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Knowledge transfer, transitional dynamics and optimal Research & Development policy in a dynamic monopoly setting

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Abstract

This paper focuses on the question whether or not a reduction of the knowledge barrier is good for welfare. Based on a dynamic monopoly setting with simultaneous investment decisions in process as well as in product Research & Development (R&D), we show that a reduction of the knowledge barrier has ambiguous welfare consequences: Due to a lower knowledge barrier, product quality and welfare increase in the short-run. However, this may not necessarily be the case in the long-run. One reason is that a positive long-lasting knowledge barrier shock triggers the monopolist to sub-optimally lower its product R&D investments today and in the future at the cost of future product quality. This in turn may reduce welfare. Accordingly, to realize the first-best level of product quality, the long-run optimal R&D subsidy rate for product innovations increases with a reduction of the knowledge barrier.

Keywords

Process and product innovation, learning by doing, knowledge spillovers, optimal taxation, dynamic monopoly analysis

JEL Classifications

D4, D6, D9, C4, L1, O3

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1 Introduction

Knowledge and its management is of crucial importance for a firm's inventiveness and, hence, is essential for its sustainable competitive advantage. In general, knowledge itself has both, an internal as well as an external dimension. For instance, firms can increase their own knowledge stock by either investing in product or process R&D, or both. However, theoretically, a firm can also exploit external sources of knowledge produced by its competitors. Ironically, new knowledge - accessed externally or internally - also generates types of knowledge barriers which may hinder the firm to exploit the accumulated and existing stock of knowledge (Caldwell (1967)).

The knowledge barrier is also subject of research in knowledge management.¹ In general and applied to an economic setting, the knowledge barrier refers to any impediment that prevents existing knowledge to be used elsewhere. This interpretation is inline with Attewell's (1992) view on knowledge barriers as the absence of ability accessing existing knowledge. Knowledge barriers are relevant within the firm as well as with respect to knowledge external to the firm. The former one is also known as internal stickiness (Szulanski (1996,2003)).² Inline with this somehow vague concept of knowledge barriers, we consider anything that prevents existing knowledge to be applied, transferred, shared or diffused. We take account of these barriers in our theoretical approach at several instances whenever existing knowledge affects the economic environment of the firm. We distinguish explicitly between barriers in accessing internal and external existing knowledge.

Following the knowledge management literature, the reduction of knowledge barriers goes hand in hand with improving knowledge transfer and knowledge sharing. While the latter is usually focusing on what is happening inside the firm, the former is often applied to knowledge at a higher aggregated level containing also inter-business relations (Choo and Alvarenga Neto (2010)). To reduce knowledge barriers, the relevant economic literature suggests to foster the firm-specific, underlying learning process and, thereby to increase the firms' absorption capacities (Cohen and Levinthal (1990)). Lichtenthaler (2009) considers a firm's absorption capacity as important to transform knowledge into innovations for improved products and production processes. Inline with the concept of "a learning economy" proposed by Lundvall and Johnson (1994), we argue that a *conditio sine qua non* to reduce the firm's knowledge barriers is learning. Linking this to the knowledge management literature, learning includes sharing and transferring knowledge from where it is originating towards where it can contribute to productivity. This interpretation is inline with Demircioglu and Audretsch (2017) pointing out that learning can be seen as a critical dimension reducing the knowledge barrier. However, the authors also highlight that more research is needed to explore the link between knowledge barriers and innovation. Our paper not only exactly adds to this issue, but goes a step further by asking

¹A comprehensive review of the knowledge management review is beyond the scope of this section. We just report on the influential contributions in this strand of the literature.

²An example could be the successful implementation of a new technology by one department within the firm while the other department is unable to do so due to lack of expertise and lack of knowledge that particular problems have already been solved at other instances inside the firm.

whether or not a reduction of the knowledge barrier is beneficial for welfare.

In our setting, we focus on a very abstract form of learning without specifying the learning process itself. For instance, a direct effect of learning is reflected in a reduction of innovation costs. This might happen if a firm successfully makes use of its own accumulated knowledge through sharing it within the firm. As a second effect, learning also increases the accumulation of knowledge within the firm which is again related to knowledge sharing. Further, learning makes it also easier to benefit from external knowledge spillovers which would typically be seen as a knowledge transfer.

(Strategic) investment decisions (in knowledge) always have a dynamic dimension as today's decisions in most cases affect the conditions for future investments.³ In recent years, some efforts have been made to discuss the inherently *dynamic* nature of product and process improvement, ranging from the specific patterns of R&D races, to the timing of innovative ventures, to spillover effects. For example, Lambertini and Mantovani (2009) focused on the dynamic behavior of a multiproduct monopolist investing in process and product innovation, whereas Lambertini and Orsini (2015), Li and Ni (2016) or more recently, Zhong and Zhang (2018) consider a monopolist investing in cost-reducing and quality-enhancing endeavors. Common to the latter mentioned strand of literature is the result that due to the monopoly power on product quality, the firm distorts product quality downwards compared to what is optimal from a social point of view. However, none of the before mentioned literature has analyzed the *dynamic adjustment* of welfare due to a potential unforeseen shock in the knowledge barrier sphere represented by unforeseen changes at one of the several instances where knowledge is shared or transferred. Consequently, also none of the mentioned contributions discussed the optimal policy design aiming to increase product quality towards the optimal level, when external and internal knowledge transfers and sharing are present.

Process and product innovations differ without doubt. We distinguish them by introducing a product innovation specific cost component to the monopolist. Additionally, a product innovation boils down to an increase in the consumers marginal utility. Inline with the literature, we refer to this as an improvement in the monopolist's product quality. Such innovations are subject to a scale independent production cost component. This will give rise to scale effect in product quality which is exploited by the monopolists market power. As the existing literature almost entirely focus on the role of scale in process innovations (see e.g. Cohen and Klepper (1996a,b)), little attention has been paid to the role of scale in product innovations.

This paper exactly fills the mentioned gaps. First, we combine the dynamic models introduced by Lambertini and Orsini (2015), Li and Ni (2016) and Zhong and Zhang (2018) and, *inter alia*, solve the control problem for the monopolist's offered equilibrium product quality. We show that this obtained hyperbolic equilibrium is a degenerate saddle implying that there exists a path to which the economy converges in the long-run. We find that product quality increases with a decreasing knowledge barrier. Next, we compute the social optimal solution. We find that the social optimal level

³In contrast, recently, Deng and Hendrikse (2018) investigate the role of social interactions for product quality in a static environment.

of product quality is higher due to under-investments in product quality in the decentralized setting. However, the level of process R&D is socially optimal, which is inline with the literature.⁴ Hence, in order to establish the first-best solution, we propose a policy imposing a product R&D subsidy scheme which is completely lump-sum financed by the consumers.

The paper makes the following important points. (i) We find that the higher the optimal, long-run subsidy-rate is, the lower is the knowledge barrier. For instance, a lower knowledge barrier allows for better accessing existing knowledge. This increases product quality *today* but, and this is important, for a given depreciation rate, product quality decreases in the *future* as private R&D endeavors decrease. Obviously, a higher subsidy then counteracts the lower private incentive to invest in own product R&D due to a low knowledge barrier. Further, only in the long-run where product quality is constant, the optimal subsidy-rate is constant as well. During the transitional phase towards the steady-state, the optimal subsidy is time-varying. (ii) Performing a numerical simulation study, we show that welfare may considerably differ in the short-run from its long-run counterpart. This latter finding has important policy implications: If policy makers focus on the short-run welfare effects of lowering the knowledge barrier, the policy recommendation which may be optimal in the long-run can be biased in the short-run. Hence, welfare effects in the short-run may differ considerably from those in the long-run as speculated by Audretsch, Baumol and Burke (1995). To clarify on this issue, we run several numerical simulations of our model over its transition towards the long-run equilibrium. We find heterogeneity with respect to the effects of knowledge barrier reductions across different types of barriers as well as during the short and the long-run. Additionally, we find R&D policy to provoke different welfare reactions in the short and long-run as well. As will be seen, market size plays an important role in all the mentioned effects.

The plan of the paper is as follows: Section 2 introduces the model and solves for the monopolist's long-run equilibrium. The stability properties of this equilibrium are worked out in Section 3. Section 4 provides our welfare analysis, whereas Section 5 establishes the welfare optimal R&D policy. Section 6 focuses on the transitional dynamics of welfare effects due to knowledge barrier shocks and R&D subsidies. Section 7 finally concludes.

2 The model

2.1 Consumers

In what follows, we derive a dynamic extension of the static Mussa and Rosen (1978) framework. Let us consider a single-product monopoly which sells a non-durable good of quality $q(t)$ at a price $p(t)$ over a continuous time span $t \in [0, \infty^+]$. Following Lambertini (2018), Lambertini and Orsini (2015) or Zhong and Zhang (2018), the marginal willingness to pay $\theta \in [\Theta - 1, \Theta]$ with $\Theta > 1$ is

⁴This latter finding is due to the fact that the monopoly price does not depend on the marginal cost of production and, further, market demand does not respond to changes in marginal costs.

uniformly distributed with a density function $d, d \geq 1$. Hence, the total mass of consumers is $d \geq 1$ which is at the same time also representing market size. Assuming further that the market is fully covered at any time $t \in [0, \infty^+]$ ⁵, the net-utility of a representative consumer purchasing a single unit of the good at quality $q(t)$ is given by:

$$\mathcal{U} = \theta q(t) - p(t) \geq 0. \quad (1)$$

Under full market coverage, we have that $x(t) = d$ and the profit-maximizing price $p^m(t)$ completely extracts the surplus from the poorest consumer, i.e. $p^m(t) = (\Theta - 1)q(t)$.

