party based on its share of list votes, and also where a disproportionate ratio of list votes to the number of constituencies won can lead to overhang mandates. Furthermore, the state level offers us the chance to look into the effect of district size and of the effective number of parties on disproportionality. With regard to the number of parties, it is necessary to study not only the ratio of CDU votes and seats to SPD votes and seats, but also the shares of each respective party with regard to both the percentages of votes and of seats against the votes and seats of all other parties. This is the subject of the following section.

Disproportionality in the Plurality Tier of the German Electoral System: The State Level

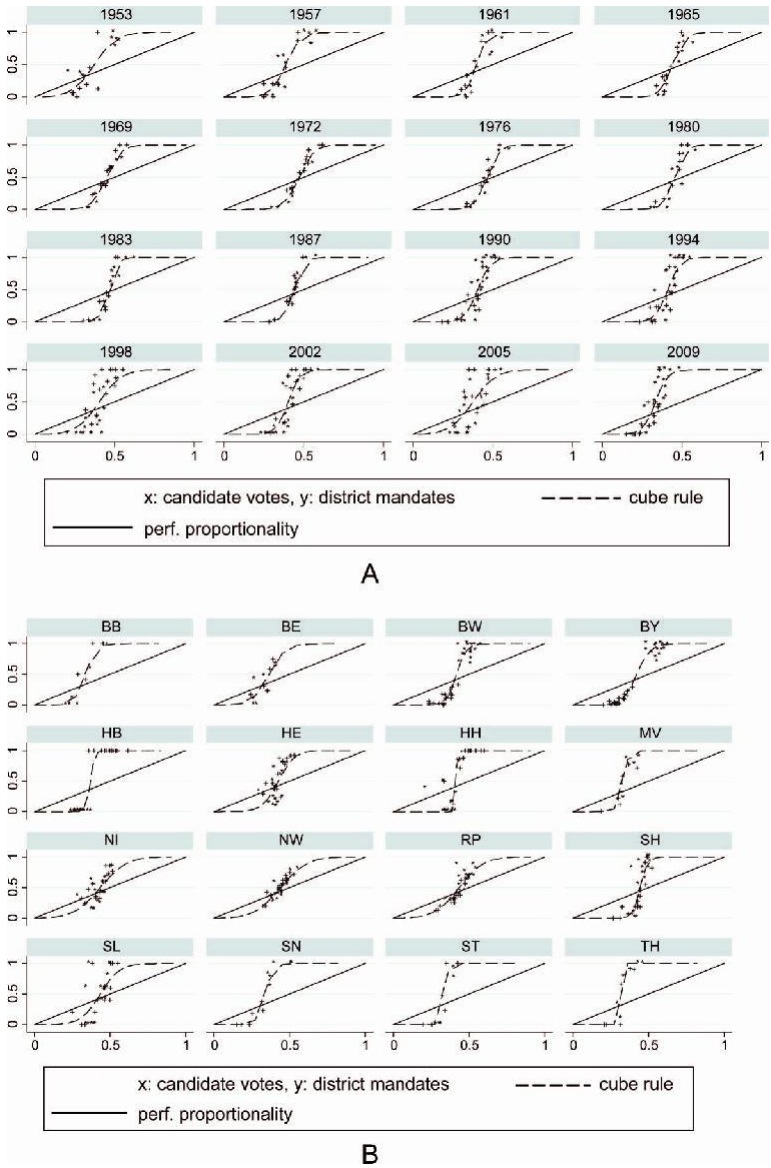
Bundestag elections. Looking at the nexus between a party's share of candidate votes and its share of district mandates at the state level allows us to pay particular attention to the influence on disproportionality exerted by the 'district size'. In this dimension, we see significant variance among the states – from two seats in Bremen in the Bundestag elections of 2002 and 2005 to the 73 district mandates in North Rhine-Westphalia in the elections from 1965 to 1976. From the literature we know that district size is important for the degree of disproportionate conversion of votes into seats (Taagepera and Shugart 1989). However, just how greatly it actually influences proportionality or disproportionality can only be answered empirically.

