Robust Capacitated Vehicle Routing Problem with Uncertain Demands

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Abstract

Every decision-maker has to cope with uncertainty in the world that is changing more and more rapidly. The presence of uncertain inputs distinguishes today's decision-making problems from traditional mathematical optimization problems. For decision-making problems under uncertainty at least one of the inputs of the problem is uncertain and thus needs to be modeled as stochastic variables. Failing to recognize and deal with uncertainty may result in poor decisions. Modeling a decision-making problem under uncertainty as a deterministic problem, however, may lead to solutions, which are sensitive to perturbations in the inputs, and thus may be infeasible, suboptimal, or even both. This circumstance motivates the interest in building solutions to decision-making problems that are less affected by uncertainties, i.e., robust solutions. Nevertheless, protection often comes associated with the so-called *Price of Robustness*. This price corresponds to the reduced quality of the solutions as decision-makers aim for safety.

The aspiration to create models, which are more appropriate for real applications in which stochasticity is a major issue, is present in different optimization problems, such as the Vehicle Routing Problem (VRP). For several years, the VRP was handled with the assumption that all inputs of the problem were deterministic and known in advance. However, since in real-world problems stochasticity arises in many situations, decision models for this problem changed in order to incorporated uncertainties into them, giving rise to one of the categories of the VRP, the Stochastic Vehicle Routing Problem (SVRP). The SVRP can be further classified into *static* and *dynamic*. The term *static* refers to the fact that all routing decisions and corrective actions are preprocessed decisions, i.e., they are computed

before the execution of the route plan. In contrast, the term *dynamic* refers to situations, where all decisions are made in an ongoing manner, whenever new events arise. Examples can be the revelation of the true value of the stochastic input or a vehicle breakdown. Although modifying the route plan dynamically allows for additional savings, it requires technical support and may lead to loss of driver familiarity. Thus, the Static and Stochastic VRP (SSVRP) has received more interest.

The decision-making problem under uncertainty addressed in this thesis is the Static and Stochastic Capacitated Vehicle Routing Problem with Stochastic Demands (SSCVRPSD). In this problem, an initial a priori route plan is designed. During plan execution, corrective actions, i.e. *detours-to-depot*, are applied as each customer's demand is revealed, if the true total demand of a route exceeds the vehicles' capacities. The goal is to calculate a robust a-priori route plan, that is, an a priori route plan that will only undergo small changes (few corrective actions) when the true demands are revealed during its implementation. For that, we propose a mathematical formulation and a solution approach. The mathematical decision model is based on a Mean Absolute Deviation (MAD) objective function. This objective function combines two conflicting objectives, minimization of the expected planned transportation cost (optimality) and minimization of the mean absolute deviation of the second-stage transportation cost (robustness). In this MAD model, the variability term is multiplied by a parameter of decision-maker's choice ω, used to obtain a spectrum of route plans that can be more or less robust. In this manner, the proposed formulation not only delivers flexibility to the decision-maker to define desired safety levels, i.e. the level of robustness, but also allows to trade off cost minimization and protection against fluctuation in the stochastic demands. Since the MAD formulation takes the structure of a multi-objective optimization problem, we propose a respective method called Robust Multi-Objective (RoMO) solution approach.

For evaluating the efficiency of RoMO, we develop a benchmark dataset and seven performance measures. In this stage, since we want to compare solutions of different degree of robustness, we select different values of $\omega \in \{0, 1, 5, 10\}$. We compare RoMO's performance with those of two other solution approaches: Deterministic Approach (DA) and

Robust Simulation-Based (RoSi) approach. DA is the most commonly used approach to deal with decision-making problems under uncertainty, thought it does not consider the effect of the uncertain inputs on the feasibility and optimality of the solutions. In this approach, the values of the stochastic inputs are assumed to be equal to their expected values so that the linear programming formulation of the problem can be solved. On the other hand, RoSi has similar goals as RoMO. It designs route plans of different degree of robustness based on a decision-maker's choice of parameter ω. The computational experiments show that the proposed approach provides significant improvements over the deterministic approach and over RoSi in some cases. Moreover, in a second stage, we parametrize our MAD formulation and change ω to $\omega \in [0, 1]$, to provide different weighting combinations for prioritization of optimality and/or robustness. In this way, the parametrized decision model provides a way of quantifying the trade-off between the two conflicting objectives, and to calculate the route plan that best trades off optimality and robustness, i.e. the route plan with the lowest real transportation cost.

Zusammenfassung

Entscheidungsträger müssen mit sich immer schneller verändernden Unsicherheiten in der Welt umgehen. Das Vorhandensein unsicherer Umstände und Daten unterscheidet heutige Entscheidungsprobleme von traditionellen mathematischen Optimierungsproblemen. Für Entscheidungsprobleme unter Unsicherheit ist mindestens einer der Eingangswerte des Problems unbekannt oder unsicher und muss daher als stochastische Variable modelliert werden. Wenn Ungewissheiten nicht erkannt und entsprechend beachtet werden, kann dies zu Fehlentscheidungen führen. Die Modellierung eines Entscheidungsproblems unter Unsicherheit als deterministisches Problem kann jedoch zu Lösungen führen, die empfindlich auf Varianzen in den Eingaben reagieren, und daher möglicherweise unzulässig, suboptimal oder beides sein. Dieser Umstand motiviert das Interesse, robuste Lösungen für Entscheidungsprobleme zu entwickeln, die weniger von Unsicherheiten betroffen sind. Diese Robustheit ist allerdings oft mit dem sogenannten *Price of Robustness* verbunden. Dieser Preis beschreibt eine Verschlechterung der Lösungsgüte, bei steigender Robustheit der Lösung.