2.2 In-house product and process innovation

We assume that the monopoly carries out the entire R&D activities in-house. At any $t \in [0, \infty^+]$, the monopoly has to set the levels of product and process innovations simultaneously. Following Chenavaz (2012), product innovations increase the cumulative level of product quality $q(t)$, while investments in the production process decrease the cumulative production costs $c(t)$ accordingly. Hence, the dynamics of $q(t)$ and $c(t)$ can be fully described by the following system of differential equations:

$$\dot{q}(t) \equiv \frac{\partial q(t)}{\partial t} = [k(t) - \delta]q(t), \quad (2)$$

$$\dot{c}(t) \equiv \frac{\partial c(t)}{\partial t} = -[h(t) - \eta]c(t), \quad (3)$$

with $k(t)$ and $h(t)$ as the R&D efforts in product and process improvements, respectively. $\delta \in (0, 1)$ is the constant depreciation rate on product quality, while $\eta \in (0, 1)$ represents the production technology's obsolescence rate.

Both, $q(t)$ and $c(t)$, are state variables in our problem. Increasing product quality at one particular instance in time also has influence on quality's entire future path. An increment in quality is therefore an investment with instantaneous as well as future returns. If $k(t) = 0$ for $t \in [0, \infty^+]$ would be chosen, quality would decay down to zero in the very long run. The same property applies in general to the costs $c(t)$. Of course, the sign of effects demands for a different interpretation. Without steady investments into cost reductions beyond the obsolescence rate, costs $c(t)$ would diverge to infinity. Hence, production costs as well as the level of product quality require maintenance in the long run.

We assume the monopoly's total cost function $C(t)$ at any $t \in [0, \infty^+]$ reads as:

$$C(t) = c(t)x(t) + \alpha k^2(t) + \beta h^2(t) + \gamma q(t)^2, \quad (4)$$

⁵The assumption of a fully covered market can be justified with a demand size which is known *a priori* due to a full information assumption which enables the firm to identify the position $\Theta - 1$ of the marginal consumer.

with $x(t)$ as the firm's output. The second, third and fourth term of equation (4) show the R&D costs in product and process R&D as well as the costs implied by the chosen product quality, respectively. We assume these costs to follow a quadratic representation to reflect their distinctive character when compared with ordinary marginal costs in production. The choice of a quadratic representation is inline with the related literature. While marginal costs $c(t)$ are independent of scale in general, the other terms reflect increasing marginal cost contributions in the respective dimensions. Costs increase more than proportional in the chosen gross investments into quality improvement and cost reductions as well as the level of quality at which production takes place. These three components are giving rise to a technology with quasi-fixed costs. This also justifies the monopolistic market environment chosen in this contribution.

As costs depend on the level of quality $q(t)$ in a quadratic way, product innovations have a distinguishing feature in our set-up as compared with process innovations. This term will introduce a scale effect as will be seen further down below. The larger the market for the monopolist's product, the higher will be the chosen level of quality.

2.3 Knowledge accumulation

We follow Li and Ni (2016) or Zhong and Zhang (2018) and stipulate that knowledge is created by process and product innovations and accumulates according to the following rules:

$$A_k(t) = \bar{A}_k + \sigma_k \int_0^t k(s) ds - \omega_k \int_0^t A_k(s) ds, \quad (5)$$

$$A_h(t) = \bar{A}_h + \sigma_h \int_0^t h(s) ds - \omega_h \int_0^t A_h(s) ds. \quad (6)$$

$\bar{A}_i > 0$ for $i = \{h, k\}$ represents an externally given knowledge stock. $\omega_i \in (0, 1)$ denote the depreciation rates on accumulated knowledge. This mirrors the view of Jorgenson (1973), Griliches (1998) and others that new ideas replace older ones as they become obsolete. $\sigma_i \in (0, 1)$ denote the knowledge accumulation rates arising from investments in product and process innovations. σ_i are part of our approach modeling the knowledge barrier: Larger values for σ_i reduce the knowledge barrier. At this particular instance, we try to reflect the within firm dimension of the knowledge barrier. This specific barrier limits the amount of the firm's knowledge stock that can be gained from the innovative actions representing pieces of knowledge required to raise quality or reduce costs. This additionally refers to knowledge sharing within the firm familiar from the literature on knowledge management. At this point, we also link learning abilities with knowledge barriers as mentioned in the introduction. Increasing σ_i may represent an increase in learning efforts.⁶ As such, learning effort

⁶As mentioned in the introduction, we desist from modeling a specific learning process as this would not deliver new insights for the paper's topic and, hence, is clearly beyond the scope of this paper. However, an avenue for further research would be to introduce different forms of (endogenous) learning to the model and discuss the influence of (endogenous)

is helpful in reducing the knowledge barrier at this particular point in the firm's knowledge creation process.

2.4 External knowledge spillover

There is by now a voluminous literature pointing to the importance of knowledge spillovers for R&D endeavors, both at the firm as well as at the macroeconomic level (among others, see Audretsch (1995), Audretsch and Feldman (1996) or Acs et al. (2013)). It is straightforward to incorporate knowledge spillovers in our dynamic setting which, at the same time, also represent knowledge transfers from external sources as discussed in the knowledge management literature. We assume that the external, and hence, exogenous sources of knowledge and their internal counterparts are perfect substitutes. Unlike in Klarl (2014), there is no knowledge spillover across product and process innovations which implies that the monopolists cannot exploit the process R&D knowledge stock to increase product quality and vice versa.⁷ We follow Cellini and Lambertini (2009) and re-express the state dynamics of $q(t)$ and $c(t)$ in the following way:

$$\dot{q}(t) \equiv \frac{\partial q(t)}{\partial t} = [k(t) - \delta + \kappa_k \tilde{k}(t)]q(t), \quad (7)$$

$$\dot{c}(t) \equiv \frac{\partial c(t)}{\partial t} = -[h(t) - \eta + \kappa_h \tilde{h}(t)]c(t), \quad (8)$$

where the parameter κ_i , $i = \{h, k\}$ measures the positive knowledge transfers that the monopolist receives from external knowledge sources. For simplicity, we assume that external investments $\tilde{k}(t)$ and $\tilde{h}(t)$ grow at an exogenous, constant net rate $g_i - \varsigma_i$, $i = h, k$, where $g_i \in (0, 1)$ ($\varsigma_i \in (0, 1)$) represent the external knowledge accumulation (depreciation) rate. This is the second instance where we introduce the knowledge barrier to our problem. Obviously, a larger value of κ_i can be interpreted as a reduction in this dimension of the knowledge barrier leading to a better transfer of knowledge from external sources towards the firm.

2.5 Internal learning by doing

We already take account of knowledge sharing within the monopolist's firm at the stage of accumulating knowledge (see equations (5) and (6)). This, however, represents knowledge sharing in a rather incomplete way as the monopolist might also draw on the level of accumulated knowledge. This will happen whenever something can be learned from the accumulated experiences represented in the model by $A_h(t)$ and $A_k(t)$. As we believe that this is reasonably the case, we follow Thompson (2010) and Clarke et al. (1982), among others, in assuming that learning from in-house knowledge stocks

learning on product quality and (transitional) welfare.

⁷This may be seen as a strict assumption but reduces the analytical complexity of our model without losing important insights.

immediately reduces R&D costs. Hence at any $t \in [0, \infty^+]$, the monopolist's total cost function has to be adjusted in the following manner (see Li and Ni (2016)):

$$C(t) = c(t)x(t) + \alpha k^2(t) - \tau_k \Gamma_k(t) + \beta h^2(t) - \tau_h \Gamma_h + \gamma q(t)^2, \quad (9)$$

where $\tau_i \in (0, 1)$, $i = \{h, k\}$ denote the cost reducing impact of the monopolist's accumulated knowledge in excess of the externally available knowledge stock. This excess knowledge is given by the knowledge-gaps

$$\Gamma_k(t) \equiv A_k(t) - \bar{A}_k, \quad (10)$$

$$\Gamma_h(t) \equiv A_h(t) - \bar{A}_h. \quad (11)$$

The impact of the knowledge-gaps on total production costs is limited again by the knowledge barrier. As both, internally accumulated and external knowledge stocks affect production costs, τ_i refers to the ability of knowledge sharing and transfer. We interpret this as a learning rate from firm specific knowledge acquired by product and process innovations translating into lower firm specific production costs. Obviously and ceteris paribus, larger knowledge-gaps lead to larger cost-saving potentials. Thus, besides κ_i and σ_i , τ_i are the last elements representing the various aspects of the knowledge barrier in our set-up.

