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The implications of automation for economic growth when investment decisions are irreversible

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Abstract

This paper discusses automation embedded into a standard growth model without exogenous growth when investment decisions for physical and automation capital are irreversible. The imposed non-negativity constraints on physical and automation capital induces an imbalance effect between the growth rate of output and the fraction between physical and automation capital. The paper shows that this imbalance effect leads (i) to transitional dynamics off the steady state while (ii) retaining perpetual growth of the AK style in the steady state without exogenous technological progress. We also show that the resulting transition path does not have to be on the saddle path of the system without the nonnegativity constraints.

Keywords

Automation, perpetual economic growth, irreversibility of investment decisions

JEL Classifications

O40

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1 Introduction

In recent contributions, inter alia Prettner (2019), Lankisch et al. (2019) and Geiger et al. (2018) show that the introduction of automation into the standard neoclassical growth framework of Solow (1956) leads to perpetual growth even in the absence of exogenous technological progress. Steigum (2011) arrives at a corresponding result in an optimal growth model. The reason for this finding is that these settings are equivalent to a standard AK framework, as automation capital is a perfect substitute for human labor. Labor together with automation capital complements physical capital and, hence, off-sets the diminishing returns of physical capital.

As a consequence, these models do not exhibit transitional dynamics. Most important, the implicit assumption made in these models is that investment decisions in physical and automation capital are reversible, which seems to be implausible. However new results emerge, if we allow for the constraints that gross investment in physical and automation capital must each be nonnegative. In other words, this papers makes the more realistic assumption that investment decisions in physical and automation capital are irreversible. Hence, it is not allowed to disinvest automation or physical capital. This paper shows that the resulting, non-symmetric imbalance effect between the growth rate of output and the fraction between physical and automation capital leads (i) to transitional dynamics off the steady state while (ii) retaining perpetual growth of the AK style in the steady state without exogenous technological progress. We also show that the resulting transition path does not have to be on the saddle path of the system without the nonnegativity constraints.

This paper is organized as follows. Section 2 presents the analytical framework. Section 3 analyzes the equilibrium, and Section 4 introduces and discusses the implications of the nonnegativity constraints for gross investment in automation and physical capital. Section 5 concludes.

2 The analytical framework

We consider a closed economy. The infinite-lived households maximize utility, as given by

$$\Omega = \int_0^\infty \exp[-(\rho - n)t] \ln(c(t)) dt, \tag{1}$$

where $\rho > 0$ is the rate of time preference and $c(t) := \frac{C(t)}{L(t)}$ denotes per capita consumption. Population $L(t)$ grows with the exogenous rate $\frac{\dot{L}(t)}{L(t)} = n$.

The individual's output, $Y(t)$, is represented by a Cobb-Douglas technology:

$$Y(t) = F(L(t), P(t), K(t)) = A(t)(L(t) + P(t))^{1-\alpha} K(t)^\alpha, \ \alpha \in (0,1), \tag{2}$$

where, like Prettner (2019) we assume that automation capital $P(t)$ is a perfect substitute for labor $L(t)$. Automation capital comprises robots, 3D printers etc. $A(t) \equiv 1$ stands for the level of technology, which we normalize to one[1].

The economy's resource constraint is:

$$Y(t) = F(L(t), P(t), K(t)) = C(t) + I(t)_K + I(t)_P, \tag{3}$$

where $I(t)_K$ and $I(t)_P$ are gross investment in physical and automation capital, respectively. The changes in the two capital stock are governed by

$$\begin{aligned} \dot{K}(t)_P &= I(t)_P - \delta P(t), \delta \in (0,1), & (4) \\ \dot{K}(t)_K &= I(t)_K - \delta K(t), \delta \in (0,1). & (5) \end{aligned}$$

For simplicity, we assume identical depreciation rates for both stocks of capital. We would like to point out that the general findings in this paper are not affected by this simplifying assumption.

The central assumption in this model is that, ex ante, even though the households can decide whether to invest in physical capital $K(t)$ or automation capital $P(t)$, once the decision is made, it is irreversible[2]. Technically, these irreversibility constraints translate into the following nonnegativity constraints:

$$\begin{aligned} I(t)_P &\geq 0, & (6) \\ I(t)_K &\geq 0. & (7) \end{aligned}$$

In other words, it is not allowed to disinvest robot or physical capital. Finally, we assume that a fraction $\tau \in [0,1]$ of the economy's total investments $I(t) = I(t)_K + I(t)_P$ goes into automation capital investment $I(t)_P$.