Das Bestreben, geeignete Modelle für reale Anwendungen aufzustellen, in denen Stochastik ein Hauptproblem darstellt, findet sich in verschiedenen Optimierungsproblemen, beispielsweise dem Vehicle Routing Problem (VRP). Für das VRP wurde lange Zeit davon ausgegangen, dass alle Eingaben des Problems deterministisch und im Voraus bekannt waren. Da in Realität jedoch viele Situationen mit Unsicherheiten verknüpft sind, haben sich hierfür Entscheidungsmodelle etabliert, welche Unsicherheiten einzubeziehen. Hieraus entstand eine neue Kategorie von VRP: das Stochastic Vehicle Routing Problem (SVRP). Das SVRP kann weiter in *static* und *dynamic* untergliedert werden. Der Begriff *static* bezieht sich auf

die Tatsache, dass alle Routenentscheidungen und Korrekturmaßnahmen bereits im Vorfeld berechnet werden und vor Ausführung des Routenplans bekannt sind. Im Gegensatz dazu bezieht sich der Begriff *dynamisch* auf Situationen, in denen alle Entscheidungen fortlaufend getroffen werden, wenn neue Ereignisse auftreten. Beispiele solcher Ereignisse können die Feststellung des wahren Wertes einer stochastischen Variable oder das Ausfallen von Fahrzeugen sein. Eine kontinuierliche Anpassung des Routenplans ermöglicht zwar zusätzliche Einsparungen, erfordert jedoch technische Unterstützung und kann zum Verlust der Vertrautheit des Fahrers führen. Daher ist in letzter Zeit das Static and Stochastic VRP (SSVRP) weiter in den Fokus gerückt.

Das in dieser Arbeit betrachtete Entscheidungsproblem ist das Static and Stochastic Capacitated Vehicle Routing Problem with Stochastic Demands (SSCVRPSD). Bei diesem Problem wird ein anfänglicher a-priori Routenplan erzeugt. Während der Planausführung werden die tatsächlichen Kapazitätsbedarfe der Kunden enthüllt. Korrekturmaßnahmen, z.B. *Umwege-zum-Depot*, werden immer dann angewendet, wenn der Gesamtbedarf einer Route die Kapazitäten der Fahrzeuge übersteigt. Das Ziel ist die Berechnung eines robusten a-priori Routenplans, d.h. Plans der nur geringfügigen Änderungen unterworfen ist (wenige Korrekturmaßnahmen), wenn die tatsächliche Nachfrage während der Planumsetzung aufgedeckt wird. Hierzu schlagen wir eine mathematische Formulierung und einen Lösungsansatz für diese Formulierung vor. Das mathematische Entscheidungsmodell basiert auf einer *Mean Absolute Deviation*-Zielfunktion (MAD). Diese Zielfunktion kombiniert zwei widersprüchliche Ziele: Minimierung der erwarteten geplanten Transportkosten (Optimalität) und Minimierung der mittleren absoluten Abweichung der Transportkosten der zweiten Stufe (Robustheit). In diesem MAD-Modell wird der Variabilitätsterm mit einem Entscheidungsparameter ω multipliziert, welcher es Entscheidungsträgern ermöglicht, ein Spektrum von Routenplänen zu erstellen, die mehr oder weniger robust sein können. Auf diese Weise gibt die vorgeschlagene Formulierung dem Entscheidungsträger nicht nur Flexibilität bei der Definition der gewünschten Sicherheitsniveaus, d.h. des Robustheitsniveaus, sondern ermöglicht auch einen Kompromiss zwischen Kostenminimierung und Schutz vor stochastischen Unsicherheiten. Da die MAD-Formulierung die Struktur eines multikri-

teriellen Optimierungsproblems annimmt, schlagen wir eine entsprechende Methode vor, die als Robust Multi-Objective (RoMO) Lösungsansatz bezeichnet wird.

Zur Bewertung der Effizienz von RoMO entwickeln wir ein Benchmark-Dataset und sieben Leistungskennzahlen. In einem ersten Schritt variieren wir $\omega \in \{0, 1, 5, 10\}$, um Lösungen mit unterschiedlichem Robustheitsgrad zu erhalten. Wir vergleichen die Leistung von RoMO mit zwei anderen Lösungsansätzen: Deterministic Approach (DA) und Robust Simulation-Based (RoSi). DA ist der am häufigsten verwendete Ansatz zur Lösung von Entscheidungsproblemen unter Unsicherheit. Dabei wird jedoch nicht berücksichtigt, wie sich die Unsicherheit in den Eingangsparametern auf die Zulässigkeit und Optimalität der Lösungen auswirkt. Bei diesem Ansatz wird angenommen, dass die Werte der stochastischen Eingangsparameter ihrem Erwartungswert entsprechen, so dass das Problem mittels linearer Programmierung gelöst werden kann. Demgegenüber hat RoSi ähnliche Ziele wie RoMO. Beide Ansätze generieren Pläne für unterschiedliche Robusteheitsgrade, basierend auf dem aktuell gewählten Wert für ω. Die Experimente zeigen, dass der vorgeschlagene Ansatz gegenüber dem deterministischen Ansatz und in einigen Fällen gegenüber RoSi erhebliche Verbesserungen bietet. In einem zweiten Schritt wurde die MAD-Formulierung überarbeitet, sodass durch eine Variation von $\omega \in [0, 1]$ verschiedene Gewichtungskombinationen für die Priorisierung von Optimalität und / oder Robustheit generiert werden. Auf diese Weise bietet das parametrisierte Entscheidungsmodell die Möglichkeit, Kompromisse zwischen den beiden sich widersprechenden Zielen zu quantifizieren. Hierdurch kann der Routenplan mit dem besten Trade-off zwischen Optimalität und Robustheit berechnet werden, welcher im Endeffekt die niedrigsten realen Transportkosten aufweist.