2.6 The monopolist's problem

The objective of the monopolist is to set the optimal level of investments in product and process R&D which in turn maximizes the discounted profit flow, $\Pi(t)$. Thus, the monopolist's problem can be expressed as:

$$\begin{aligned}
 \max_{\{k(t), h(t)\}} \Pi(t) &\equiv \int_0^{\infty^+} \pi(t) \exp[-\rho t] dt & (12) \\
 \text{s.t.} & \\
 \pi(t) &= [(\Theta - 1)q(t) - c(t)]d - [\alpha k^2(t) - \tau_k \Gamma_k] - [\beta h^2(t) - \tau_h \Gamma_h] - \gamma q(t)^2, \\
 \dot{q}(t) &= [k(t) - \delta + \kappa_k \tilde{k}(t)]q(t), \quad \dot{c}(t) = -[h(t) - \eta + \kappa_h \tilde{h}(t)]c(t), \\
 \dot{A}_k(t) &= \sigma_k k(t) - \omega_k A_k(t), \quad \dot{A}_h(t) = \sigma_h h(t) - \omega_h A_h(t), \\
 \Gamma_k(t) &= A_k(t) - \bar{A}_k, \quad \Gamma_h(t) = A_h(t) - \bar{A}_h, \\
 \dot{\tilde{k}}(t) &= [g_k - \varsigma_k] \tilde{k}(t), \quad \dot{\tilde{h}}(t) = [g_h - \varsigma_h] \tilde{h}(t), \\
 \tilde{k}(0) &= \tilde{k}_0 > 0, \quad \tilde{h}(0) = \tilde{h}_0 > 0, \\
 q(0) &= q_0 > 0, \quad c(0) = c_0 \in (0, (\Theta - 1)q(0)), \\
 A_k(0) &= \bar{A}_k > 0, \quad A_h(0) = \bar{A}_h > 0, & (13)
 \end{aligned}$$

and the appropriate transversality conditions which are specified below. In order to guarantee that the market is fully covered even at $t = 0$, we have to impose an initial condition on the marginal costs, $c(0)$. This imposition guarantees that the marginal costs are strictly lower than the spending of the poorest consumer existing in the market.

2.7 Equilibrium

The firm's current value Hamiltonian \mathcal{H} is given by:

$$\begin{aligned}
 \mathcal{H} &= \{(\Theta - 1)[q(t) - c(t)]d - [\alpha k^2(t) - \tau_k \Gamma_k] - [\beta h^2(t) - \tau_h \Gamma_h] - \gamma q(t)^2 \\
 &+ \lambda_1(t)[(k(t) - \delta + \kappa_k \tilde{k}(t))q(t)] \\
 &+ \lambda_2(t)[-(h(t) - \eta + \kappa_h \tilde{h}(t))c(t)] \\
 &+ \lambda_3(t)[\sigma_k k(t) - \omega_k A_k(t)] \\
 &+ \lambda_4(t)[\sigma_h h(t) - \omega_h A_h(t)] \\
 &+ \lambda_5(t)(g_k - \varsigma_k) \tilde{k}(t) \\
 &+ \lambda_6(t)(g_h - \varsigma_h) \tilde{h}(t)\},
 \end{aligned}$$

where λ_i for $i = \{1, \dots, 6\}$ are the costate variables associated with $q, c, A_k, A_h, \tilde{k}$ and \tilde{h} . Henceforth, we shall omit the indication of the time argument t for the sake of brevity whenever no confusion can arise. Here, we assume that the initial condition for $c(0)$ is fulfilled right from the beginning. The

first order conditions (FOCs) for the controls k, h and costate equations are:

$$k : \quad -2\alpha k + \lambda_1 q + \lambda_3 \sigma_k = 0, \quad (14)$$

$$h : \quad -2\beta h - \lambda_2 c + \lambda_4 \sigma_h = 0, \quad (15)$$

$$\lambda_1 : \quad \dot{\lambda}_1 = (\rho - k + \delta - \kappa_k \tilde{k})\lambda_1 - (\Theta - 1)d + 2q\gamma, \quad (16)$$

$$\lambda_2 : \quad \dot{\lambda}_2 = (\rho + h + \kappa_h \tilde{h} - \eta)\lambda_2 + d, \quad (17)$$

$$\lambda_3 : \quad \dot{\lambda}_3 = (\rho + \omega_k)\lambda_3 - \tau_k, \quad (18)$$

$$\lambda_4 : \quad \dot{\lambda}_4 = (\rho + \omega_h)\lambda_4 - \tau_h, \quad (19)$$

$$\lambda_5 : \quad \dot{\lambda}_5 = -\kappa_k q \lambda_1 - (g_k - \varsigma_k - \rho)\lambda_5, \quad (20)$$

$$\lambda_6 : \quad \dot{\lambda}_6 = +\kappa_h c \lambda_2 - (g_h - \varsigma_h - \rho)\lambda_6. \quad (21)$$

The corresponding transversality conditions are given by:

$\lim_{t \rightarrow \infty} \lambda_1(t)q(t) \exp(-\rho t) = 0$, $\lim_{t \rightarrow \infty} \lambda_2(t)c(t) \exp(-\rho t) = 0$, $\lim_{t \rightarrow \infty} \lambda_3(t)A_k(t) \exp(-\rho t) = 0$, $\lim_{t \rightarrow \infty} \lambda_4(t)A_h(t) \exp(-\rho t) = 0$, $\lim_{t \rightarrow \infty} \lambda_5(t)\tilde{k}(t) \exp(-\rho t) = 0$, and $\lim_{t \rightarrow \infty} \lambda_6(t)\tilde{h}(t) \exp(-\rho t) = 0$.

The transversality conditions make sure that neither knowledge nor product and process innovations are accumulated by an inefficiently high or low degree as time goes by.

From equations (14)-(15) we obtain for the optimal controls.⁸

$$k^* = \max \left\{ 0, \frac{\lambda_1 q + \left(\frac{\tau_k}{\rho + \omega_k} \right) \sigma_k}{2\alpha} \right\}, \quad (22)$$

$$h^* = \max \left\{ 0, \frac{-\lambda_2 c + \left(\frac{\tau_h}{\rho + \omega_h} \right) \sigma_h}{2\beta} \right\}, \quad (23)$$

as well as the control equations by differentiating (22) and (23) with respect to time:

$$\dot{k} = \frac{\dot{q}\lambda_1 + q\dot{\lambda}_1}{2\alpha}, \quad (24)$$

$$\dot{h} = -\frac{\dot{c}\lambda_2 + c\dot{\lambda}_2}{2\beta}. \quad (25)$$

Using (7), (18) and (14) in (24) and (8), (19) and (15) in (25) gives:

⁸See Appendix A for details on the derivations.

$$\dot{k} = \rho \left(k - \frac{\tau_k \sigma_k}{2\alpha(\rho + \omega_k)} \right) - \frac{(\Theta - 1)dq}{2\alpha} + \frac{2q^2\gamma}{2\alpha}, \quad (26)$$

$$\dot{h} = \rho \left(h - \frac{\tau_h \sigma_h}{2\beta(\rho + \omega_h)} \right) - \frac{cd}{2\beta}. \quad (27)$$

The system consisting of equations (26)-(27) and (7)-(8) establishes a system of differential equations which allows us to study the dynamic relationship between R&D investments in product and process innovations, product quality, and production costs when external knowledge spillovers and in-house learning by doing are present. This system ([S1]) can be formally comprised as:

$$[S1] \begin{cases} \dot{k} = \rho \left(k - \frac{\tau_k \sigma_k}{2\alpha(\rho + \omega_k)} \right) - \frac{(\Theta - 1)dq}{2\alpha} + \frac{2q^2\gamma}{2\alpha}, \\ \dot{h} = \rho \left(h - \frac{\tau_h \sigma_h}{2\beta(\rho + \omega_h)} \right) - \frac{cd}{2\beta}, \\ \dot{q} = (k - \delta + \kappa_k \tilde{k})q, \\ \dot{c} = -(h - \eta + \kappa_h \tilde{h})c. \end{cases} \quad (28)$$

For the sake of simplicity and keeping analytical complexity at a reasonable minimum, we assume that \tilde{k} and \tilde{h} are constant over time. This requires either $g_i - \zeta_i = 0$ or $g_i = 0$ and $\zeta_i = 0$ for $i = \{k, h\}$ and does not affect the overall findings this paper makes. Requiring that the steady state is time-invariant, i.e. $\dot{c} = \dot{q} = \dot{k} = \dot{h} = 0$, solving system [S1] together with $\dot{A}_k = \dot{A}_h = 0$ delivers six possible sets of solutions of which only one is economically sensible. We hereby exclude corner solutions, such as zero product quality in the steady state⁹. This solution ([Sol1]) reads as:

$$[Sol1] \begin{cases} q^{sts} = \frac{\sqrt{(\rho + \omega_k)(8\gamma\rho\sigma_k\tau_k + (\rho + \omega_k)(d^2(\Theta - 1)^2 - 16\alpha\gamma\delta\rho + 16\alpha\gamma\kappa_k\tilde{k}\rho))} + d(\Theta - 1)}{4\gamma}, \\ c^{sts} = \frac{\rho}{d} \left[2\beta(\eta - \tilde{h}\kappa_h) - \frac{\sigma_h\tau_h}{\rho + \omega_h} \right], \\ k^{sts} = \delta - \tilde{k}\kappa_k, \\ h^{sts} = \eta - \tilde{h}\kappa_h, \\ A_k^{sts} = \frac{\sigma_k(\delta - \tilde{k}\kappa_k)}{\omega_k}, \\ A_h^{sts} = \frac{\sigma_h(\eta - \tilde{h}\kappa_h)}{\omega_h}, \end{cases} \quad (29)$$

and non-zero given steady state values \tilde{h}^{sts} and \tilde{k}^{sts} . Reflecting [Sol1], the constraints

$$\tau_h < \bar{\tau}_h \equiv \frac{2\beta(\eta - \tilde{h}\kappa_h)(\rho + \omega_h)}{\sigma_h}, \quad (30)$$

$$\tau_k > \bar{\tau}_k \equiv \frac{2\alpha(\delta - \tilde{k}\kappa_k)(\rho + \omega_k)}{\sigma_k} \quad (31)$$