In the following, ρ and β are estimated for the 16 Bundestag elections since 1953 based on a dataset that includes the percentages of candidate votes for the CDU/CSU and the SPD as well as their percentage of district mandates at the state level for each of the elections. The variable Candidate Vote Share indicates the percentages of the total number of valid candidate votes that were won by the SPD and the CDU/CSU, respectively, while the variable District Mandate Share represents the percentages of the total number of district mandates awarded to each of these parties. Furthermore, the dataset contains information on the effective number of parties at the state level in each Bundestag election (Effective Parties) (see Laakso and Taagepera 1979) as well as data on the total number of district mandates to be won at the state level (District Size). The voting data is based on the results of the Bundestag

elections at the constituency level (aggregated at the state level), as they were made available by Germany's Federal Election Office (Bundeswahlleiter). The data on district mandates at the state level has been taken from the Datenhandbücher zur Geschichte des Deutschen Bundestags (Schindler 1999; Feldkamp 2005, 2006). The number of observations is based on a total of 16 elections (the 1949 election is excluded), of which there was one election with nine states (1953),[7] nine with ten states (1957–87),[8] and six with 16 states (since 1990). Candidate votes and district mandates are stipulated for the CDU/CSU and SPD respectively, so that the dataset included a total of $2 \times (9 + 9 \times 10 + 6 \times 16) = 39$ observations.[9] When considering individual Bundestag elections or individual states, the number of observations is limited accordingly: in the case of Bundestag elections, to a minimum of 18 and a maximum of 32 observations; in the case of individual states, to a minimum of 12 and a maximum of 32 observations.

When we observe the Bundestag elections at the state level over the course of time and based on the modified Cube Rule formula (see above), several important regularities can be seen in the graphic depiction. In Figure 4, the two major political parties are represented by their own symbolic markers (CDU/CSU (*), SPD (+)), which enables us to follow the relative success of both parties in each of the national elections. Compared to the national level (see Figures 1 and 3), we first see a clearly more disproportionate distribution of seats at the state level. The entire value scale of the y-axis is covered with states in which the CDU/CSU and the SPD obtain either all or none of the district mandates. At the same time, the presupposition is confirmed that a relatively high disproportionality is evident in the early Bundestag elections and then again after 1990, whereas the elections for the sixth to the eleventh Bundestag elections (1969 to 1987) show less 'polarised' results, the slope of the curve is flatter, and the line for perfect proportionality is crossed near the 50 per cent mark for vote share. Overall, the deviation from the line of an exactly proportional distribution of votes and seats at the state level is far more pronounced, and ρ always lies over 3 (the range of values for each of the Bundestag elections lies between 3.5 and 9.3; see Table 2a below). In addition, we can see that the supposedly more important effect in converting candidate votes into district mandates at the state level lies in a shift to the left of the curve – that is, in a β that is clearly different from zero (which varies between 2.8 and 12.8 percentage points for individual Bundestag elections, see Table 2a below). It is here – so it appears – that a greater number of parties makes a noticeable difference. Finally, the effect of a consolidation of the German party system until 1961 can be seen in Figure 4 in the less dispersed scattering. After 1990 there are nearly two different functions of the votes-to-seats ratio – and the estimated Cube Rule function lies in the middle of them (see Figure 4, specially the Bundestag elections of 1998 and 2005). Besides, after German unification, distribution is often polarised once again, whereby the CDU/CSU or SPD win all or no district mandates in a state.

FIGURE 4
(A) CANDIDATE VOTES (X-AXIS) AND SHARE OF DISTRICTS (Y-AXIS), CDU/CSU AND SPD IN BUNDESTAGS ELECTIONS 1953–2009
(B) CANDIDATE VOTES (X-AXIS) AND SHARE OF DISTRICTS (Y-AXIS), CDU/CSU AND SPD IN BUNDESTAGS ELECTIONS AT THE STATE LEVEL 1953–2009



Notes: BB = Brandenburg; BE = Berlin; BW = Baden-Württemberg; BY = Bavaria; HB = Bremen; HE = Hesse; HH = Hamburg; MV = Mecklenburg-West Pomerania; NI = Lower Saxony; NW = North Rhine-Westphalia; RP = Rhineland-Palatinate; SH = Schleswig-Holstein; SL = Saarland; SN = Saxony; ST = Saxony-Anhalt; TH = Thuringia.