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Abbreviations

2-OPT LS	2-Optimal Local Search
AMP	Adaptive Memory Programming
B&P	Branch-and-Price
BPP	Bin Packing Problem
CCP	Chance-Constrained Program
CVRP	Capacitated Vehicle Routing Problem
C&W	Clark and Wright
CP	Current Problem
CVRPDPD	Capacitated Vehicle Routing Problem with Divisible Pickups and Deliveries
CVRPSPD	Capacitated Vehicle Routing Problem with Simultaneous Pickup and Delivery
DA	Deterministic Approach
DD	Detour-to-Depot
DDARP	Dynamic Dial-a-Ride Problem
DDVRP	Dynamic and Deterministic Vehicle Routing Problem
DVRP	Dynamic Vehicle Routing Problem
DSVRP	Dynamic and Stochastic Vehicle Routing Problem
GA	Genetic Algorithms
GPS	Global Positioning Systems
HVRP	Heterogeneous Vehicle Routing Problem

IG	Iterated Greedy
LS	Local Search
LNS	Large Neighbourhood Search
MAD	Mean Absolute Deviation
MA|PM	Memetic Algorithm with Population Management
MC-CVRPSD	Multicompartment Capacitated Vehicle Routing Problem with Stochastic Demands
MDVRP	Multi-Depot Vehicle Routing Problem
MV	Mean-Variance
MSA	Multiple Scenario Approach
MTZ	Miller-Tucker-Zemlin
PDF	Probability Distribution Function
PSO	Particle Swarm Optimization
PR	Preventive Restocking
RVRP	Robust Vehicle Routing Problem
RO	Robust Optimization
ROC	Robust Optimization Criterion
RoMO	Robust Multi-Objective Solution Approach
RoSi	Robust Simulation-Based Solution Approach
SA	Simulated Annealing
SAA	Sample Average Approximation
SP	Stochastic Programming
SPR	Stochastic Programming with Recourse
SCVRPSC	Stochastic Capacitated Vehicle Routing Problem with Stochastic Customers
SCVRPSD	Stochastic Capacitated Vehicle Routing Problem with Stochastic Demands
SDCVRP	Static and Deterministic Capacitated Vehicle Routing Problem
SDVRP	Static and Deterministic Capacitated Vehicle Routing Problem

SSCVRP	Static and Stochastic Capacitated Vehicle Routing Problem
SSCVRPSD	Static and Stochastic Capacitated Vehicle Routing Problem with Stochastic Demands
SSCVRPSD-DC	Static and Stochastic Capacitated Vehicle Routing Problem with Stochastic Demands and Duration Constraints
SSCVRPTW	Static and Stochastic Capacitated Vehicle Routing Problem with Time Windows
SSVRP	Static and Stochastic Vehicle Routing Problem
SSVRPTW	Static and Stochastic Vehicle Routing Problem with Time Windows
SVRP	Stochastic Vehicle Routing Problem
SVRPSST	Stochastic Vehicle Routing Problem with Stochastic Service Times
SVRPSTT	Stochastic Vehicle Routing Problem with Stochastic Travel Times
VNS	Variable Neighbourhood Search
VRP	Vehicle Routing Problem
VRPB	Vehicle Routing Problem with Backhauls
VRPSC	Vehicle Routing Problem with Stochastic Customers
VRPSST	Vehicle Routing Problem with Stochastic Service Times
VRPSTT	Vehicle Routing Problem with Stochastic Travel Times

To all my sister in science.

Alone we are strong,

but together

we are fearless.

1   INTRODUCTION

For several years, optimization cared about finding the "optimal" solution for decision-making problems under certainty subject to either single or multiple objectives. These optimization studies operated under the assumption that all inputs necessary to solve the problem were known and available. As researchers attempted to better represent real-life problems faced by companies, uncertainty was incorporated by modeling the uncertain input as a stochastic variable. Uncertainty arises in many industrial problems. A decision-making problem under uncertainty occurs when decision-makers cannot predict with confidence what the outcome of their decisions will be. In some decision-making problems the degree of uncertainty is slight and may be overlooked if decisions can be made without rising extra costs or service. In many other cases, omission of uncertainty during decision-making may cause negative consequences.

Formulating a decision-making problem under uncertainty as a deterministic problem is commonly called deterministic optimization approach. In this approach, one chosen instance of the input data is supplied to a mathematical model in order to calculate the "optimal" decision. The chosen instance expresses the most likely estimate of the realization of the data in the future [KY97]. That is, the value of the uncertain input in the chosen instances is inferred to be equal to its expected (nominal) value. Such an approach does not take into consideration the influence of the uncertain input on the quality and feasibility of the solutions [BS04], and may therefore design solutions that are sensitivity to perturbations in the stochastic input. These solutions may then become suboptimal or even infeasible [BBC10]. This situation motivates the concern of building solutions that are less

affected by the various uncertainties encountered in a decision-making problem, i.e., robust solutions.

Several communities have their own interpretation of the term robustness and propose different approaches to achieve it [SCPC15]. For this reason, the decision-maker must first determine what it means for her/him to have a robust solution. Is it a solution whose feasibility must be guaranteed? Or whose objective value must be guaranteed? [GMT14]. Yet, what most decision-makers refer to as a robust solution is a solution resisting as much as possible to fluctuations on the stochastic inputs [SCPC15]. There are mainly two approaches to achieve robustness in decision-making problems under uncertainty, Stochastic Programming (SP) and Robust Optimization (RO). SP divides the set of decisions into two groups. Decisions that have to be taken before the realization of the stochastic inputs are called first-stage decisions and the period when these decisions are made is called the first-stage. Decisions that are made after the realization are called second-stage decisions and the corresponding period is called the second-stage [BL11]. In SP, feasibility constraints are relaxed and can thus be violated by perturbations on the inputs. First-stage decisions for which the constraints do not hold as the real value of the stochastic input is revealed, can afterwards be repaired by corrective/recourse actions (second-stage decisions). In stochastic programming models, the goal is to minimize the cost of first-stage plus the cost of second-stage decisions [AJ15]. For SP a solution is said to be robust when it needs less corrective actions for dealing with the real value of the stochastic input. We thus refer to this robustness as *recoverable robustness*. On the other hand, RO assumes only a single period when all decisions have to be taken before the uncertain input is realized. Instead of seeking to immunize the solution in some probabilistic sense to stochastic uncertainty, RO aims at *strict robustness*, i.e., RO cares about designing a solution that is feasible for any realization of the uncertainty in a given set [BBC10].