⁹The explicit expressions for all solutions are available from the authors upon request. Only the real valued solutions are economically meaningful and are discussed here.

need to be satisfied simultaneously in order to obtain positive and real-valued values for process as well as for product R&D investments.¹⁰ To understand the conditions, it is helpful to rewrite the critical values $\bar{\tau}_h$ and $\bar{\tau}_k$ by using [Sol1] for the steady-state investments as

$$\begin{aligned}\bar{\tau}_h \frac{\sigma_h}{\rho + \omega_h} &= 2\alpha k^{sts}, \\ \bar{\tau}_k \frac{\sigma_h}{\rho + \omega_h} &= 2\beta h^{sts}.\end{aligned}$$

The critical values are the values for τ_h and τ_k that would exactly balance the long-run marginal benefits and marginal investment costs for process and product innovations. The left hand side of each of the equations represents respectively the quality contribution or cost reduction through the long-run knowledge contribution of a marginal unit of investment $\frac{\sigma_i}{\rho + \omega_i}$, $i = \{h, k\}$ and its knowledge transfer/sharing effect via $\bar{\tau}_i$. The right hand side of the equations clearly represent the costs of a marginal unit of investments into process and product innovation. At the critical values for τ_h and τ_k , marginal long-run benefits and marginal investment costs need to equal each other. However, (31) implies τ_k to be larger than its critical value, i.e. marginal long-run benefits outweigh marginal investment costs. This is due the additional negative contribution of the quality level on overall costs C and the additional positive effect as a higher quality level increases the monopolists revenues at the total scale of production. In the optimum, the former contribution is outweighing the latter. $\tau_k > \bar{\tau}_k$ balances the net additional contributions of investments into product innovations. If the condition would be violated, the monopolist wouldn't have an incentive to invest into product innovations in steady-state.

τ_h needs to be smaller than $\bar{\tau}_h$. This applies as there is an additional positive contribution of process innovations. They reduce the monopolist's marginal production costs, and hence, contribute to profits at the total scale of production. This is causing the profit maximizing allocation to tolerate a knowledge sharing/transfer effect below its critical value $\bar{\tau}_h$. If the condition would be violated, the monopolist could drive marginal costs in production down to or even below zero.

As the obtained steady-state results are similar to Lambertini and Orsini (2015) or Zhong and Zhang (2018), we desists from discussing the properties of the steady-state but refer to this relevant literature. However, for what follows, it is worth noting that the constant steady-state product quality q^{sts} given by [Sol1] increases with a reduction of the knowledge barrier and the size of the market d .

3 Stability analysis

In this section we show that our equilibrium point is a degenerate saddle-point. This implies that starting from a historical point, there exists a path on which the economy converges to a steady-state. To show this, we first linearize our model around [Sol1]. Let x be the (6×1) vector $x' =$

¹⁰The condition for τ_k is a sufficiency condition guaranteeing a real valued quality level respectively of Θ and d .

$[q, c, k, h, A_k, A_h]$ and let us define $\mathcal{J}_{6 \times 6} = Df(x^{Sol1})$ as the Jacobian matrix of the system evaluated at the equilibrium point [Sol1]. Hence, the linearized counterpart of [S1] reads as $\dot{x} = \mathcal{J}x$ with $x \in \mathbb{R}^6$.

$$\mathcal{J} \equiv \begin{bmatrix} \frac{\partial \dot{c}}{\partial c} & \frac{\partial \dot{c}}{\partial q} & \frac{\partial \dot{c}}{\partial A_k} & \frac{\partial \dot{c}}{\partial A_h} & \frac{\partial \dot{c}}{\partial k} & \frac{\partial \dot{c}}{\partial h} \\ \frac{\partial \dot{q}}{\partial c} & \frac{\partial \dot{q}}{\partial q} & \frac{\partial \dot{q}}{\partial A_k} & \frac{\partial \dot{q}}{\partial A_h} & \frac{\partial \dot{q}}{\partial k} & \frac{\partial \dot{q}}{\partial h} \\ \frac{\partial \dot{A}_k}{\partial c} & \frac{\partial \dot{A}_k}{\partial q} & \frac{\partial \dot{A}_k}{\partial A_k} & \frac{\partial \dot{A}_k}{\partial A_h} & \frac{\partial \dot{A}_k}{\partial k} & \frac{\partial \dot{A}_k}{\partial h} \\ \frac{\partial \dot{A}_h}{\partial c} & \frac{\partial \dot{A}_h}{\partial q} & \frac{\partial \dot{A}_h}{\partial A_k} & \frac{\partial \dot{A}_h}{\partial A_h} & \frac{\partial \dot{A}_h}{\partial k} & \frac{\partial \dot{A}_h}{\partial h} \\ \frac{\partial \dot{k}}{\partial c} & \frac{\partial \dot{k}}{\partial q} & \frac{\partial \dot{k}}{\partial A_k} & \frac{\partial \dot{k}}{\partial A_h} & \frac{\partial \dot{k}}{\partial k} & \frac{\partial \dot{k}}{\partial h} \\ \frac{\partial \dot{h}}{\partial c} & \frac{\partial \dot{h}}{\partial q} & \frac{\partial \dot{h}}{\partial A_k} & \frac{\partial \dot{h}}{\partial A_h} & \frac{\partial \dot{h}}{\partial k} & \frac{\partial \dot{h}}{\partial h} \end{bmatrix} \quad (32)$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -c \\ 0 & 0 & 0 & 0 & q & 0 \\ 0 & 0 & -\omega_k & 0 & \sigma_k & 0 \\ 0 & 0 & 0 & -\omega_h & 0 & \sigma_h \\ 0 & \frac{4q\gamma - (\Theta - 1)d}{2\alpha} & 0 & 0 & \rho & 0 \\ -\frac{d}{2\beta} & 0 & 0 & 0 & 0 & \rho \end{bmatrix}.$$

For positive values of [Sol1], the six Eigenvalues of \mathcal{J} associated with [Sol1] are real-valued.¹¹ It can be shown that four of them are negative valued, whereas two of them are positive. We can show that [Sol1] fulfills the properties of a hyperbolic equilibrium point. In general, a hyperbolic equilibrium point is a fixed point that does not have any center manifolds.¹² For this fixed point, it can be shown that there exists a locally stable and unstable manifold. Further, according to the Hartman-Grobman theorem (see Teschl (2012), chapter 9.2. and 9.3), the behavior of a dynamical system in the domain near a hyperbolic equilibrium point is qualitatively the same as the behavior of its linearization near this equilibrium point due to its topological equivalence. Hence, from the Hartman-Grobman theorem and the stable manifold theorem we can deduce that [Sol1] is a hyperbolic equilibrium point which is a (degenerate) saddle (see also Buitier (1984)).

4 Welfare

In this section we focus on the welfare consequences of an unanticipated shock to the knowledge barrier. We assume a benevolent social planner running the firm and, hence, being in charge of choosing the R&D investment paths for product and process innovation that maximize the discounted, intertemporal social welfare. Social welfare ($sw(t)$) is defined as the sum of consumer surplus ($cs(t)$)

¹¹See Appendix A for details related to the Eigenvalues.

¹²This is due to the fact that none of our Eigenvalues is zero valued.

and the profit stream ($\pi(t)$). For $cs(t)$, we obtain:

$$cs(t) := \int_{\Theta-1}^{\Theta} (\theta q(t) - p(t)) d\theta = \frac{q(t)}{2}. \quad (33)$$

Thus, $sw(t)$ can be re-expressed as:

$$sw(t) := \frac{q(t)[1 + 2d(\Theta - 1) - 2\gamma q(t)]}{2} - c(t)d - [\alpha k^2(t) - \tau_k \Gamma_k(t)] - [\beta h^2(t) - \tau_h \Gamma_h(t)]. \quad (34)$$

Therefore, the planner's current value Hamiltonian is given by:

$$\begin{aligned} \mathcal{H} = & \left\{ \frac{q(t)[1 + 2d(\Theta - 1) - 2\gamma q(t)]}{2} - c(t)d - [\alpha k^2(t) - \tau_k \Gamma_k(t)] - [\beta h^2(t) - \tau_h \Gamma_h(t)] \right. \\ & + \lambda_1(t)[(k(t) - \delta + \kappa_k \tilde{k}(t))q(t)] \\ & + \lambda_2(t)[-(h(t) - \eta + \kappa_h \tilde{h}(t))c(t)] \\ & + \lambda_3(t)[\sigma_k k(t) - \omega_1 A_k(t)] \\ & + \lambda_4(t)[\sigma_h h(t) - \omega_2 A_h(t)] \\ & + \lambda_5(t)(g_k - \varsigma_k) \tilde{k}(t) \\ & \left. + \lambda_6(t)(g_h - \varsigma_h) \tilde{h}(t) \right\}, \end{aligned} \quad (35)$$

where $\lambda_i(t)$ for $i = \{1, \dots, 6\}$ are the costate variables associated with $q, c, A_k, A_h, \tilde{k}$ and \tilde{h} . Applying the same apparatus as during the derivation of the decentralized solution, again, we can establish a system of differential equations which allows us to study the dynamics between R&D investments in product and process innovations, product quality, and production costs.¹³ This system ([S2]) can be formally written as:

$$[S2] \begin{cases} \dot{k} = \rho \left(k - \frac{\tau_k \sigma_k}{2\alpha(\rho + \omega_k)} \right) - \frac{(\Theta-1)dq}{2\alpha} + \frac{q}{2\alpha} \left(2q\gamma - \frac{1}{2} \right), \\ \dot{h} = \rho \left(h - \frac{\tau_h \sigma_h}{2\beta(\rho + \omega_h)} \right) - \frac{cd}{2\beta}, \\ \dot{q} = (k - \delta + \kappa_k \tilde{k})q, \\ \dot{c} = -(h - \eta + \kappa_h \tilde{h})c. \end{cases} \quad (36)$$