TABLE 2A
ESTIMATED ρ AND β, BUNDESTAGELECTIONS

Election	ρ	β	Adj. R^2
1953	3.679059	0.126763	0.8743
1957	5.5299	0.1124117	0.9326
1961	7.812453	0.1024887	0.9437
1965	6.805042	0.0598372	0.954
1969	5.605289	0.048757	0.974
1972	5.438842	0.028691	0.9804
1976	6.855029	0.0373722	0.969
1980	7.314088	0.0467738	0.9441
1983	9.318454	0.0349524	0.9673
1987	8.056896	0.0614341	0.9785
1990	6.203744	0.094211	0.8989
1994	6.361608	0.0880403	0.8817
1998	3.590299	0.0963051	0.8483
2002	6.760531	0.0968296	0.8927
2005	3.536517	0.1248424	0.8049
2009	6.501548	0.1663592	0.8907

TABLE 2B
ESTIMATED ρ AND β, GERMAN STATES

State	ρ	β	Adj. R^2
Schleswig-Holstein	10.31204	0.0554426	0.8763
Hamburg	55.33912	0.0861433	0.934
Lower Saxony	2.924057	0.0655062	0.9088
Bremen	17.31048	0.1305711	0.948
North Rhine-Westphalia	2.891581	0.0581984	0.9744
Hesse	4.688254	0.0731961	0.8692
Rhineland-Palatinate	3.124953	0.0652	0.9509
Baden-Württemberg	7.662721	0.0828948	0.9593
Bavaria	5.570434	0.0782112	0.9837
Saarland	3.98983	0.0641099	0.7789
Berlin	4.647363	0.1357335	0.8873
Brandenburg	7.184596	0.1625417	0.9515
Mecklenburg-Western Pomerania	13.10875	0.1692866	0.9365
Saxony	9.787505	0.1620606	0.975
Saxony-Anhalt	19.78164	0.160993	0.9251
Thuringia	45.85739	0.178481	0.9194

Likewise, the elections in which a party was successful can be clearly identified, such as the unification election of 1990 in which the CDU/CSU was particularly successful in winning district mandates, compared to the election of 1998 in which it was the SPD that was able to win the vast number of district mandates. Once again the pattern becomes evident in which the general electoral success or failure of a party, measured on its gain or loss of list votes, is reflected in the plurality tier to an overproportionate degree. This, in turn, translates into the high percentage of representatives

who hold district mandates within the majority parliamentary party of their respective governmental coalition.

The states. When we shift focus to the individual states, we find the presupposition confirmed that 'district size', meaning the number of constituencies in each state, clearly affects the proportionality of the conversion of votes into seats. Bremen represents an extreme case because it has three constituencies in some elections and only two in others (which were always won by the SPD). As a result, the curve for this state is extraordinarily steep. The city-state of Hamburg offers evidence of a similar situation.

The increase in comparatively small constituencies that occurred following German unification is identified in the literature as one of the reasons for the frequent appearance of overhang mandates. Likewise, a higher probability of 'polarised' seat distribution (100:0) can also be reported from several of the new German states, particularly Saxony-Anhalt and Thuringia. It is also characteristic of the new German states that the estimated function crosses the diagonal of perfect proportionality long before the 50 per cent mark of the share of candidate votes. This can be attributed especially to the impact of the large percentage of votes cast for the PDS/Die Linke (Manow 2010). It is hardly a surprise to find that the CDU/CSU is particularly successful in the former West German states of Baden-Wurttemberg, Bavaria, and even Rhineland-Palatinate when it comes to winning constituency races. In Hesse, the SPD is more successful, but the states of Lower Saxony, Schleswig-Holstein, and also North Rhine-Westphalia offer an inconsistent picture. Figure 4B offers an overview of the 16 states. Once again, the parties are separately designated with CDU/CSU (*) and SPD (+).