Imposing protection by creating solutions that are (recoverable or strictly) robust conducts to the so-called *Price of Robustness* introduced by Bertsimas and Sim [BS04]. This price corresponds to the cost payed to allow for certain deviations within the stochastic variables and is usually defined as the difference between the cost of the robust solution

and the cost of the nominal solution (deterministic optimization approach). This deterioration in the "optimal" value of the robust counterpart with respect to the "optimal" value of the deterministic problem is caused by the presence of the additional hard constraints imposing robustness. The *Price of Robustness* is a consequence of restricting the set of feasible solutions to the (in general smaller) set of robust solutions [MDE17]. Decision-makers that are willing to take more risk accept less protection and acquire a reduced price. In contrast, decision-makers that have higher risk aversion seek higher protection, but are subjected to a higher *Price of Robustness*. Solutions with high price are often too conservative in the sense that we have to give up too much of optimality for the nominal problem in order to ensure robustness [BS04]. A solution will hardly remain both robust and optimal for all realization of the uncertainty [MVZ95]. Hence, there exists a trade-off between optimality and robustness.

One decision-making problem in which this context is particularly true is the Vehicle Routing Problem (VRP). The classical VRP is the decision-making problem under certainty, where all inputs needed to solve the problem are at hand. The decision-making problem under uncertainty is called Stochastic Vehicle Routing Problem (SVRP), where at least one of the necessary inputs is unknown and uncertain at the planning (first) stage.

The classical VRP was initially introduced by Dantzig and Ramser [DR59] and is one of the most important and studied problems in combinatorial optimization. It lies at the heart of distribution management and is faced by thousands of companies and organizations engaged in the delivery and collection of goods or people each day [CLSV07]. Real-life examples of VRP include courier delivery, waste collection, dial-a-ride services (taxis, transportation of elderly and handicapped people), as well as the routing of school buses, snow plow trucks, and maintenance engineers [GWF13]. A general definition of the goal of this optimization problem is based on the Capacitated Vehicle Routing Problem (CVRP), which is a category of the classical problem where constraints on the capacity of the vehicles are included. That is why the names VRP and CVRP are used interchangeably. The definition is given as calculate a route plan, i.e. a set of routes, to attend a set of customers with a given vehicle fleet at minimum transportation cost, such that

- the requirements of all customers are met;

- each route starts and ends at the depot;

- each customer is attended by a single vehicle;

- the total load allocated to each vehicle does not exceed its capacity [KB85].

Real-world applications have demonstrated that the use of computerized solution approaches for solving VRPs produces great savings in the transportation costs. One of the reasons for the success of the application of optimization techniques in this problem is the development of decision models that are able to consider almost all the characteristics of VRPs arising in real-world situations [TV14]. As a result, a large number of VRP categories has been proposed, such as VRP with Backhauls (VRPB), CVRP with Simultaneous Pickup and Delivery (CVRPSPD), Dynamic VRP (DVRP), and Stochastic VRP (SVRP).

The Stochastic CVRP (SCVRP) arises when one or more inputs are uncertain and therefore modeled as stochastic variables within the problem. In contrast to the assumptions of the classical CVRP, in the real world one or more of the elements of the VRP are uncertain. Uncertainty exists because of the time gap that separates the stages when route plans are planned and executed [TV14]. Uncertainty may come from different sources. The four most common stochastic inputs studied in the literature are:

- Customer's demand: Demands are considered stochastic when the loads to either be picked up or delivered at customers are uncertain. The SCVRP with Stochastic Demands (SCVRPSD) is the most studied stochastic CVRP. There are many real-world application problems that are best modeled as SCVRPSD. One example is the routing of forklifts in a cargo terminal [Ber92].

- Travel time: Stochastic travel times are included in the model when traveling times between customers are uncertain. Traveling speed are subjected to the traffic jam, road maintenance, and/or weather conditions. In this way, a number of real world problems can be modeled as a SVRP with Stochastic Travel Time (SVRPSTT). One industrial example is Road Feeder Services (RFS), where a cargo operator transport products from/to airplane(s) to/from terminal(s) [DKV11].

- Presence of customers: Customers are treated as stochastic variables when customers are either present or absent with a given probability. A real-life application of SCVRP with Stochastic Customers (SCVRPSC) occurs in patient transportation, where transportation requests originate from both institutions, e.g., hospitals and individuals (patients) [BLMN10].

- Service time: Service times are modeled as stochastic inputs when the service times at customers are uncertain. A real-world problem that can be regarded as a SVRP with Stochastic Service Times (SVRPSST) is the optimization of ground handling operations in airport aprons, where the aircrafts need specific operations before/after departure/landing, and the vehicles are required for the same operations by different aircrafts [ACDFDG16]

Because of the importance of the time horizon in the decision-making process, the SCVRP can be studied from two perspectives, a static and a dynamic one. From a static perspective, all routing decisions are made before the implementation of the solution (*preprocessed decisions*). This perspective gives rise to the Static and Stochastic CVRP (SSCVRP). The aim of the SSCVRP is to calculate a priori robust route plan that will go through small changes during its implementation [BSL96]. From a dynamic perspective, the Dynamic and Stochastic SCVRP (DSCVRP) arises and all routing decisions are made in a online manner as soon as a dynamic event happens (*online decisions*), for example, the revelation of one of the stochastic inputs [PGGM13a]. The goal of the DSCVRP is to design a route plan in an ongoing fashion, communicating to the vehicles which customer to serve next as soon as it becomes idle [SM09]. Addressing the problem in a dynamic way assumes the existence of communication between the dispatcher (where the route plan is created, e.g., the headquarter of the company) and vehicles/drivers, because the dispatchers have to periodically inform to the drivers about the new customers assigned to them [MGRD05]. In practice, the capability to modify a solution is limited as a result of when the information on the dynamic event becomes available and the amount of work in computing new route plans [Ord10]. In other words, periodically taking new decisions may be infeasible when the rate of dynamic events is high and the available time for making them is short. Holding a robust route

plan, that can be slightly modified, has practical advantages as it can lead to better training of drivers who become familiar with a certain area or plan and are, thus, better prepared to manage uncertain situations or emergencies [Ord10]. Thus, solving the SCVRP with a static strategy has received more interest.