Again requiring that the steady-state is time-invariant, we obtain six possible sets of solutions of which only one is economically sensible. Further, this solution is a degenerate saddle-point where the same rational applies as in the case analyzed in Section 3. Comparing the social optimal outcome with the decentralized one, with the exemption of the product quality, q , and investments in product quality, k , it turns out that this solution coincides with [Sol1]. The optimal quality level, q^m , turns out to be:

¹³See Appendix B for details on the derivations.

$$q^m = \frac{1 + 2d(\Theta - 1) + \frac{\sqrt{(\rho + \omega_k)(32\gamma\rho\sigma_k\tau_k + (\rho + \omega_k)((2d(\Theta - 1) + 1)^2 - 64\alpha\gamma\delta\rho + 64\alpha\gamma\kappa_k\bar{k}\rho))}}{\rho + \omega_k}}{8\gamma}. \quad (37)$$

Inspecting the expressions for q^m and q^{sts} in [Sol1] and (37) reveals that q^m is always exceeding q^{sts} as long the latter is positive valued (condition (31)). Thus, we observe a general under-supply of product quality in the decentralized economy. Compared to the benevolent social planner, the monopolist chooses a non-optimal level of product R&D investments.

This can be easily seen by solving each of the \dot{k} equations in [S1] and [S2] for the stationary solution of k given one particular level of quality q . Denote the solutions by $k^{sts}(q)$ and $k^m(q)$, we find the difference $\Delta k(q) := k^m(q) - k^{sts}(q) = \frac{q}{4\alpha\rho} > 0$. This is identical to the result in Lambertini and Orsini (2015).

The driving force behind the quality under-supply in the decentralized setting is the absence of the consumer surplus in the monopolist's objective function. On the contrary, the level of process innovations is socially efficient as the monopoly price p is not a function of the marginal costs and that the market demand is invariant to changes in the latter.

5 Optimal R&D policy

In the last section, we have shown that from a welfare point of view, the monopolist's profit incentives cause a downward distortion of product quality. Hence, the immediate question arises, as to which policy can establish the optimal level of product R&D that establishes the optimal product quality level in the decentralized setting. We assume the policy maker to introduce a lump-sum financed R&D subsidy tax scheme with a subsidy rate $\xi(t) \in (0, 1)$. Hence, the costs for setting up a R&D lab reduce to $\alpha k^2(1 - \xi)$. We would like to emphasize here that the subsidy rate might well vary with t . It is straightforward that this finding implies that short-run welfare effects (during transition) of a public financed R&D subsidy significantly differs from its long-run counterpart. We will return to this point later.

Solving the monopolists profit maximization problem, we can show that the (second-best) subsidy-adjusted equilibrium level of R&D investments in product quality, k^{sub} , are given in steady-state as:¹⁴

$$k^{sub} = \frac{q(\rho + \omega_k)(d(\Theta - 1) - 2\gamma q) + \rho\sigma_k\tau_k}{2\alpha\rho(1 - \xi)(\rho + \omega_k)}. \quad (38)$$

Hence, comparing equation (38) with the optimal level of product R&D, k^m , and solving for the subsidy

¹⁴See Appendix C for details in the derivations.

rate ξ by using the optimal solution for q^m , we arrive at the optimal product R&D subsidy rate, ξ^{opt} :

$$\xi^{opt}(\sigma_k, \tau_k, \kappa_k, \cdot) = \frac{q^m(\rho + \omega_k)}{q^m(\rho + \omega_k)(2d(\Theta - 1) - 4\gamma q^m + 1) + 2\rho\sigma_k\tau_k} = \quad (39)$$

$$= \frac{2d(\Theta - 1)\rho + \omega_k(2d(\Theta - 1) + 1) + \rho + \sqrt{\Psi}}{32\alpha\gamma\rho(\rho + \omega_k)(\delta - \kappa_k\tilde{k})}, \quad (40)$$

with $\Psi \equiv (\rho + \omega_k)(32\gamma\rho\sigma_k\tau_k + (\rho + \omega_k)((2d(\Theta - 1) + 1)^2 - 64\alpha\gamma\rho(\delta - \kappa_k\tilde{k})))$. Note that for $\bar{\tau}_k < \tau < \bar{\bar{\tau}}_k$ with $\bar{\bar{\tau}}_k \equiv \frac{4\alpha(\rho + \omega_k)(\delta - \kappa_k\tilde{k})(8\alpha\gamma\rho(\delta - \kappa_k\tilde{k}) + d(1 - \Theta))}{\sigma_k}$ (see equation (31)), it always holds that $\xi^{opt} \in (0, 1)$. Furthermore, Ψ is increasing in d and therefore $\xi^{opt}(\sigma_k, \tau_k, \kappa_k, \cdot)$ is increasing in market size. The optimal subsidy policy is, thus, more pronounced if the monopolist is active in smaller markets with less demand. Policy should target its subsidy policy on R&D in product innovations towards those firms contributing to consumers' surplus but suffering at the same time from low market volumes.

Taken together, these findings guide the following proposition:

Proposition

1. The subsidy rate $\xi^{opt}(\sigma_k, \tau_k, \kappa_k, \cdot)$ positively depends on the rate of knowledge accumulation, σ_k , the product innovations' learning rate, τ_k , as well as on the knowledge spillover rate, κ_k .
2. The subsidy rate never turns into a tax (i.e. $\xi < 0$) as long as $k \geq 0 \Leftrightarrow \delta \geq \tilde{k}\kappa_k$ provided that $\Theta > 1$.
3. Only along the steady-state, the subsidy rate is constant over time as the steady-state level of product quality is constant as well. However, during transition, the optimal subsidy rate is time-varying in a non-linear fashion with the level of product quality.

Proof: See Appendix D.

Hence, the last proposition tells that a reduction of the knowledge barrier implies a higher R&D subsidy given the product quality is below the social optimal. A reduction in the knowledge barrier related to knowledge spillovers κ_k reduces, ceteris paribus, steady-state investments in product innovations. This happens as the monopolist drives down own (costly) investments and partially replaces them by (cost less) external knowledge. As external knowledge is carrying a zero price, the reduction outweighs the additional external knowledge available due to the decreased knowledge barrier.

This has an additional effect on the impact of a subsidy on the quality level chosen by the monopolist. As investments in product innovations are reduced in case of a lower knowledge barrier, the implied cost reduction for the monopolist is more than proportionate as marginal investments cost are increasing. The discussed subsidy scheme targets exactly these costs and hence becomes less effective as its marginal impact is reduced by the non-linearity in investment costs (quadratic in our case).

A reduction in the knowledge barrier related to σ_k and τ_k has similar effects on the effectiveness of the subsidy scheme. The lower knowledge barrier increases the social optimum quality level (see (37)) which causes due to quadratic investment cost more than proportionate higher costs in production. Consequently, the optimal subsidy rate has to increase to provide an appropriate incentive for the monopolist.

6 Transitional dynamics

The main impetus of this paper is to persuade the reader that unexpected, knowledge barrier related shocks, such as shocks to the learning rate or to knowledge spillover have not only long-run, but also short-run welfare consequences. Hence, a policy maker with a pure long-run focus arrives at a misleading policy design for the short-run when it comes to a mitigation of such shocks only in the long-run.

To make this last point clear, we shock the decentralized economy and investigate the short and long-run welfare consequences. We do so by following the transitional dynamics considering different assumptions with respect to the market size d . As argued above, market size is an important factor influencing the optimal R&D policy. To simulate the entire welfare transition path, we employ the relaxation algorithm suggested by Trimborn et al. (2008). Whether or not welfare increases or decreases depends the (competing) forces of the consumer and producer surplus, respectively. Hence, the impetus of this simulation study is not to calibrate the model to real-world data, which is clearly beyond the scope of this contribution, but to persuade the reader that a welfare analysis focusing exclusively on the long-run would be incomplete.¹⁵ Due to the fact that process R&D investment incentives are optimally chosen in the decentralized economy, we only focus on those parts of the knowledge barrier related to product R&D investments and the subsidy policy targeting product quality.¹⁶

6.1 Parameter values

The chosen parameter space has to meet the parameter restrictions worked out above which ensure an economically meaningful equilibrium for our economy. Table (1) shows the parameter choice as well as the shock size expressed as percentage changes, respectively.