3 Equilibrium

For the moment, let us ignore the nonnegativity constraints imposed on I_K and I_P[3]. The households maximize the utility function (1) subject to the two constraints represented by equations (4) and (5) and subject to the economy-wide resource constraint given by equation (3).

Using per capita notation $x := \frac{X}{L}$ for $X = \{P, K, I, Y, C\}$, the present-value Hamiltonian reads as

[1]The reason for the normalization is that we want to avoid that the long-run growth potential due to automation is confounded by a second engine of economic growth, such as technological progress which is embodied by $A(t)$.

[2]This implies the realistic idea that old units of robot capital $P(t)$ cannot be converted into physical capital $K(t)$ and vice versa.

[3]Where appropriate, throughout the rest of the paper, we omit time indices for clarity.

$$\begin{aligned}\mathcal{H}(c,\tau,k,p,\lambda_1,\lambda_2) &:= \ln(c)\exp[-(\rho-n)t]+ \\ +\lambda_1[(1+p)^{(1-\alpha)}k^\alpha - c - \tau i - (\delta+n)k] &+ \lambda_2[\tau i - (\delta+n)p]\end{aligned}, \tag{8}$$

where λ_1 and λ_2 are shadow prices associated with $\dot{k}$ and $\dot{p}$, respectively.

The first-order conditions for an interior optimum are

$$\mathcal{H}(\cdot)_c = 0 \Leftrightarrow c^{-1}\exp[-(\rho-n)t] = \lambda_1, \tag{9}$$

$$\mathcal{H}(\cdot)_\tau = 0 \Leftrightarrow \lambda_1 = \lambda_2, \tag{10}$$

$$-\mathcal{H}(\cdot)_k = -[\alpha(1+p)^{(1-\alpha)}k^{\alpha-1} - (\delta+n)]\lambda_1 = \dot{\lambda_1}, \tag{11}$$

$$-\mathcal{H}(\cdot)_p = -[(1-\alpha)(1+p)^{-\alpha}k^\alpha]\lambda_1 - (\delta+n)]\lambda_2 = \dot{\lambda_2}, \tag{12}$$

plus the transversality conditions

$$\lim_{t\to\infty}(\lambda_1 k) = \lim_{t\to\infty}(\lambda_2 p) = 0. \tag{13}$$

Using conditions (9)-(12), the growth rate of per capita consumption reads as

$$\gamma_c := \frac{\dot{c}}{c} = \alpha k^{\alpha-1}(1+p)^{1-\alpha} - \delta - \rho, \tag{14}$$

where $\alpha k^{\alpha-1}(1+p)^{1-\alpha} - \delta$ represents the net marginal product of physical capital.

Further, condition (10) says that, in equilibrium, the compensation of physical capital owners has to be equal to the compensation of automation capital owners which is $(1-\alpha)(1+p)^{-\alpha}k^\alpha - \delta$. Thus, for sufficiently large p and k, we obtain the constant ratio[4]

$$\xi := \frac{1+p}{k} \approx \frac{p}{k} = \frac{1-\alpha}{\alpha}. \tag{15}$$

Rewriting equation (15) and multiplication with L leads to: $P = \left(\frac{1-\alpha}{\alpha}\right)K - L$, which shows that a higher stock of physical capital K raises (reduces) the rate of return on investment in automation (physical) capital P (K)[5].

After we have substituted expression (15) into (14), we can show that $\frac{\dot{c}}{c}$ is constant and equal to

$$\hat{\gamma}_c = \alpha^\alpha(1-\alpha)^{1-\alpha} - \delta - \rho, \tag{16}$$

and positive as long as $\alpha^\alpha(1-\alpha)^{1-\alpha} > \delta + \rho$ which holds in the following. Moreover, using (15) in

[4] If the depreciation rates on the two kinds of capital differ, the equality between net marginal products of robot and physical capital still holds. However, the solution cannot be written as a closed-form expression in terms of the model's underlying parameters.

[5] From this we conclude that $P = \max\left[0, \left(\frac{1-\alpha}{\alpha}\right)K - L\right]$.

equation (3), we directly observe that our economy is equivalent to a standard AK setting:

$$Y = \left[\frac{1-\alpha}{\alpha}\right]^{1-\alpha} K. \tag{17}$$

Now, we focus on the steady state at which consumption, production and both kinds of capital and investment grow at the same rate. We can state the following proposition.