Solving the static and stochastic CVRP involves two choices, one on the mathematical formulation and the other on the solution approach. A mathematical formulation refers to translating the real-world problem into a mathematical model. A solution approach (e.g. exact methods, heuristics, and metaheuristics) is a set of routines that are followed in order to solve the mathematical model. Similarly to any other decision-making problem under uncertainty, the most commonly used modeling techniques for the SVPR are SP and RO. Regarding solution approaches, a large set of efficient solution approaches have already been created for the classical CVRP. This is not yet the case for the stochastic problem, which is a more complex decision-making problem due to the uncertainty introduced by the random behaviour of the inputs [JFG$^+$11]. Several studies on the SSCVRP have, thus, tried to reduce the stochastic capacitated vehicle routing problem to its deterministic counterpart, so that the resulting mathematical model can be solved with any well-established algorithm for the deterministic problem [GWF13] [SOM08] [SS09].

Current stochastic programming models for the SSCVRPSD are Chance-Constrained Programming (CCP) and Stochastic Programming with Recourse (SPR). In CCP, an optimal route plan is calculated to satisfy the constraints on the vehicle capacity with a given probability. On the other hand, in SPR the vehicle capacity constraints are relaxed and included in the objective function, assuming that route failures induced by fluctuation on the demands during the execution of the a priori route plan can be adjusted by recourse actions [SCPC15]. A route failure in the SSCVRPSP happens when the total demand on a route exceeds the capacity of the associated vehicle during the implementation of the a priori route plan. The main difficulty associated with SP models is the need to provide the probability distributions that govern the stochastic demands. In contrast, RO acknowledge the uncertain demands without making specific assumptions on the probability distributions, instead they are assumed to belong to a deterministic uncertainty set [MPB05]. In present robust

optimization settings for the SSCVRPSD, the min-max model has been generally adopted. The robust min-max model seeks a solution that optimizes the worst over all demand scenarios in the uncertainty set, leading to the design of an overly conservative route plan. To address the issue of over conservative robust route plans, robust optimization models have assumed that stochastic demands belong to different structures of uncertainty set, cf., e.g., [GWF13]. Yet, these two approaches are not the only methods used to produce the so-desirable robustness (be that recoverable or strict) in the context of the SSCVRPSD. When dealing with this problem other strategies, such as allocation of vehicle capacity [JFG+11], have been applied. Nevertheless, to the best of our knowledge, no study about the SSCVRPSD has considered the issue of trading off optimality and robustness.

Optimality and robustness are two conflicting objectives. Optimality refers to planned (first-stage) transportation cost, that is, the transportation cost of the a priori route plan, while robustness regards to second-stage transportation cost. The real transportation cost, i.e., the cost that are really afforded by companies, is the sum of the first and second-stage transportation cost. As we aim at optimality more customers will generally be served in individual routes, leading to lower planned transportation cost. But the savings in the transportation cost due to larger routes will tend to be offset by more frequent route failures and, therefore, higher second-stage transportation cost will be incurred thanks to, for example, an additional vehicle that is dispatched to complete an unfinished route [SG83]. The real transportation cost is then higher. In turn, as we focus on robustness, fewer customers will normally be attended in single routes, designing small routes. Thus, more vehicles are required. In this case, route failures tend to not occur and the planned transportation cost equals the real transportation cost. But as the transportation cost of the route plan increases with the number of routes, the real transportation cost is higher in this case too. There is a trade-off between cost minimization and protection, and a balance must be set for these two objectives. This is the goal of this thesis. The question is *How to support decision-making process in the SSCVRPSD so that a solution that calculates the best trade-off between planned transportation cost and second-stage transportation cost, i.e. a route plan with low real transportation cost, can be obtained?* The presence of this trade-off is perceptive, but a decision

model that provides a manner for quantifying the trade-off and determining a route plan that best trades off expected transportation cost and safety is still missing.

In other decision-making problems under uncertainty, mathematical models that seek to trade off robustness and optimality are based on using variance as a measure of robustness. The objective function of the model turns into a utility function that embodies the expected solution cost (first-stage cost), called optimality, and a variability in the mean value (second-stage cost), called robustness [MVZ95], establishing a mean-variance trade-off. However, variance gives equal weight to deviations above and below the expected value. The variance as a measure of variability that uses absolute values and thus avoids the issue of negative differences between deviations and the mean. In this way, the Mean-Variance (MV) objective function is unable to notice the incremental and/or reductional trend on the second-stage costs. In some decision-making problems, it is plausible to focus on mean-variance tradeoffs, but in other it is not [LWN$^+$03]. In the SSCVRPSD, the MV objective function may calculate a route plan that are overly conservative. Here, we propose a mathematical model for the SSCVRPSD that uses mean absolute deviation instead of the variance as a measure of robustness, called Mean Absolute Deviation (MAD) formulation, and a solution method for solving such decision model. Since the proposed formulation takes the structure of a multi-objective optimization problem, we call the solution method Robust Multi-Objective (RoMO) solution approach. Similar to the work of Mulvey et al. [MVZ95], the variability term is multiplied by a parameter of decision-maker's choice ω to be used to obtain a spectrum of solutions that trades off optimality for robustness. In this way, the mathematical formulation and the solution approach introduced in this thesis acknowledge the need for the decision-maker to incorporate a measure of her/his level of risk aversion in the objective function.

For achieving our goal, the remainder of this thesis is structured as follows: In Chapter 2, we lay the foundation for the addressed decision-making problem. The decision-making problem studied in this thesis is the most famous SVRP, the static and stochastic capacitated vehicle routing problem with stochastic demands. Because of its ability to incorporate different aspects of real-world problems, several VRP classes have been presented

and different ways have been proposed to categorize them. In this chapter, we introduce a classical definition of the CVRP and propose a new classification for the VRP. Moreover, instead of covering just the decision-making problem, we explore the SVRP regarding the **source of stochasticity** present in the problem, the type of **decision policy** involved in the decision-making process, the **modeling technique** applied to formulate the problem, and the **solution method** used to solve the mathematical model.