Determinants of Disproportionality – District Size and the Effective Number of Parties

In the following, degree of disproportionality is treated as a factor in need of explanation, that is, as a dependent variable. One peculiarity in this two-step regression design needs to be considered: we are dealing here with an estimated variable acting as a dependent variable, which in turn is to be estimated as a function of two independent variables: district size and the effective number of parties. One problem that this poses is the possible heteroscedasticity of the dependent variables that could be caused, for example, by different sample sizes (Lewis and Linzer 2005). In the case presented here, state election parameters are estimated in part on the basis of 32 observations ('old' German states), and in part on the basis of 12 observations only ('new' German states), while national election parameters are based on at least 18 and at most 32 observations. At first glance, one possible solution for the varying estimation accuracy of the dependent

variable would be a Weighted Least Square Regression, in which the dependent variable is weighted with the inverse of its standard error. As Lewis and Linzer have shown, this 'therapy' is often worse than the actual 'illness' because it treats the entire error of estimates for the second regression as being caused solely by the error of the first regression. Yet a part of the error term of the second regression is 'genuine' uncertainty, which can indeed be homoscedastically distributed (Lewis and Linzer 2005: 346). One of the solutions suggested by Lewis and Linzer consists of an OLS estimate with heteroscedastic consistent standard errors – a procedure that is particularly applicable when, as in the case here, the uncertainty of the estimate of y in relation to its variation is not large and does not vary greatly between the samples. Therefore, in the following regressions White's standard errors are reported. Tables 2a and 2b show the values of the ρ -and β-parameters for the Bundestag elections and the German states as estimated by the Cube Rule function.

I first regress the β-parameter for each of the Bundestag elections and each state on the average number of parties and on the average district size (Table 3). With German unification, comparably small 'districts' – measured against the number of constituencies/district mandates per state – entered the electoral system, which lowered the average size of the secondary districts (Cox 1997: 49) from the Bundestag elections of 1990. The effective number of parties has developed – as measured by the absolute number of candidate votes – from 3.7 in 1953 to 2.2 in the 1972 Bundestag elections, back to 3.7 in the most recent (2009) election. In a second step I regress the estimated ρ -parameters – again for the Bundestag elections and for each of the German states – on the district size and on the effective number of parties (see Table 4).

The findings reported in Tables 3 and 4 are very clear. The variable Effective Parties has a very robust and strong influence on the left shift of the Cube Rule function. When the effective number of parties rises from 2 to 3, for example, β increases by 7.5 (Bundestag elections) or 8.3 percentage points (German states). Both the β-parameter and the Effective Parties coefficient from Table 3 are to be interpreted directly. β indicates where the Cube Rule function crosses the line of a perfectly proportional vote-to-seat ratio. As becomes evident in Table 2b, a high percentage of votes won by a strong 'third party' such as the PDS/Die Linke in the new German states is reflected here in a β that lies well over 10 per cent. In other words, vote shares between 30 and 40 per cent are often more than enough to win a district. We can ascertain an increase in this parameter since the mid 1980s – in conjunction with the establishment of the Green Party in the German party system. It is the effective number of (district) parties that explains the vast amount of variance for this dependent variable. For the ρ-parameter on the level of the states, the theoretically expected 'arithmetic effect' in the conversion of votes into seats is confirmed – the smaller the number of constituencies per German state, the steeper the curve of the Cube Rule

TABLE 3
DETERMINANTS OF DISPROPORTIONALITY – THE β PARAM ETER

	β – Bundestag elections	β – German states
Effective parties	0.075 (0.000)***	0.083 (0.000)***
District size	70.002 (0.026)**	70.000 (0.367)
Constants	70.079 (0.032)**	70.139 (0.001)***
Observations	32	32
Adj. R^2	0.942	0.766

*Note: p-values in parentheses; *significant to the 10% level; **significant to the 5% level; ***significant to the 1% level.*

TABLE 4
DETERMINANTS OF DISPROPORTIONALITY – THE ρ PARAMETER

	ρ – Bundestag elections	ρ – German states
Effective parties	71.692 (0.010)***	0.957 (0.887)
District size	0.010 (0.916)	70.322 (0.045)**
Constants	10.695 (0.001)***	16.903 (0.472)
Observations	32	32
Adj. R^2	0.191	0.086

*Note: p-values in parentheses; *significant to the 10% level; **significant to the 5% level; ***significant to the 1% level.*

function. In other words, as the number of constituencies increase, the value of ρ decreases, and the slope of the curve for the Cube Rule function becomes flatter and therefore closer to the diagonal line of perfect proportionality. However, the number of districts per state and the effective number of district parties explains the variance in the case of the ρ-parameter to a much lesser degree than in the case of β.