Proposition 1. *The unique long-run growth rate of the economy is*

$$\frac{\dot{y}}{y} = \frac{\dot{i}}{i} = \frac{\dot{p}}{p} = \frac{\dot{k}}{k} = \frac{\dot{c}}{c} := \hat{\gamma}_c = \alpha^{\alpha}(1-\alpha)^{1-\alpha} - \delta - \rho. \tag{18}$$

Proof. The transversality conditions holds for $\rho > n$. Further, it can be shown that on the steady state, $\frac{c}{k} = \frac{\rho - n}{\alpha} > 0$. Hence, c and k grow at the same constant rate. Equation (15) implies that on the steady state, p and k grow at the same rate. Moreover, as $\frac{y}{k} = \left(\frac{1+p}{k}\right)^{1-\alpha}$, y and k grow with the constant rate. Finally, using the economy-wide budget constraint (3), on the steady state, i grows at the constant rate $\hat{\gamma}_c$.

We note that Proposition 1 implies a gross savings rate $1 - \frac{c}{y} = 1 - (\rho - n)(1-\alpha)^{-(1-\alpha)}\alpha^{-\alpha}$ in steady-state. Geiger et al. (2018) [equation (5)] and Lankisch et al. (2019) [equation (15)] derive the steady-state growth rate in a Solow model, and hence, with an exogenous savings rate. Using the endogenous steady-state growth rate just derived in the results of Geiger et al. (2018) and Lankisch et al. (2019) for the case of only one type of labor yields exactly our result in Proposition 1. This shows how our approach complements the existing literature by adopting endogenous savings (Steigum (2011)) and introducing them into the analysis previously done within Solow type models.

4 The constraints of nonnegative gross investment in automation and physical capital

Suppose the economy starts with the two capital stocks $p(0)$ and $k(0)$. The analysis in section 3 implies that for $\frac{p(0)}{k(0)} = \frac{1-\alpha}{\alpha}$, all variables grow at the constant rate $\hat{\gamma}_c$ from the beginning ($t = 0$). Hence, there is no transitional dynamics. In contrast to the analysis conducted in section 3, where we have ignored the nonnegativity constraints on investments, the results are different if these constraints are binding. In particular, we show that the dynamics of the neoclassical growth model apply during transition while leaving the long-run growth property ($\hat{\gamma}_c > 0$) in steady-state unaffected. Even if exogenous technological progress is equal to zero like in our setting, we find the potential for perpetual growth. This is because of the absence of diminishing returns in the steady-state related to a broad definition of capital consisting of physical and automation capital implied by the AK structure of our economy (see equation (17)). In the following, we consider the following alternative cases: $\frac{p(0)}{k(0)} > \frac{1-\alpha}{\alpha}$ and $\frac{p(0)}{k(0)} < \frac{1-\alpha}{\alpha}$.

4.1 Case 1: $\xi(0) := \frac{p(0)}{k(0)} > \frac{1-\alpha}{\alpha}$

Suppose we have $\xi(0) := \frac{p(0)}{k(0)} > \frac{1-\alpha}{\alpha}$. In this case, households reduce p and raise k by discrete amounts, so that $I_P \geq 0$ is binding in a finite interval $[0, T]$, whereas $i_k > 0$. As $I_P = 0 \Leftrightarrow \tau = 0$ together with $\frac{\dot{p}}{p} = -\delta$, we have that $\frac{\dot{c}}{c} = \alpha\xi^{1-\alpha} - \delta - \rho$ together with $\frac{\dot{p}}{p} = -(\delta + n)$ and $\frac{\dot{k}}{k} = \xi^{1-\alpha} - c - \delta - n$. Critical for our analysis is to make sure that the AK balanced growth path derived in the preceding section is reached in finite times. To show this, the first order condition (10) should be rewritten as the complementary slackness condition for investment in automation capital:

$$\lambda_1 - \lambda_2 \geq 0,\ (\lambda_1 - \lambda_2)\tau = 0. \tag{19}$$

For $\tau = 0$, complementary slackness tells that $\lambda_1 > \lambda_2$. However, as $\lim_{k\to\infty} \lambda_1 = \infty$, capital would be over-accumulated until depreciation on capital exceeds its marginal product and consumption goes down to zero (see equation (9)). Hence, the situation where $\tau = 0$ cannot prevail forever. In turn, this implies that $\lambda_1 - \lambda_2$ shrinks towards zero[6] in finite times, where $\tau > 0$ with $\lambda_1 = \lambda_2$, and, thus, the economy is on its balanced growth path. A similar argument can be made for Case 2 discussed below.