In Chapter 3, we cover the second part of the literature review. We introduce **approaches adopted for handling uncertainties**, mainly stochastic demands, and achieving robustness in the decision-making problem. As we try to cope with uncertainties in decision-making problems, the goal of producing robust solutions arises, i.e. solutions that are less affected by changes on the stochastic inputs. Nevertheless, there have been different interpretations on the term robustness. Therefore, we present not only the definitions proposed by the stochastic programming and robust optimization approaches but also our own **definition of robustness**. Apart from SP and RO, other approaches have been applied in the literature to cope with the SSCVRPSD acknowledging robustness. A review on these papers is presented. This chapter is concluded by pointing out what is different between our goal and the aim of these papers.

In Chapter 4, we present the structure of the MAD formulation for the SSCVRPSD and the stages included in the RoMO solution approach to deal with the proposed mathematical decision model. The MAD formulation is similar to a stochastic programming with recourse model and assumes that the probability distribution of the uncertain demands can be estimated. The parameter of choice ω included in the second term of the MAD objective function allows decision-makers to see what possible trade-offs between robustness and optimality exists and to choose a solution that is consistent with her/his willingness to take risk. Like other works, RoMO also makes use of efficient and well-known heuristics for the classical CVRP to solve the stochastic problem. The conclusions of this chapter are based on the contribution of our work to the literature on robust formulations for the decision-making problem.

In Chapter 5, we demonstrate the effectiveness of our solution approach by solving the

instances of a developed benchmark dataset and comparing RoMO solutions with solutions obtained via the **deterministic optimization approach** and solutions obtained via another approach that similarly to RoMO also aims at producing robustness. The deterministic optimization approach is the benchmark method to solve decision-making problems under uncertainty, even though it does not consider the effect of the stochastic inputs on the solution's feasibility and optimality. The instances differ mainly regarding number of customers and include locations of the depot and the customers, transportation cost, demands, and vehicle capacity. For comparison purposes, we adopt $\omega \in \{0, 1, 5, 10\}$ to generate solutions of different degree of robustness and introduce seven performance measures, **probability of route failure**, **reliability of the route plan**, **expected** and **real number of routes**, **expected** and **real transportation cost**, and *Price of Robustness*. Monte Carlo simulation and the probability distribution that models the demands are used to estimate some of them. Moreover, we discretize ω and parametrize the MAD objective function in order to perform a trade-off analysis. We conclude this chapter by stressing how our solution approach performs and its ability to find a solution that best trades off first and second-stage transportation cost. Morover, we highlight how RoMo improves decision-making in optimization problems under uncertainty.

Finally, we conclude the thesis in Chapter 6 by summarizing our contributions and outlining directions for future research.

2 | THE VEHICLE ROUTING PROBLEM

2.1 INTRODUCTION

The literature review is devoted to two parts, to the decision-making problem, and to approaches for handling uncertainties in the decision-making problem. The first part is covered in this chapter and the second in Chapter 3. The decision-making problem is the static and stochastic capacitated vehicle routing problem with stochastic demands. Nevertheless, since this decision-making problem is one of several variants of the vehicle routing problem and the frontier between the categories (and all principles related to them) is somehow fuzzy, rather than restricting this chapter to the SSCVRPSD, we provide literature on the stochastic vehicle routing problem. Moreover, this chapter uses the CVRP a number of times as the primary example as this is a reasonably simple variant that makes it easy to introduce the necessary concepts.

2.2 DEFINITION

Several definitions of the vehicle routing problem have been introduced. In this section, we present a classical definition and use it throughout the thesis. In the most general sense, the VRP can be defined as the problem of designing an optimal route plan to attend a set of geographically dispersed customers under the limitations of constraints. Because conditions vary from one problem to another, objective function and constraints are changeable. Typically, an optimal route plan means the one with minimal transportation cost (distribution

costs and/or vehicle acquisition costs), however, different goals can be targeted [KB85]. For instance, service improvement is a common objective for service industries, as customer satisfaction is often crucial. The most common constraints are on the route duration, time window, and vehicle capacity [TV14]. Duration constraints guarantee that the total length of every route does not exceed a defined threshold. Time window constraints ensure that each customer is attended within a predefined time interval. Capacity constraints certify that the total demand of any route does not exceed the vehicle capacity. When capacity constraints are added to the classical problem, then the most studied version of the VRP emerges, the capacitated vehicle routing problem. In this variant, the goal is to minimize the travel times while serving all customers using a fleet of identical vehicles of limited capacities located at a single depot [Lap09]. Vehicles collect and/or deliver (goods) at all customers. Delivery and collection problems are symmetrical with one another and equivalent from a modeling point of view [GLS96]. CVRP belongs to the class of NP-hard combinatorial optimization problems, i.e., there is no known algorithm that can solve it in polynomial time. If the size of the set of customers grows, the number of solutions to the problem grows exponentially.