In total, we run four simulations each considering three different values for d . They show the welfare consequences of an unanticipated positive knowledge spillover shock ($\Delta\kappa_k > 0$), an unanticipated positive knowledge accumulation rate shock ($\Delta\sigma_k > 0$), an unanticipated positive learning

¹⁵Of course it is an interesting task to apply this model for a specific industry and discussing the transitional dynamic responses of shocking a specific parameter value. We leave this as a task for future research.

¹⁶The results from the analysis investigating the knowledge barrier related to process innovations can be obtained from the authors upon request. In general, the qualitative findings for welfare by a reduction of the process R&D-related knowledge barriers are similar.

Benchmark economy				
Evolution of product quality $q(t)$				
$\delta = 0.02$	$\kappa_k = 0.21$	$\tilde{k}(t) = \tilde{k} = 0.02$	$\bar{A}_k = 0.0001$	
Evolution of production cost $c(t)$				
$\eta = 0.90$	$\kappa_h = 0.10$	$\tilde{h}(t) = \tilde{h} = 0.70$	$\bar{A}_h = 0.0001$	
Evolution of product R&D investments $k(t)$				
$\tau_k = 0.10$	$\sigma_k = 0.20$	$\alpha = 0.80$	$\omega_k = 0.01$	$\gamma = 0.05$
Evolution of process R&D investments $h(t)$				
$\tau_h = 0.11$	$\sigma_h = 0.12$	$\beta = 0.90$	$\omega_h = 0.01$	
Structural parameter				
$\rho = 0.06$	$\Theta = 1.10$	$d \in \{1.10, 1.20, 1.30\}$	$\xi = 0.00$	
Shock size (in percentage points)				
$\Delta\xi = 0.10$	$\Delta\tau_k = 0.10$	$\Delta\sigma_k = 0.10$	$\Delta\kappa_k = 0.10$	

Table 1: Parameter calibration for the benchmark economy

rate shock ($\Delta\tau_k > 0$) as well as the welfare response of an unanticipated positive R&D subsidy shock ($\Delta\xi > 0$). Without loss of generality we assume that \tilde{h} and \tilde{k} are constant over time as in the theoretical analysis above. It is important to note that we also assume that the shocks last forever, i.e. a shocked parameter does not return to its original value. For all simulation runs, we report the relative difference as compared with the benchmark economy (Table 1). According to our stability analysis, after the shock, the economy converges towards its new steady-state. Finally, for all scenarios, the calibration guarantees that product quality is below the optimal level.

6.2 Results from simulation

Figure (1) shows the welfare change during transition. For every shock scenario, Table (2) shows the welfare change in the short- and in the long-run. Throughout transition, shocks to τ_k and σ_k have positive effects on welfare (see panels (b) and (c)). Increasing σ_k enhances the accumulation of knowledge while increasing τ_k improves on internal learning. Both imply cost reducing effects in the simulated scenarios which have positive effects on welfare. Looking at the effects taking account of market size uncovers that effects are stronger if the market size is smaller.

Adjustment after an unanticipated knowledge spillovers shock follows a different pattern (panel (d) in Figure (1)). As κ_k increases, the monopolists enjoys a higher rate of knowledge spillovers or external knowledge transfers. The subsequent reaction will be to reduce investments in product innovation which then leads to lower knowledge accumulation. Therefore, the knowledge gap will decline and, hence, production cost will increase. In the short-run, we see a positive effect on welfare. This happens as the reduction in investments have an instantaneous effect of reducing costs. R&D costs are quadratic in \bar{k} and lower investment, thus, initially reduce the total costs of the monopolist. After a while, however, the negative consequences of lower investments become visible. The negative effect

Welfare changes: short-run vs. long-run						
Shock	Short-run (sw(0))			Long-run		
	$d = 1.1$	$d = 1.2$	$d = 1.3$	$d = 1.1$	$d = 1.2$	$d = 1.3$
$\Delta \xi = 0.10$	-0.00072	-0.00064	-0.00057	0.05437	0.04858	0.043551
$\Delta \tau_k = 0.10$	102.371	96.8549	91.7787	108.109	102.023	96.4438
$\Delta \sigma_k = 0.10$	-0.05505	-0.04829	-0.04257	4.50211	4.11803	3.77754
$\Delta \kappa_k = 0.10$	0.00396	0.00379	0.00363	-0.32550	-0.31154	-0.29833

Table 2: Short vs. long-run absolute welfare changes relative to the new steady-state

on accumulated knowledge now outweighs the instantaneous cost reduction and overall production costs increase. The total welfare effect now becomes negative. Again, we find effects to be more pronounced in smaller markets.

The welfare effects of the subsidy are negative in the very short-run and positive afterwards as expected as can be seen from panel (a) in Figure (1)). The reason is again to be found in the time lag between an investment in product quality R&D and its effect on production costs. Investment implies instantaneous (quadratic) cost while cost reductions will be significant only after corresponding knowledge accumulation took place. Once this has happened, welfare is improving however. Looking at the market size, once more smaller markets imply stronger effects.

To sum up, the purpose of this simulation study is to show that, first, shocks to the knowledge barrier might have heterogeneous effects and, second, that the introduction of product innovation R&D subsidies potentially have very distinct effects in the short and in the long-run. Besides this, market size is a crucial factor influencing the results. If market size d is small, the quasi fixed cost implied by R&D influence the behavior of the monopolist to a great extent as they are large relative to total turnover. This needs to be kept in mind during policy making together with the finding that initial short-run effects of policy might be considerably different from their long run counterparts. Although our analysis considers a stationary d , this might of special importance to policy considering innovative ideas suffering initially from a small market size.

7 Conclusion

Based on a dynamic monopoly setting with simultaneous investment decisions in process as well as in product R&D, the main impetus of the paper is to show that a reduction of the knowledge barrier due to long-lasting shocks has ambiguous welfare consequences: With the "standard" focus solely on the long-run, welfare effects are ambiguous as welfare directly depends on the calibrated parameter space. If the focus is both on the short- and on the long-run, a simulation study highlights that short-run welfare effects may completely differ from their long-run counterparts. This is due to the fact that a knowledge barrier shock triggers the monopolist to sub-optimally lower its product R&D investments today at the cost of lower future levels of knowledge. Hence, this finding has an

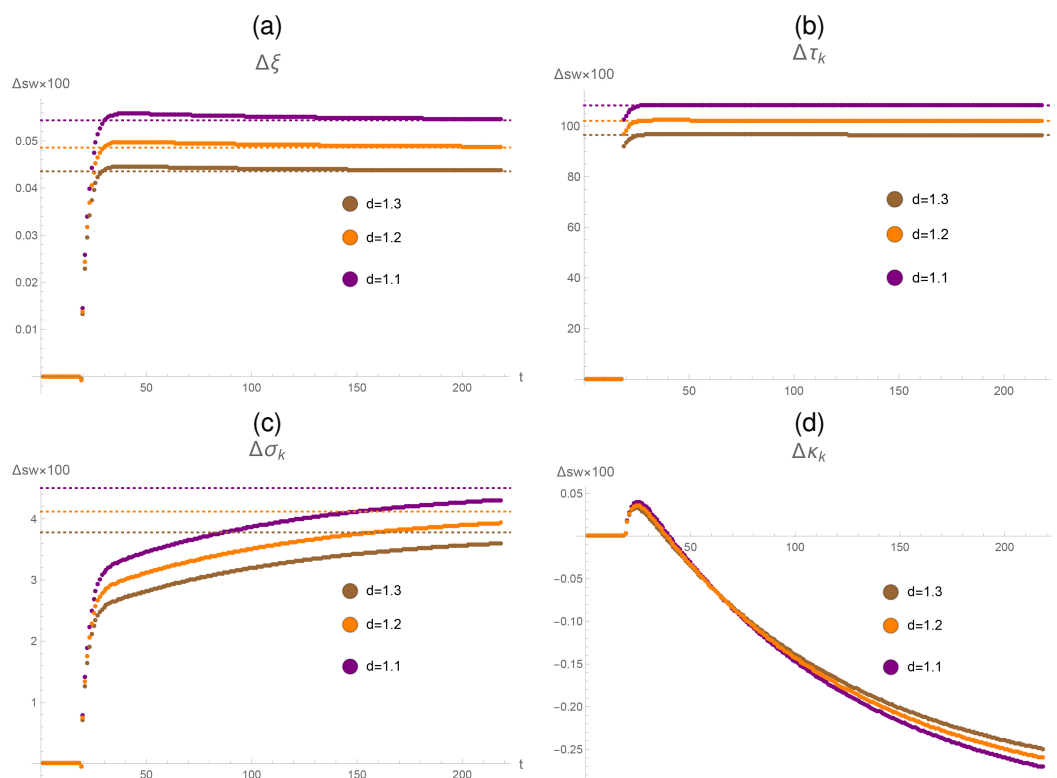


Figure 1: Welfare change at time t defined as the change of welfare relative to the new steady-state welfare level

important policy-implication: If a policy maker focuses on the short-run welfare effects of a decrease of the knowledge barrier, the policy recommendation which might be optimal in the long-run could be biased in the short-run.

In this contribution we focus on the product R&D sphere as, in contrast to the process R&D investment sphere, the optimal solution for the product quality differs from the social optimal one. Hence, we discuss the implementation of a consumer financed R&D subsidy which aims to realize the social optimal product quality level. We find that in the short-run, this R&D subsidy rate is time-varying with the level of product quality and only constant in the long-run. We also find that the R&D subsidy rate is higher, the lower the knowledge barrier is. Market size is a crucial factor for policy making as well. Smaller markets are benefiting relatively more from subsidy policies. At the same time, innovative monopolies are also in general stronger affected by knowledge barriers. This stresses our argument from the introductory section that product innovations should be discussed along with process innovations in the context of market size and should not be left out of this discussion.