Introducing the additional variable $\chi := \frac{c}{k}$ allows us to graphically discuss the transitional dynamics of $\{c, p, k\}$ in the (ξ, χ) space. From the Proof of Proposition 1, we know that both variables, ξ and χ are constant in the steady state. The transition equations for ξ and χ are:

$$\gamma_\chi := \frac{\dot{\chi}}{\chi} = (\alpha - 1)\xi^{-(\alpha-1)} - \rho + n + \chi, \tag{20}$$

$$\gamma_\xi := \frac{\dot{\xi}}{\xi} = -\xi^{1-\alpha} + \chi. \tag{21}$$

Using similar arguments as those employed by Ladrón-de-Guevara et al. (1997) or Gomez (2003) show that the economy converges to the interior balanced growth path with $\gamma_k > \hat{\gamma}$ and $\gamma_p = -(\delta + n) < 0$. Thus, as the economy evolves, ξ decreases. At time T, we observe an equalization of the net returns of automation and physical capital, and the constraint $I_P \geq 0$ becomes nonbinding. From $t > T$, the solution is given by $\xi(t) = \hat{\xi}$ and $\chi(t) = \hat{\chi}$ and, according to Proposition 1, all variables with the exemption of χ and ξ grow at the constant rate $\hat{\gamma}$. From equation (9) we observe that consumption is continuous as the shadow price λ_1 is continuous. As both types of capital are continuous as well, ξ and χ are also continuous. The resource constraint (3) implies that the expression $i_k + i_p$ is continuous as well[7]. Hence, whenever the balanced growth path is reached, consumption reaches its steady-state value without jumping.

[6]Linearization of equations (11) and (12) around $(\xi - \hat{\xi})$ shows that $\frac{\dot{\lambda}_1}{\lambda_1} < 0$ and $\frac{\dot{\lambda}_2}{\lambda_2} > 0$. Hence $\lambda_1 - \lambda_2$ shrinks towards zero.

[7]This does not exclude that both, i_k and i_p jump, which in fact they do.

The left panel of figure (1) represents the phase diagram in the (ξ, χ) space for Case 1. It is obvious that $\dot{\xi} = 0$ directly implies $\chi = \xi^{1-\alpha}$, whereas $\dot{\chi} = 0$ requires that $\chi = (1-\alpha)\xi^{1-\alpha} + \rho - n > 0$ as $\rho > n$. The stable $\dot{\xi} = 0$ locus as well as the unstable $\dot{\xi} = 0$ locus are both continuously increasing with ξ but at a decreasing rate. For $\alpha \in (0,1)$, evaluated at $\xi = 0$, it is obvious that the slope of the $\dot{\xi} = 0$ locus is larger than the slope of the $\dot{\chi} = 0$ locus. Moreover, as for $\xi = 0$, the $\dot{\chi} = 0$ locus is above the $\dot{\xi} = 0$ isocline, we have one unique positive steady state at (ξ^*, χ^*). The arrows indicate that the steady state is (locally) a saddle point[8].

Proposition 2 helps us to characterize the transition path towards $(\hat{\xi}, \hat{\chi})$.

Proposition 2. *The assumption* $\hat{\gamma} > 0$ *implies that*

1. $(\hat{\xi}, \hat{\chi})$ *is below the* $\dot{\chi} = 0$ locus
2. $\hat{\xi} > \xi^*$
3. $\hat{\chi} = \chi^*$

Proof. (2.1) $\dot{\chi} = 0$ evaluated at $(\hat{\xi}, \hat{\chi})$ implies that $\hat{\chi} = (1-\alpha)\hat{\xi}^{1-\alpha} + \rho - n = \hat{\xi}^{1-\alpha} - (\hat{\gamma} - \delta - n)$. From the resource constraint we have that $\hat{\chi} = \hat{\xi}^{1-\alpha} - (\hat{\gamma} - \delta - n)(1+\xi) < \hat{\xi}^{1-\alpha} - (\hat{\gamma} - \delta - n)$. (2.2) As $\hat{\gamma} > 0$, it directly follows that $\hat{\xi}^{1-\alpha} > \frac{\delta+\rho}{\alpha} > \frac{\rho-n}{\alpha} = \xi^{*1-\alpha}$. (2.3) From (2.1) we have that $\hat{\chi} = \hat{\xi}^{1-\alpha} - (\hat{\gamma} - \delta - n)(1+\xi)$. Now claiming that $\hat{\chi} = \chi^*$ implies that $\hat{\xi}^{1-\alpha} - (\hat{\gamma} - \delta - n)(1+\xi) = \frac{\rho-n}{\alpha}$ which reduces to $\hat{\gamma} = \hat{\gamma}$ after using the fact that $\hat{\xi} = \frac{1-\alpha}{\alpha}$.