The CVRP is represented on a fully connected undirected graph $G = (N, A)$, where $N = (0, 1, 2, 3 \ldots n)$ is the set of nodes (a single depot denoted as node 0) and $A = \{(i, j)|i, j \in N, i \neq j\}$ is the set of arcs. Therefore, there are $|A| = n(n + 1)/2$ arcs in the graph. The deterministic amount of goods that has to be delivered to (or/and collected at) customer $i \in N$ is denoted as customer's demand and is given by d_i, therefore, $d_0 = 0$. The fleet of vehicles $K = \{1, 2, ..., k\}$ is considered to be homogeneous, that is, there are k vehicles of capacity C each. The coefficient c_{ij} represents the transportation cost between node i and j. This coefficient is calculated using $c_{ij} = \sqrt{(i_x - j_x)^2 + (i_y - j_y)^2}$, where i_x and i_y are the x and y coordinates of the customer i, respectively. It is assumed that the coefficients satisfy the triangular inequality, i.e. $c_{ij} + c_{jk} \geq c_{ik}$ and that the graph is symmetric, i.e. $c_{ij} = c_{ji}$. Note that an instance of the problem is defined by a complete weighted graph $G = (N, A, c_{ij})$ together with the size of the fleet of vehicles $|K|$ and the vehicle capacity C. The decision variables x_{ij} indicate whether or not a vehicle travels from node i to node

j. A solution y to the CVRP, called route plan, consists of a set of K feasible routes, which means that the number of routes is equal to the number of vehicles used in the plan. The transportation cost of a route plan y is expressed by $J(y)$. A feasible route p is performed by one vehicle which leaves the depot, serves a subset $R = \{i_1, i_2, \ldots, i_r\} \subseteq N$ of customers, whose total demand does not exceed C, and returns to the depot, i.e. $p = (i_0, i_1, \ldots, i_r, i_{r+1})$ and $p[0] = p[r+1] = 0$. Let $S \subseteq N$ be an arbitrary subset of nodes. $m(S)$ denotes the minimum number of vehicles necessary to attend S. The value of $m(S)$ can be calculated by solving a Bin Packing Problem (BPP) with item set S and bins of capacity Q. For S, let $\delta(S) = \{(i, j) : i \in S, j \notin S \text{ or } i \notin S, j \in S\}$. It became a standard to define $\delta(i) := \delta(\{i\})$ for singleton sets $S = \{i\}$ [TV14]. Although several formulations have been proposed for the CVRP (cf. [TV02]), in this works we use an integer two-index compact CVRP formulation proposed by Laporte et al. [LND85] defined as follows

Definition 2.1 (Capacitated Vehicle Routing Problem)

$$\min_{y} \quad J_0(y) := \min \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij} \tag{2.1}$$

$$\text{s.t.} \quad \sum_{(i,j) \in \delta(i)} x_{ij} = 2 \qquad\qquad i \in N \backslash \{0\}, \tag{2.2}$$

$$\sum_{(i,j) \in \delta(0)} x_{ij} = 2K, \tag{2.3}$$

$$\sum_{(i,j) \in \delta(S)} x_{ij} \geq 2m(S) \qquad\qquad S \subseteq N \backslash \{0\}, S \neq \emptyset, \tag{2.4}$$

$$x_{ij} \in \{0, 1\} \qquad\qquad i, j \notin \delta(0), \tag{2.5}$$

$$x_{ij} \in \{0, 1, 2\} \qquad\qquad i, j \in \delta(0). \tag{2.6}$$

In this model, constraints (2.2) mean that each client is visited by one incoming and one outgoing vehicle. Constraints (2.3) ensure that K routes are designed. Connectivity of the route plan y and the vehicle capacity requirement are imposed by constraints (2.4) by forcing a sufficient number of edges to enter each subset of vertexes. Since the BPP is NP-hard, $m(S)$ may be approximated from below by any BBP lower bound, such as

$\left\lceil \sum_{i \in S} d_i / C \right\rceil$ [CLSV07]. Constraints (2.5) and (2.6) define the decision variables. In this formulation if $i, j > 0$, then x_{ij} can only take the values 0 or 1. However, if $i = 0$, x_{ij} can also be equal to 2 when a vehicle visits a single customer [LLH02].

2.3 CLASSIFICATION

There exists a large number of VRP categories because different setting and attributes/constraints and combinations of both can be added to the classical vehicle routing problem. The Capacitated Vehicle Routing Problem with Divisible Pickups and Deliveries (CVRPDPD), Multi-Depot Vehicle Routing Problem (MDVRP) and Heterogeneous Vehicle Routing Problem (HVRP) are just a few examples. Hence, different classifications of the VRP have been proposed. Toth and Vigo [TV14] classify it according to the network structure (arc routing and node routing), type of transportation requests, constraints that determine whether or not a route is feasible, fleet composition and location, inter-route constraints related with solution feasibility, and optimization objectives. Bodin and Golden [BG81] present a taxonomy of the VRP based on objectives, operations, costs, maximum vehicle route-times, vehicle capacity, underlying network, location of demands, nature of demands, size of vehicle fleet available, number of domiciles, and time to service a particular node or arc. Psaraftis [Psa88] use two aspects, "evolution of information" and "quality of information", to categorize VRP into four classes, *static and deterministic*, *dynamic and deterministic*, *static and stochastic*, and *dynamic and stochastic*. Evolution of information refers to the fact that in some problems the information available to the decision-maker may change during runtime and divides the problems into static when inputs are known beforehand, and into dynamic otherwise. Quality of information indicates whether possible uncertainty exists on the available data and categorizes the problems into deterministic when there is no stochastic information about the inputs, and into stochastic otherwise. Based on this classification, we propose a new one presented on Table 2.1. In the proposed classification, the VRP is categorized into the same four classes as by [Psa88]. But, we use "uncertain information" and "solution evolution" as classification aspects rather than evolution of in-

formation and quality of information. Uncertain information relates to whether there is stochastic information available to the decision-maker while solution evolution pictures whether the sequence of customers to be attended on the route plan is slightly changed or it is completely altered by the decision-maker during its execution.

Tab. 2.1: Vehicle Routing Problem Classification based on Solution Evolution and Uncertain Information.

		Uncertain Information	
		Deterministic Input	Stochastic Input
Solution Evolution	Structure Unchanged	*Static and Deterministic*	*Static and Stochastic*
	Structure Changed	*Dynamic and Deterministic*	*Dynamic and Stochastic*

In the *static and deterministic* class, all necessary data is known in advance and with certainty and routes do not change once they are being performed. These conditions apply to the classical VRP and its variants (CVRP, VRPSPD, MDVRP, etc). This class has been widely studied.