Conflict of Interest

None. The authors have nothing to declare.

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Appendix

A: Monopolist's problem

The FOCs (18) and (19) allow for a closed form solution for $\lambda_3(t)$ and $\lambda_4(t)$. Rewrite these conditions as

$$\begin{aligned}\dot{\lambda}_j(t) + f_j(t) &= g_j(t), j = \{3, 4\}, \\ \text{with} \\ f_j(t) &= -(\rho + \omega_i), \quad g_j(t) = -\tau_i, i = \{k, h\},\end{aligned}$$

we find

$$\lambda_j(t) = \lambda_j(0) \exp\left[-\int_0^t f_j(s) ds\right] + \int_0^t g_j(z) \exp\left[-\int_z^t f_j(s) ds\right] dz.$$

This delivers

$$\lambda_3(t) = \frac{\tau_k}{\rho + \omega_k} + \exp[(\rho + \omega_k)t] \left(\lambda_3(0) - \frac{\tau_k}{\rho + \omega_k} \right), \quad (41)$$

$$\lambda_4(t) = \frac{\tau_h}{\rho + \omega_h} + \exp[(\rho + \omega_h)t] \left(\lambda_4(0) - \frac{\tau_h}{\rho + \omega_h} \right). \quad (42)$$

With $\tilde{k}(t)$ and $\tilde{h}(t)$ constant, the steady-state fulfills $\dot{c}(t) = \dot{q}(t) = \dot{k}(t) = \dot{h}(t) = \dot{A}_k(t) = \dot{A}_h(t) = 0$. In a non-degenerate equilibrium, we need to have $\lim_{t \rightarrow \infty} A_i(t) = A_i^{sts}$ with $A_i^{sts} > 0$ and finite.

The transversality conditions for $A_i(t)$ therefore imply $\lim_{t \rightarrow \infty} \exp[-\rho t] \lambda_j(t) A_i(t) = A_i^{sts} \lim_{t \rightarrow \infty} \exp[-\rho t] \lambda_j(t) = 0$. Using (41) and (42) then gives

$$\begin{aligned}\exp[-\rho t] \lambda_3(t) &= \exp[-\rho t] \frac{\tau_k}{\rho + \omega_k} + \exp[\omega_k t] \left(\lambda_3(0) - \frac{\tau_k}{\rho + \omega_k} \right), \\ \exp[-\rho t] \lambda_4(t) &= \exp[-\rho t] \frac{\tau_h}{\rho + \omega_h} + \exp[\omega_h t] \left(\lambda_4(0) - \frac{\tau_h}{\rho + \omega_h} \right).\end{aligned}$$

Hence, the transversality conditions for $A_k(t)$ and $A_h(t)$ require $\lambda_3(t) = \frac{\tau_k}{\rho + \omega_k}$ and $\lambda_4(t) = \frac{\tau_h}{\rho + \omega_h}$ for all t as $\lambda_3(t)$ and $\lambda_4(t)$ otherwise explode or degenerate towards zero. Inserting these results into (16) and (17) respectively, gives the optimal controls in (22) and (23).

The Eigenvalues $\mu_i, i \in 1, \dots, 6$ of the Jacobian \mathcal{J} are given by

$$\begin{aligned} \mu_1 &= -\omega_k, & \mu_2 &= -\omega_h, \\ \mu_3 &= \frac{1}{2} \left(\rho - \frac{\sqrt{\alpha^2 \beta (\beta \rho^2 + 2c^{sts} d)}}{\alpha \beta} \right), & \mu_4 &= \frac{1}{2} \left(\rho - \frac{\sqrt{\alpha \beta^2 (\alpha \rho^2 + 2q^{sts} (d(1-\theta) + 4\gamma q^{sts})}}{\alpha \beta} \right), \\ \mu_5 &= \frac{1}{2} \left(\rho + \frac{\sqrt{\alpha^2 \beta (\beta \rho^2 + 2c^{sts} d)}}{\alpha \beta} \right), & \mu_6 &= \frac{1}{2} \left(\rho + \frac{\sqrt{\alpha \beta^2 (\alpha \rho^2 + 2q^{sts} (d(1-\theta) + 4\gamma q^{sts})}}{\alpha \beta} \right). \end{aligned}$$

Note that $\mu_1 < 0, \mu_2 < 0, \mu_3 < 0$ for $\tau_h < \bar{\tau}_h, \mu_4 < 0$ for $\tau_k > \bar{\tau}_k, \mu_5 > 0$ for $\tau_h > \bar{\tau}_h$ and $\mu_6 > 0$ for $\tau_h > \bar{\tau}_h$ where $\bar{\tau}_h, \bar{\tau}_k$ are given by (30) and (31).

B: Centralized solution

Given the Hamiltonian (35), the FOCs read as

$$k : \quad -2\alpha k + \lambda_1 q + \lambda_3 \sigma_k = 0, \quad (43)$$

$$h : \quad -2\beta h - \lambda_2 c + \lambda_4 \sigma_h = 0, \quad (44)$$

$$\lambda_1 : \quad \dot{\lambda}_1 = (\rho - k + \delta - \kappa_k \tilde{k}) \lambda_1 - (\Theta - 1)d + 2q\gamma - \frac{1}{2}, \quad (45)$$

$$\lambda_2 : \quad \dot{\lambda}_2 = (\rho + h + \kappa_h \tilde{h} - \eta) \lambda_2 + d, \quad (46)$$

$$\lambda_3 : \quad \dot{\lambda}_3 = (\rho + \omega_k) \lambda_3 - \tau_k, \quad (47)$$

$$\lambda_4 : \quad \dot{\lambda}_4 = (\rho + \omega_h) \lambda_4 - \tau_h, \quad (48)$$

$$\lambda_5 : \quad \dot{\lambda}_5 = -\kappa_k q \lambda_1 - (g_k - \varsigma_k - \rho) \lambda_5, \quad (49)$$

$$\lambda_6 : \quad \dot{\lambda}_6 = +\kappa_h c \lambda_2 - (g_h - \varsigma_h - \rho) \lambda_6. \quad (50)$$

The corresponding transversality conditions are given by:

$$\lim_{t \rightarrow \infty} \lambda_1(t) q(t) \exp(-\rho t) = 0, \lim_{t \rightarrow \infty} \lambda_2(t) c(t) \exp(-\rho t) = 0, \lim_{t \rightarrow \infty} \lambda_3(t) A_k(t) \exp(-\rho t) = 0, \lim_{t \rightarrow \infty} \lambda_4(t) A_h(t) \exp(-\rho t) = 0, \lim_{t \rightarrow \infty} \lambda_5(t) \tilde{k}(t) \exp(-\rho t) = 0, \text{ and } \lim_{t \rightarrow \infty} \lambda_6(t) \tilde{h}(t) \exp(-\rho t) = 0.$$

We note that the FOCs in this case only differ by condition (45) from the FOCs in the decentralized solution as only the term $\frac{q(t)}{2}$ reflecting consumers' surplus adds to the Hamiltonian.

Therefore, as in Appendix A, the co-states associated with $A_k(t)$ and $A_h(t)$ need to be constant at $\lambda_3(t) = \frac{\tau_k}{\rho + \omega_k}$ and $\lambda_4(t) = \frac{\tau_h}{\rho + \omega_h}$. Using these expressions for $\lambda_3(t)$ and $\lambda_4(t)$ in (43) and (44) gives the same optimal controls as before ((22), (23))

$$\begin{aligned}
 k^* &= \max \left\{ 0, \frac{\lambda_1 q + \left(\frac{\tau_k}{\rho + \omega_k} \right) \sigma_k}{2\alpha} \right\}, \\
 h^* &= \max \left\{ 0, \frac{-\lambda_2 c + \left(\frac{\tau_h}{\rho + \omega_h} \right) \sigma_h}{2\beta} \right\},
 \end{aligned} \tag{51}$$

implying

$$\dot{k} = \frac{\dot{q}\lambda_1 + q\dot{\lambda}_1}{2\alpha}, \tag{52}$$

$$\dot{h} = -\frac{\dot{c}\lambda_2 + c\dot{\lambda}_2}{2\beta}. \tag{53}$$

Using (7), (47) and (43) in (52) and (8), (48) and (44) in (53) gives the dynamic system [S2]:

$$[S2] \left\{ \begin{aligned}
 \dot{k} &= \rho \left(k - \frac{\tau_k \sigma_k}{2\alpha(\rho + \omega_k)} \right) - \frac{(\Theta - 1)dq}{2\alpha} + \frac{q}{2\alpha} \left(2q\gamma - \frac{1}{2\alpha} \right), \\
 \dot{h} &= \rho \left(h - \frac{\tau_h \sigma_h}{2\beta(\rho + \omega_h)} \right) - \frac{cd}{2\beta}, \\
 \dot{q} &= (k - \delta + \kappa_k \tilde{k})q, \\
 \dot{c} &= -(h - \eta + \kappa_h \tilde{h})c,
 \end{aligned} \right. \tag{54}$$

where we note that the only difference between [S2] and [S1] is the additional far right term in the first equation. The steady-state solution for the case $\dot{c}(t) = \dot{q}(t) = \dot{k}(t) = \dot{h}(t) = \dot{A}_k(t) = \dot{A}_h(t) = 0$ is then straightforward. Except for q , we find the same results as in case of the decentralized equation. In particular, the steady-state quality level in the centralized equilibrium, q^m is found by solving for \tilde{k} in steady-state as in Appendix A by using (55) and inserting the result into (54) in steady-state.