Hence, as the $\dot{\chi} = 0$ is flatter than the $\dot{\xi} = 0$ locus, and both loci are increasing with ξ, the stable saddle path of the system without the nonnegativity constraints must be flatter than the $\dot{\chi} = 0$ locus but increasing as well. Moreover, Proposition 2 also tells us that the transition path cannot be on the stable saddle path of the system without the nonnegativity constraints. As $\hat{\chi} = \chi^*$ and $\hat{\xi} > \xi^*$, the stable saddle path of the system without the nonnegativity constraints must be above $\hat{\chi}$ and to the right of $\hat{\xi}$. Hence, the steady state $(\hat{\xi}, \hat{\chi})$ is below the stable saddle path of the system without the nonnegativity constraints as shown in the left panel of figure (1)[9].

4.2 Case 2: $\xi(0) := \frac{p(0)}{k(0)} < \frac{1-\alpha}{\alpha}$

Similar results can be obtained for the case $\xi(0) < \frac{1-\alpha}{\alpha}$. Now, automation capital p is initially abundant relative to standard physical capital k. Now, the constraint $I_K \geq 0$ is binding, which directly implies that $\frac{\dot{K}}{K} = -\delta \Leftrightarrow \tau = 1$.

[8] As usual we can perform a first-order Taylor row expansion to evaluate the dynamics of the system (ξ, χ) around the steady state (ξ^*, χ^*). It can be shown that the corresponding Jacobian matrix reads as $Det[J] = \frac{(\alpha-1)(\rho-n)^2}{\alpha}$, which is negative for $\alpha \in (0,1)$, whereas the trace $Tr[J] = \rho - n$ is positive for $\rho > n$.

[9] The Appendix contains a numerical example for Case 1. The stable saddle path of the system without the nonnegativity constraints has been computed by means of the time elimination method proposed by Mulligan and Sala-i-Martín (1991). Given the nonnegativity constraints hold, with the same procedure we have computed the transition path towards the steady state $(\hat{\xi}, \hat{\chi})$.

The transition equations for χ and ξ can written as

$$\gamma_\chi := \frac{\dot{\chi}}{\chi} = (1-\alpha)\xi^{-\alpha} + n - \rho \tag{22}$$

and

$$\gamma_\xi := \frac{\dot{\xi}}{\xi} = \xi^{-\alpha} - \chi\xi^{-1}. \tag{23}$$

The right panel of figure (1) represents the phase diagram in the (ξ, χ) space for Case 2. The $\dot{\xi} = 0$ locus, $\chi = \xi^{1-\alpha}$, is continuously increasing but at a decreasing rate and unstable, whereas $\dot{\chi} = 0$ requires that $\chi = (1-\alpha)\xi^{1-\alpha} + \rho - n > 0$ as $\rho > n$. There is a unique steady state (ξ^*, χ^*) which is (locally) a saddle point[10]. As shown in the right panel of figure (1), for $\xi(0) < \frac{1-\alpha}{\alpha}$, the economy moves along a path where ξ and χ rise monotonically up to the point in which the returns on both types of capital are equalized. From $t > T$, the solution is given by $\xi(t) = \hat{\xi}$ and $\chi(t) = \hat{\chi}$ and, according to Proposition 1, with the exemption of χ and ξ, all variables grow at the constant rate $\hat{\gamma}$.

Proposition 3 helps us to characterize the transition path towards $(\hat{\xi}, \hat{\chi})$.