Overall the regressions strongly substantiate the claim that the effective number of district parties has a marked impact on the vote–seat translation in the plurality tier of Germany's mixed electoral system. In this system, the effective number of parties running in each constituency is greater than two because smaller parties also put up constituency candidates in what are actually hopeless races for district mandates. By doing so, they hope the presence of the candidates will positively influence their share of the list vote (Hainmüller and Kern 2008; Manow 2011). As demonstrated, the number of participating parties strongly influences the degree of disproportionality, especially in the form of a left shift of the Cube Rule function. Finally, this also increases the probability that the percentage of district mandates won by a party and its list percentage of the vote deviate considerably from one another, which consequently increases the probability that overhang

mandates will be created. I will turn to these consequences of disproportionality in the next section.

Consequences of Disproportionality

I will briefly highlight three consequences of the increasingly disproportionate translation of votes into seats in the plurality tier of Germany's mixed electoral system. These pertain to: the internal composition of the parliamentary parties, parties' nomination strategies and the probability of overhang mandates.

Disproportionality in the plurality tier affects the internal composition of the parliamentary party groups. This becomes evident if we look at their internal composition depending on their government status. In a Bundestag election, the two major parties CDU/CSU and SPD – as the only parties to win district mandates regularly – gain an overproportionate number of district mandates with an increase in votes and lose an overproportionate number of district mandates with a loss of votes. In each case it is highly probable that the successful, vote-gaining party will participate in the next government. In this way, the disproportionality of the plurality tier translates into a disproportionate complexion of parliamentary party groups, since, on a regular basis, about three-fifths of the members of parliament from the major party in government are elected in a district and only two-fifths from the party lists (Manow and Nistor 2009), whereas the prevailing ratio between list and district candidates for the entire Bundestag displays an overrepresentation of list candidates.[10] The flip side is that the largest opposition party is clearly underrepresented by district candidates in parliament.

Excluding the election of the very first Bundestag (because of the different election rules then in place), we find that 66.8 per cent of elected CDU/CSU representatives were district mandates in the years when the Christian democrats ruled and only 43 per cent in the years when the party was in the opposition. An almost identical picture appears for the SPD: if the party is part of the ruling coalition, 60.6 per cent of all SPD parliamentarians are elected in a district; when in the opposition, only 37.3 per cent hold district mandates. This is to be compared with the parliamentary average, in which 45 per cent hold district mandates and 55 per cent are list candidates. To the best of my knowledge, this distortion in representation dependent on a party's government status has not yet been closely examined in the literature on Germany's mixed electoral system. However, it overlays the better known and more extensively discussed (albeit still controversial) difference between district and list candidates (Stratmann and Baur 2002; Lancaster and Patterson 1990; Bawn and Thies 2003; Klingemann and Wessels 2001).

Given that the translation of candidate votes into seats in parliament is increasingly disproportionate, parties – if they try to maximise re-election chances for their candidates – need to protect them against two eventualities: if they are district candidates, there is the danger that they

will not be elected if the party loses votes. If they are list candidates, there is the danger that the party's disproportionate number of successful district candidates 'crowd out' its own list candidates in the event that the party gains votes. Double candidacies, i.e. running in a district and on a list, hedge against both scenarios. In light of this, it is no surprise that the share of MPs with a double candidacy has increased from 50 per cent in the 1950s to over 80 per cent today. Two other trends should be mentioned in this context. If we define a district as safe if it has been won with more than a 10 per cent margin and if we define a list position as safe if– in the past – it has always secured a Bundestag mandate, then the proportion of safe candidacies among all MPs (either a safe district or a safe list position or both) has decreased from around 66 per cent of all parliamentary seats in the 1950s to slightly above 50 per cent today (Manow and Nistor 2009). And whereas two-thirds of these safe candidacies were double ones in the past, they are almost exclusively double candidacies today (ibid.).