C: R&D policy

The Hamiltonian in this case reads as

$$\begin{aligned}
 \mathcal{H} = & \{(\Theta - 1)[q(t) - c(t)]d - [(1 - \xi(t))\alpha k^2(t) - \tau_k \Gamma_k] - [\beta h^2(t) - \tau_h \Gamma_h] - \gamma q(t)^2 \\
 & + \lambda_1(t)[(k(t) - \delta + \kappa_k \tilde{k}(t))q(t)] \\
 & + \lambda_2(t)[-(h(t) - \eta + \kappa_h \tilde{h}(t))c(t)] \\
 & + \lambda_3(t)[\sigma_k k(t) - \omega_k A_k(t)] \\
 & + \lambda_4(t)[\sigma_h h(t) - \omega_h A_h(t)] \\
 & + \lambda_5(t)(g_k - \varsigma_k)\tilde{k}(t) \\
 & + \lambda_6(t)(g_h - \varsigma_h)\tilde{h}(t)\},
 \end{aligned}$$

where λ_i for $i = \{1, \dots, 6\}$ are the costate variables associated with $q, c, A_k, A_h, \tilde{k}$ and \tilde{h} . Suppressing time arguments whenever possible, the first order conditions (FOCs) for the controls k, h and costate equations read as:

$$k : \quad -2(1 - \xi)\alpha k + \lambda_1 q + \lambda_3 \sigma_k = 0, \quad (56)$$

$$h : \quad -2\beta h - \lambda_2 c + \lambda_4 \sigma_h = 0, \quad (57)$$

$$\lambda_1 : \quad \dot{\lambda}_1 = (\rho - k + \delta - \kappa_k \tilde{k})\lambda_1 - (\Theta - 1)d + 2q\gamma, \quad (58)$$

$$\lambda_2 : \quad \dot{\lambda}_2 = (\rho + h + \kappa_h \tilde{h} - \eta)\lambda_2 + d, \quad (59)$$

$$\lambda_3 : \quad \dot{\lambda}_3 = (\rho + \omega_k)\lambda_3 - \tau_k, \quad (60)$$

$$\lambda_4 : \quad \dot{\lambda}_4 = (\rho + \omega_h)\lambda_4 - \tau_h, \quad (61)$$

$$\lambda_5 : \quad \dot{\lambda}_5 = -\kappa_k q \lambda_1 - (g_k - \varsigma_k - \rho)\lambda_5, \quad (62)$$

$$\lambda_6 : \quad \dot{\lambda}_6 = +\kappa_h c \lambda_2 - (g_h - \varsigma_h - \rho)\lambda_6. \quad (63)$$

The corresponding transversality conditions are given by:

$$\begin{aligned}
 \lim_{t \rightarrow \infty} \lambda_1(t)q(t) \exp(-\rho t) = 0, \quad \lim_{t \rightarrow \infty} \lambda_2(t)c(t) \exp(-\rho t) = 0, \quad \lim_{t \rightarrow \infty} \lambda_3(t)A_k(t) \exp(-\rho t) = \\
 0, \quad \lim_{t \rightarrow \infty} \lambda_4(t)A_h(t) \exp(-\rho t) = 0, \quad \lim_{t \rightarrow \infty} \lambda_5(t)\tilde{k}(t) \exp(-\rho t) = 0, \quad \text{and} \\
 \lim_{t \rightarrow \infty} \lambda_6(t)\tilde{h}(t) \exp(-\rho t) = 0.
 \end{aligned}$$

As in Appendices A and B, (60) and (61) together with the transversality conditions involving A_k and A_h demand λ_3 and λ_4 to be constant for all t . Their values equal those in the preceding cases.

The optimal control k^{sub} can be found by solving (56) for k and inserting $\lambda_3 = \frac{\tau_k}{\rho + \omega_k}$

$$k^{sub} = \frac{\lambda_1 q^{sub} + \frac{\sigma_k \tau_k}{\omega_k + \rho}}{2\alpha(1 - \xi)}, \quad (64)$$

with q^{sub} as the quality level resulting from the subsidy policy. Evaluating (58) in steady-state with $\dot{\lambda}_1 = 0$ gives

$$\lambda_1 = \frac{(\Theta - 1)d}{\rho} - \frac{2\gamma q^{sub}}{\rho}.$$

Inserting this into (64) gives k^{sub} as in the main text as

$$k^{sub} = \frac{q^{sub}(\rho + \omega_k)(d(\Theta - 1) - 2\gamma q^{sub}) + \rho\sigma_k\tau_k}{2\alpha\rho(1 - \xi)(\rho + \omega_k)}.$$

Returning to the case of the welfare optimum discussed in Appendix B, we find the optimal investments into quality, k^m by solving, first, (46) for λ_1 in steady-state with a constant level of quality and constant corresponding investments. Inserting this result into (43) and solving for (an interior) k^m gives

$$k^m = \frac{q^m(\rho + \omega_k)(d(\Theta - 1) - 2\gamma q^m - \frac{1}{2}) + \rho\sigma_k\tau_k}{2\alpha\rho(\rho + \omega_k)}.$$

Equalizing this expression with k^{sub} in (and demanding that the level of quality is identical to q^m) leads to the optimal tax rate $\xi^{opt}(\sigma_k, \tau_k, \kappa_k, \cdot)$ as given in the main text by equation (39). Inserting q^m given by (37) into (39) finally delivers (40).

D: Proof of Propositions

1. $\xi^{opt}(\sigma_k, \tau_k, \kappa_k, \cdot)$ **increases in σ_k , τ_k and κ_k** : This follows directly as $\xi^{opt}(\sigma_k, \tau_k, \kappa_k, \cdot)$ positively depends on Ψ which in turn depends positively on σ_k , τ_k and κ_k \square
2. $\xi^{opt}(\sigma_k, \tau_k, \kappa_k, \cdot) > 0$: From (40), this follows whenever $\delta - \kappa_k > 0$ holds. Inspecting (39) alternatively, reveals that we find a positive $\xi^{opt}(\sigma_k, \tau_k, \kappa_k, \cdot)$ in any case if $2d(\Theta - 1) - 4\gamma q^m + 1 \geq 0$. This, however, is equal to the marginal increase in welfare due to quality changes in the social optimum and, hence, cannot be negative \square
3. **Subsidy time varying off the steady-state** : Under the optimal subsidy scheme, the level of quality q and investments into product innovations k need to replicate its welfare maximizing counterparts. Equating the investments k fulfilling (64) and (51) respectively, and solving for the subsidy rate gives

$$\xi^{opt}(\sigma_k, \tau_k, \kappa_k, \cdot) = q \frac{\lambda_1^m - \lambda_1^{sub}}{\lambda_1^m q + \frac{\sigma_k}{\rho + \omega_k}},$$

where λ_1^m and λ_1^{sub} denote the shadow values of the quality level in the welfare optimum and under the subsidy scheme. Note that they are not equal as the subsidy affects this implicit price for quality. Differentiating this with respect to time leads to

$$\frac{\dot{\xi}^{opt}(\sigma_k, \tau_k, \kappa_k, \cdot)}{\xi^{opt}(\sigma_k, \tau_k, \kappa_k, \cdot)} = \frac{\dot{q}}{q} + \frac{\dot{\lambda}_1^m - \dot{\lambda}_1^{sub}}{\lambda_1^m - \lambda_1^{sub}} - \frac{\dot{\lambda}_1^m q + \lambda_1^m \dot{q}}{\lambda_1^m q + \frac{\sigma_k}{\rho + \omega_k}},$$

where \dot{q} fulfills (55). Making use of (52) gives

$$\frac{\dot{\xi}^{opt}(\sigma_k, \tau_k, \kappa_k, \cdot)}{\xi^{opt}(\sigma_k, \tau_k, \kappa_k, \cdot)} = \frac{\dot{q}}{q} + \frac{\dot{\lambda}_1^m - \dot{\lambda}_1^{sub}}{\lambda_1^m - \lambda_1^{sub}} - \frac{\dot{k}}{k},$$

where \dot{k} fulfills (52). Using the FOCs (45) and (58) together with (55), the middle term on the right hand side reads as

$$\frac{\dot{\lambda}_1^m - \dot{\lambda}_1^{sub}}{\lambda_1^m - \lambda_1^{sub}} = \rho - \frac{1}{2(\lambda_1^m - \lambda_1^{sub})}.$$

Hence,

$$\frac{\dot{\xi}^{opt}(\sigma_k, \tau_k, \kappa_k, \cdot)}{\xi^{opt}(\sigma_k, \tau_k, \kappa_k, \cdot)} = \frac{\dot{q}}{q} + \rho - \frac{1}{2(\lambda_1^m - \lambda_1^{sub})} - \frac{\dot{k}}{k}.$$

As the dynamics of the model imply transitional dynamics (see Section 3), all the terms in this result are time varying. Consequently, the optimal subsidy rate shares this property \square

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