Proposition 3. *The assumption $\hat{\gamma} > 0$ implies that*

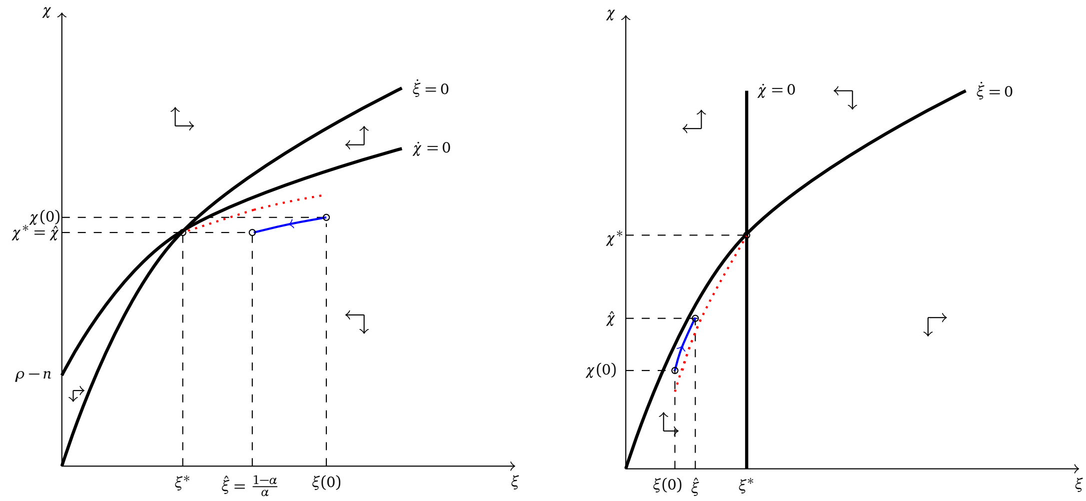
1. $(\hat{\xi}, \hat{\chi})$ *is below the* $\dot{\xi} = 0$ locus
2. $\hat{\xi} < \xi^*$
3. $\hat{\chi} < \chi^*$

Proof. (3.1) $\dot{\xi} = 0$ evaluated at $(\hat{\xi}, \hat{\chi})$ implies that $\hat{\chi} = \hat{\xi}^{1-\alpha}$. From the resource constraint we have that $\hat{\chi} = \hat{\xi}^{1-\alpha} - (\hat{\gamma} - \delta - n)(1+\xi) < \hat{\xi}^{1-\alpha}$. (3.2) As $\hat{\gamma} > 0$, it directly follows that $\hat{\xi}^{-\alpha} > \frac{\delta+\rho}{\alpha} > \frac{\rho-n}{\alpha} = \xi^{*-\alpha} \Rightarrow \xi^{*\alpha} > \hat{\xi}^{\alpha}$. (3.3) From (3.1) we have that $\hat{\chi} = \hat{\xi}^{1-\alpha} - (\hat{\gamma} - \delta - n)(1+\xi)$. Now claiming that $\hat{\chi} < \chi^*$ implies that $\xi^* > \hat{\xi}$ which is fulfilled (see 3.2). The saddle path of the system without the nonnegativity constraints is increasing and steeper than the $\dot{\xi} = 0$ locus. Proposition 3 shows that the steady state $(\hat{\xi}, \hat{\chi})$ does not have to be on the stable saddle path of the system without the nonnegativity constraints. The right panel of figure (1) plots a possible transition path. In contrast to the transition path in the left panel of figure (1), the transition path shown in the right panel can be on or off the stable saddle path of the system without the nonnegativity constraints.

4.3 Transitional behavior of output per captia: Imbalance effect

The derived results imply that there is an imbalance effect between the growth rate of output (per capita) and ξ as long as ξ deviates from its steady-state value $\hat{\xi}$. The growth rate of output per

[10] We perform a first-order Taylor row expansion to evaluate the dynamics of the system (ξ, χ) around the steady state (ξ^*, χ^*). It can be shown that the associated Jacobian matrix reads as $Det[J] = \frac{\alpha(\rho-n)^2}{\alpha-1}$, which is negative for $\alpha \in (0,1)$, whereas the trace $Tr[J] = \rho - n$ is positive for $\rho > n$.



(a) Phase diagram (Case 1) (b) Phase diagram (Case 2)

Figure 1: Transitional dynamics. Note: For Case 1, the left panel (a) shows that the transition path cannot be the saddle path of the system without the nonnegativity constraints. For Case 2, the steady state does not have to be on the stable saddle path of the system without the nonnegativity constraints as shown in the right panel (b).

capita off the steady state reads as:

$$\gamma_y = (1-\alpha)\frac{\dot{\xi}}{\xi} + \frac{\dot{k}}{k} = (\xi^{1-\alpha} - \chi)[\alpha(1-\tau) + (1-\alpha)\frac{\tau}{\xi}] - \delta - n. \tag{24}$$

Equation (24) can be linearized around $(\hat{\xi}, \hat{\chi})$.