If the share of districts won by a party is much higher than the share of seats that would be allocated to the party on the basis of its PR vote share, so-called overhang mandates occur. Take as an example the last Bundestag election from 2009. In Baden-Württemberg the Christian Democrats got 34.4 per cent of the list votes. The CDU would therefore have gained 27 of all 74 Baden-Wuerttemberg seats.[11] At the same time, however, the Christian Democrats won all but one of the total 38 districts in this state

– as a consequence 10 overhang mandates were allocated to the CDU. We see that a high disproportionality in the translation of candidate votes into district mandates should also make it more likely that the share of districts won and the share of list votes diverges – with overhang mandates being the

result. We can therefore assume that our main disproportionality measure – β – should be a good predictor for overhang mandates. Table 5 reports the

TABLE 5

b AS A PREDICTOR FOR OVERHANG MANDATES (NEGATIVE BINOMIAL-REGRESSION)

	(1)	(2)
β state level	18.389 (0.000)***	16.804 (0.010)***
β election level	23.082 (0.000)***	16.905 (0.028)**
Malapportionment		70.000 (0.001)***
Turnout		710.494 (0.063)*
Ticket splitting		0.000 (0.067)*
constant	75.833 (0.000)***	8.554 (0.183)
Nobs.	390	390

*Note: p-values in parentheses; *significant to the 10% level; **significant to the 5% level; ***significant to the 1% level; pseudo-R^2: = 0.12; Prob > x^2 = 0.000.*

results of a regression with overhang mandates as the dependent and β as the independent variable. Since my dependent variable is a count variable that can take on non-negative values only and that has an over-disbursed distribution, one cannot run a conventional OLS, but need to apply a negative binomial regression.

As Table 5 shows, the β-parameter is indeed a very strong and stable predictor for overhang mandates. This remains the case once one controls for other potential determinants of overhang mandates identified in the literature (see Behnke 2003a, 2003b) such as, for instance, turnout or the extent of ticket-splitting or potential malapportionment (see specification (2) in Table 5).[12] Moving in model 2 from the minimum to the maximum value for β increases the expected log count for overhang mandates by 0.33 (election level b) and 0.37 (state level β), respectively.

Summary and Outlook

My analysis of the plurality tier of the German mixed electoral system has identified two parameters that allow us to make a systematic longitudinal and cross-sectional comparison of how candidate votes are translated into district mandates. These two parameters – designated here as β and ρ – were determined by applying a modified version of the Cube Rule function to the German electoral system. β and ρ depict the left shift and the slope of the Cube Rule function, respectively. As has been demonstrated, they are themselves influenced by two key dimensions of the electoral system: by the effective number of district parties, on the one hand, and the number of districts per state, on the other. Since the effective number of parties competing in a constituency is greater within the plurality tier of a mixed electoral system than in a strictly plurality voting system, the β-parameter indicates that the two electoral tiers do 'contaminate' each other.

Finally, I would like briefly to address a series of implications linked to this paper's findings. One challenges the dominant opinion that the German electoral system is one of proportional representation with a rather insignificant, secondary element of personalisation. Both the PR and plurality tier in the German electoral system function differently in combination than they would separately. In order to understand these interactions better, we have to pay greater attention to the plurality element, which to date has tended to be somewhat overlooked in the political science literature on the German electoral system (but see Behnke 2007; Shikano 2007). The common argument that, with respect to proportionality, the German system is no different from 'simple' PR is only valid if we limit our analysis to the aggregate level, at the conversion of total votes won into total seats (and it is only valid as long as we ignore the – increasing number of – overhang mandates). Internally – with regard to the parliamentary parties' internal complexion – greater disproportionality is reflected in the

disproportionate shares of district and list candidates. On a regular basis, three-fifths to two-thirds of the seats in the largest governing party are district mandates. This effect has been widely overlooked in the literature, but has important implications for the representational logic of Germany's mixed electoral system. One implication is normative: list mandates are often criticised as being less responsive, less accountable to citizen demands than those directly elected in a district. But this critique has overlooked the fact that each of the leading ruling parliamentary parties consists primarily of representatives who are responsible in a very direct sense to the voters in their constituencies.

A further remark pertains to parties' electoral strategies. In the 1950s we observed quite frequent district level pre-electoral coordination between centre-right parties (CDU, Zentrum, DP, DVP) against social-democratic candidates. Today the level of party fragmentation is again high, but this time on the left. We may therefore in the future see more instances of district level coordination between social democracy, socialists (Die Linke), and the Green party. One may finally ask whether my findings provide any insights for other (mixed) electoral systems. Since we know of other electoral systems in which the number of district parties in the plurality tier does not converge into two (e.g. Canada; see also the last British elections), my analysis should have relevance for these systems, too. This, however, would be a topic for another article.