For $\xi(0) > \hat{\xi}$ (Case 1), we obtain

$$\gamma_y = \hat{\gamma} + (1-\alpha)^{1-\alpha}\alpha^{1+\alpha}(\xi - \hat{\xi}) - \alpha(\chi - \hat{\chi}). \tag{25}$$

Equation (25) implies that the iso-growth lines for γ_y in the (ξ, χ) space are linear with slope $(1-\alpha)^{1-\alpha}\alpha^{\alpha} < 1$. The grey lines in figure (2a) represent the iso-growth lines for a specific example[11]. We further draw the transition path $\chi(\xi)$, which is numerically computed using the time elimination method suggested by Mulligan and Sala-i-Martín (1991). As shown with Proposition 2, this path is not on the stable saddle path of the system without the nonnegativity constraints. This path is positively sloped but flatter than the stable $\dot{\chi} = 0$ locus in the neighborhood of the steady state[12]. In the vicinity of the steady state, we observe that γ_y is positively related to ξ. Hence, if $\xi(0) > \hat{\xi}$ and ξ decreases towards $\hat{\xi}$, the fall in $(\xi - \hat{\xi})$ dominates the fall in $(\chi - \hat{\chi})$ in terms of its effects on γ_y. Hence, the growth rate of output per capita monotonically decreases towards its minimum $\hat{\gamma}$.

[11] We use standard values from the literature, e.g. $\alpha = 0.3; n = 0.01; \delta = 0.05; \rho = 0.09$.

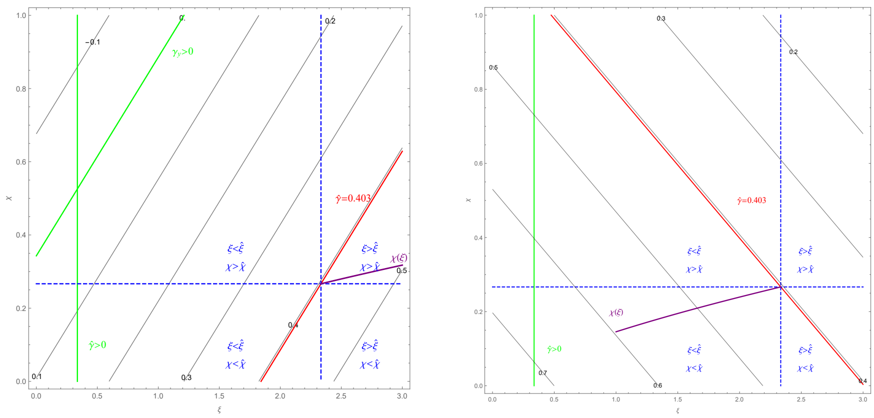
[12] Moreover, the slope of the $\dot{\chi} = 0$ locus is smaller than the slope of the iso-growth lines as $(1-\alpha)^{2-\alpha}\alpha^{\alpha} < (1-\alpha)^{1-\alpha}\alpha^{\alpha}$.

A similar analysis can be conducted for the case $\xi(0) < \hat{\xi}$ (Case 2). In this case, the linearized version of equation (24) reads as

$$\gamma_y = \hat{\gamma} + \alpha \left[\frac{\rho - n}{1-\alpha} - \frac{(1-\alpha)^{1-\alpha}\alpha^{1+\alpha}}{(1-\alpha)} \right] (\xi - \hat{\xi}) - \alpha(\chi - \hat{\chi}). \tag{26}$$

To allow for positive growth and bounded utility, the slope in square brackets of equation (26) is negative and for reasonable parameter values smaller than one, as shown in figure (2b). Moreover, we draw the numerically computed, positively sloped transition path $\chi(\xi)$, which is not necessarily on the stable saddle path of the system without the nonnegativity constraints (see Proposition 3). For $\xi(0) < \hat{\xi}$, the rise in $(\xi - \hat{\xi})$ reinforces the negative effect on γ_y originating from the rise in $(\chi - \hat{\chi})$.

Hence, on either side of the steady state $\hat{\xi}$, the growth rate γ_y rises with the magnitude of the gap between ξ and $\hat{\xi}$. However, this rise is not symmetric. Valid for both cases, this imbalance effect vanishes only if the constraints - that gross investment in physical and automation capital must each be nonnegative - is redundant, which implies that $(\xi(0) = \hat{\xi}, \chi(0) = \hat{\chi})$.



(a) Case 1 (b) Case 2

Figure 2: Imbalance Effect. Iso-growth lines are presented in grey (and red colors for the steady state growth rate). The numbers in the figure show the growth rates associated with the iso-growth lines. The transition path is drawn in purple.

5 Conclusion

We have analyzed the implications of automation for economic growth by allowing for a so far neglected constraints that gross investment in physical and automation capital must each be nonnegative. This establishes the realistic assumption that physical capital and automation capital investments are irreversible. As a consequence of this assumption, an imbalance effect between the growth rate of output and the fraction between physical and automation capital results.