Acknowledgements

I am indebted to Peter Selb for his valuable STATA tips and helpful comments. I am also grateful for the useful comments I received from Andre´ Kaiser, Susumu Shikano, Thomas Zittel and two anonymous reviewers. My thanks go to Daniel Gallasch for his support in collecting and preparing the data and to Nick Borgwardt for his help with the Cube Rule function. Many thanks also to Dona Geyer, who translated a first version of this paper into English.

Notes

1. The Cube Rule can most easily be described with the following formula: $S_1/(1 - S_1) = (V_1/(1 - V_1))^3$ with S representing the share of seats for party 1 and V its share of votes (Taagepera and Shugart 1989).
2. Overhang mandates in recent Bundestag elections: 1990: 6; 1994: 16, 1998: 13, 2002: 5, 2005: 16; 2009: 24.
3. Without the first Bundestag election of 1949, in which voters only had one vote.
4. The Cube Rule used for this and all the other estimations in this paper follows the transformation by King and Browning (1987: 1253):

$$Seatshare = \left\{1 + e\left[-\rho \ln\left(\frac{Voteshare}{1 - Voteshare}\right)\right]\right\}^{-1} \quad (1)$$

5. Measured as the effective number of parties (Laakso and Taagepera 1979).
6. STATA command nl (nonlinear).
7. Without Saarland.
8. The Berlin representatives were sent by the Berlin Abgeordnetenhaus according to the size of the parliamentary parties in the Bundestag.
9. In other words, included in the data are one state with 15 elections (Saarland), 9 states with 16 elections each (former West Germany without Berlin), and 6 states with 6 elections each (the new states created from the former GDR territory and Berlin), in each case also for the CDU/CSU and the SPD. Thus we get $2 \times (15 + 9 \times 16 + 6 \times 6) = 360$.
10. Owing to the appointments of substitutes for vacated seats and, in the past, to the representatives nominated by the Berlin state parliament and in 1990 by the East German parliament, the Volkskammer.
11. Due to the 5 per cent threshold parties that pass it receive already a slightly over-proportionate share of seats.
12. The extent of ticket-splitting was operationalised as the average difference between candidate and list votes per district; my measure for malapportionment it simply the number of eligible voters per district, turnout is coded as the difference between (valid and invalid) votes and number of eligible voters.

References

Bawn, Kathleen (1999). 'Voter Responses to Electoral Complexity: Ticket Splitting, Rational Voters and Representation in the Federal Republic of Germany', British Journal of Political Science, 29:3, 487–505.

Bawn, Kathleen, and Michael F. Thies (2003). 'A Comparative Theory of Electoral Incentives: Representing the Unorganized under PR, Plurality and Mixed-Member Electoral Systems', Journal of Theoretical Politics, 15:1, 5–32.

Behnke, Joachim (2003a). 'Ein integrales Modell der Ursachen von Überhangmandaten', Politische Vierteljahresschrift, 44, 41–65.

Behnke, Joachim (2003b). 'Von Überhangsmandaten und Gesetzeslücken', Aus Politik und Zeitgeschichte, B52/2003, 21–8.

Behnke, Joachim (2007). Das Wahlsystem der Bundesrepublik Deutschland. Logik, Technik und Praxis der Verhältniswahl. Baden-Baden: Nomos.

Cox, Gary W. (1997). Making Votes Count. Strategic Coordination in the World's Electoral Systems. New York: Cambridge University Press.

Cox, Karen E., and Leonard J. Schoppa (2002). 'Interaction Effects in Mixed-Member Electoral Systems: Theory and Evidence from Germany, Japan and Italy', Comparative Political Studies, 35:9, 1027–53.

Feldkamp, Michael F. (2005). Datenhandbuch zur Geschichte des Deutschen Bundestages, Ergänzungsband 1994–2003, ed. W.D.d.D. Bundestages. Baden-Baden: Nomos.

Feldkamp, Michael F. (2006). 'Deutscher Bundestag 1987 bis 2005: Parlaments- und Wahlstatistik', Zeitschrift für Parlamentsfragen, 1, 3–19.

Ferrara, Federico, Eric S. Herron, and Misa Nishikawa (2005). Mixed Electoral Systems: Contamination and Consequences. New York: Palgrave.

Grofman, Bernd N. (1983). 'Measures of Bias and Proportionality in Seats–Votes Relation-ships', Political Methodology, 9, 295–327.

Gschwend, Thomas (2003). Ticket Splitting and Strategic Voting in Mixed Electoral Systems, ed. by M.Z.f.E. Sozialforschung. Vol. 61, Arbeitspapiere. Mannheim.

Gudgin, Graham, and Peter J. Taylor. 1979. Seats, Votes, and the Spatial Organization of Elections. London: Pion Ltd.

Hainmülller, Jens, and Holger Lutz Kern (2008). 'Incumbency as a Source of Spillover Effects in Mixed Electoral Systems: Evidence from a Regression Discontinuity Design', Electoral Studies, 27, 213–27.

Herron, Erik S., and Misa Nishikawa (2001). 'Contamination Effects and the Number of Parties in Mixed-Superposition Electoral Systems', Electoral Studies, 20, 63–86.

Kendall, M.G., and A. Stuart (1950). 'The Law of Cubic Proportions in Electoral Results', British Journal of Sociology, 1, 183–97.

Kendall, M.G., and A. Stuart (1952). 'La Loi de Cube dans les Elections Britanniques', Revue Francaise de Science Politique, 2, 270–76.

King, Gary, and Robert X. Browning (1987). 'Democratic Representation and Partisan Bias in Congressional Elections', American Political Science Review, 81:4, 1251–73.

Klingemann, Hans-Dieter, and Bernhard Wessels (2001). 'The Political Consequences of Germany's Mixed-Member System: Personalization at the Grass Roots?', in M.S. Shugart and M.P. Wattenberg (eds.), Mixed-Member Electoral Systems. The Best of Both Worlds? Oxford: Oxford University Press, 279–96.

Laakso, Markku, and Rein Taagepera (1979). 'Effective Number of Political Parties. A Measure with Application to West Europe', Comparative Political Studies, 12:1, 3–27.

Lancaster, Thomas D., and William D. Patterson (1990). 'Comparative Pork Barrel Politics Perceptions from the West-German-Bundestag', Comparative Political Studies, 22:4, 458–77.

Lewis, Jeffrey B., and Drew A. Linzer (2005). 'Estimating Regression Models in which the Dependent Variable is Based on Estimates', Political Analysis, 13:3, 345–64.

Manow, Philip (2010). 'Dimensionen der Disproportionalität. Erststimmen und Direktmandate in den Bundestagswahlen von 1953 bis 2009', Politische Vierteljahresschrift, 51:3, 433–55.

Manow, Philip (2011). 'Quasi-Incumbency Advantages – or: Why do Small Parties Enter Hopeless District Races? Evidence from the German Case' (under review).

Manow, Philip, and Martina Nistor (2009). 'Wann ist ein Listenplatz sicher? Eine Untersuchung der Bundestagswahlen von 1953 bis 2002', Zeitschrift für Parlamentsfragen, 40:3, 588–605.

Rodden, Jonathan (2010). 'The Geographic Distribution of Political Preferences', Annual Review of Political Science, 13, 321–40.

Schindler, Peter (1999). Datenhandbuch zur Geschichte des Deutschen Bundestages 1949–1999. Baden-Baden: Nomos.

Schreiber, Wolfgang (1994). Handbuch des Wahlrechts zum Deutschen Bundestag. Kommentar zum Bundeswahlgesetz. 5. völlig überarbeitete Auflage, ed. Köln Berlin. Bonn and München: Heymanns.

Shikano, Susumu (2007). Interactive Mechanism of Mixed-Electoral Systems: A Theory-Driven Comparative Analysis via Computational Modelling and Bayesian Statistics. Mannheim: Habilitationsschrift.

Stratmann, Thomas, and Martin Baur (2002). 'Plurality Rule, Proportional Representation and the German Bundestag: How Incentives to Pork-Barrel Differ Across Electoral Systems', American Journal of Political Science, 46:3, 506–14.

Taagepera, Rein, and Matthew Soberg Shugart (1989). Seats and Votes. The Effects and Determinants of Electoral Systems. New Haven, CT: Yale University Press.