This effect leads (i) to transitional dynamics off the steady state while (ii) retaining perpetual growth of the AK style in the steady state without exogenous technological progress. We also show that the resulting transition path does not have to be on the saddle path of the system without the nonnegativity constraints. The continuity of consumption induced by the continuity of the shadow prices is sufficient to determine the transition path of this economy.

6 Appendix

In this Appendix, we provide a numerical example for Case 1. In particular, we employ the time-elimination method (see Mulligan and Sala-i-Martín (1991)) to determine the transition path. Moreover, to avoid redundancy, we only focus on Case 1, although the procedure is identical for both cases. Again, suppose that $\frac{p(0)}{k(0)} > \frac{1-\alpha}{\alpha}$. Between $[0, T]$, the dynamics of the economy is completely described by system (21)-(20). As ξ and χ are continuous, at T we have $\xi(T) = \hat{\xi}$ and $\chi(T) = \hat{\chi}$. If we know the time path for ξ in $[0, T]$, we can determine the time path for χ in $[0, T]$ since we know the policy function $\chi(\xi)$ in $[0, T]$. We employ the efficient time-elimination method for working out the policy function $\chi = \chi(\xi)$. The policy function can be computed by solving the initial value problem

$$\frac{d\chi}{d\xi} = \frac{\gamma_\chi[\xi(t), \chi(t)]\chi(t)}{\gamma_\xi[\xi(t), \xi(t)]\xi(t)}, \tag{27}$$

with the boundary condition $\chi(\xi) = \hat{\chi}$. Starting from $\chi(\hat{\xi}) = \hat{\chi}$, we can solve this first-order differential equation (27) numerically to determine the rest of the policy function.

After we have computed the policy function $\chi(\xi) = \hat{\chi}$, we can obtain the time paths of ξ in $[0, T]$ by solving the following initial value problem:

$$\frac{\partial \xi(t)}{\partial t} = \gamma_\xi[\xi(t), \chi(\xi(t))]\xi(t), \tag{28}$$

with the initial condition $\xi(0) = \frac{p(0)}{k(0)}$. Next, T is obtained when the net returns on automation and physical capital are equalized. For $t > T$, we have that the optimal time path is given by $\xi(t) = \hat{\xi}(t)$. It is straightforward to obtain the time path for χ in $[0, T]$ by direct substitution of the policy function $\chi(\xi) = \hat{\chi}$ in $\chi(t) = \chi(\xi(t))$. The time paths are continuous. Knowing the optimal time paths for $\chi(t)$ and $\xi(t)$, the time paths of the remaining variables can be computed quite simply. Other computational methods, such as backward integration (Brunner and Strulik (2002)) can be used as an alternative method to compute the transition paths.

We used the following well established values from the relevant literature to calibrate our model: $\alpha = 0.3; n = 0.01; \delta = 0.05; \rho = 0.09$. Figure (3) clearly shows that the steady state $(\hat{\xi}, \hat{\chi})$ is not on the stable saddle path of the system without the nonnegativity constraints. The saddle path of the system without the nonnegativity constraints, visualized as a dotted-red line, is also computed with the time elimination method pushed forward by Mulligan and Sala-i-Martín (1991). We find that $(\hat{\xi}, \hat{\chi}) = (2.33333; 0.266667)$ and $(\xi^*, \chi^*) = (0.151341; 0.266667)$.

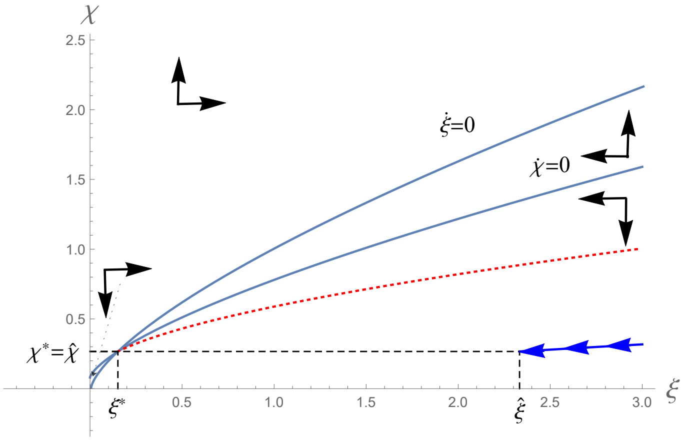


Figure 3: Phase diagram for the model when $\xi(0) > \hat{\xi}$ (Case 1)

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