The *dynamic and deterministic* class, also called online or real-time VRP [TV14], is characterized by parts or all inputs being unknown and revealed dynamically during the execution of the route plan. No historical information about the inputs is known [PWK16]. For such problems, solutions are designed in an ongoing fashion, requiring technical support, such as mobile phones and Global Positioning Systems (GPS), for real-time communication between the driver/vehicles and the decision-maker [PGGM13a]. A Dynamic and Deterministic VRP (DDVRP) is usually solved on an event-driven basis, i.e., every time a piece of information becomes available. Nonetheless, for practical applications with time-critical decisions the task of evaluating every possible decision at each time step is extremely challenging, cf., e.g., [PGGM11]. The most prominent problem in this class is the Dynamic Dial-a-Ride Problem (DDARP) and one real-life application is the transportation of patients. In this application, a service provider offers demand-responsive transportation for the purpose of hospital consultations, medical treatments, daycare activities and/or rehabilitation therapies. The set of nodes N is split into the pickup (origins) and delivery locations (destinations). Each customer calls a dynamic transportation request, that is, a trip between an origin and a destination of choice that needs to be executed as soon as possible. Origin and destination could, for instance, be the patient's home and a hospital, respectively. A service

level requirement is linked to each trip [MBCB17]. Examples of works on this problem are [KLPB11, BLMN10].

In the *static and stochastic* setting, one (or some) input is modeled as a random variable and its true values are revealed during the execution of the route plan. The solution is designed beforehand and only minor changes are applied to the a priori route plan during its implementation. Applications in this class do not require any technical support. The most studied problem in this class is the CVRP with Stochastic Demands (CVRPSD). One real-life application of this problem is a waste collection system. In this example, suitable vehicles transport solid waste from points of disposal (collection bins) to a point of treatment. Therefore, in this system, customers correspond to collection bins and a single depot corresponds to a point of treatment. The volume of waste at any collection bin, which expresses customer's demand, cannot be a priori known with certainty. Because all waste has to be collected and the capacity of the vehicles are constrained, the vehicles may require being unloaded at the treatment location and then continue to attend the remaining collection bins. This problem was studied by [Ism09, Ism08].

Last, in the *dynamic and stochastic* class, also called real-time or online VRP, similar to the *static and stochastic* category one or (some) input is unknown and revealed dynamically and stochastic knowledge about it is available. Both classes call for the design of an a priori route plan. Nevertheless, in the static case the a priori route plan is slightly updated as the true input values are acknowledged and in the dynamic case the a priori solution may be completely modified. Therefore, the help of technical support may be needed in this category. In this class, the dynamic and stochastic capacitated vehicle routing problem is the most studied problem and one real-world application is a grocery delivery service. In this problem, vehicles transport perishable goods from a retailer to customers' home. The customer selects products on a website and then chooses a time window for the delivery to take place. The retailer estimates the number of customers that can be serviced within a time window. This number is typically based on a combination of factors, including fleet size and historical data on the delivery times. The time window is made unavailable to customers as soon as the capacity is reached [Pil12]. E-grocery delivery service problems

were studied by [AGP12, CS05].

Literature on problems that lie in the third and four classes, that is, problems that hold stochastic information, has grown in the recent years. The main reason is that in most of the practical applications all information necessary to formulate the problem is not known and readily available, and integration of stochastic information can increase the look–ahead capability, reliability, and robustness of an solution approach [TV14]. We continue this chapter focusing in the SVRP, i.e., in the *static and stochastic* and *dynamic and stochastic* classes.

2.4 STOCHASTIC VARIANT

SVRPs may be catalogued based on four aspects: **source of stochasticity**, **decision policy**, **modeling technique**, and **solution method**. A source of uncertainty is any input that is uncertain, inexact, noise or likely to change in the future [SCPC15]. A decision policy defines whether the structure of a solution is completely updated over time. A modeling technique is a framework used to model the problem and leads to how the problem will be solved. Finally, a solution method describes how to solve the mathematical model, which heavily depends on the modeling method [Ord10]. In the next sections, we further describe each of these aspects and for each we present studies for exemplification. The studies classified according to the aspects are shown in Figure 2.1.

2.4.1 SOURCES OF UNCERTAINTY

The stochasticity can be incorporated into the problem through different aspects. The most studied sources of stochasticity are: presence of customer, customer's demand, travel time, and service time. Following, for each one of these sources of uncertainty we present characteristics, examples and real-world applications of SVRPs that include it.

2.4.1.1 STOCHASTIC CUSTOMERS

The Vehicle Routing Problem with Stochastic Customers (VRPSC) occurs when the set of customers to be visited is not known with certainty [TV14]. In such cases, each customer $i \in N$ has a probability P_i of being present, but has a deterministic demand d_i. A real-life application of this problem is courier services found in the local operations of international shipping services [HLL06]. In this application, the parcel is collected at different customer locations and brought back to a central depot for further processing and shipping. Service requests appear dynamically and are assigned to apt vehicles and historical information about them is available [GGPT99]. Hvattum et al. [HLL06], motivated by a problem faced by one of the leading distribution companies in Norway, tackle the dynamic and stochastic VRP where customers can call in orders during daily operations and historical data about the customers' locations is available. They divide the time horizon into a prespecified number of intervals v. The route plan is reoptimized every time interval considering the currently known customers and scenarios constructed using the stochastic knowledge in order to accommodate new requests. Waters [Wat89] studies the vehicle routing problem with some customers that may not need to be served during the execution of the a priori route plan. He proposes and compares three strategies to adapt a priori solutions to handle customers that do not need to be visited. In the first strategy, vehicles continue performing their a priori route plan. In the second strategy, vehicles keep the a priori route plan, but they skip customers that do not require to be served. In the last strategy, vehicles collect all customers that have to be visited and reoptimize the remaining solution every time an previously absent customer is revealed.

2.4.1.2 STOCHASTIC DEMANDS

The most common stochastic parameter studied in the literature is customer demand. Apart from the waste collection system described before, another real-life application occurs in the delivery of petroleum products. In this application, a supplier of petroleum products must design a route plan to replenish the inventory of a set of gas stations [DR59]. The amount of products that each gas station needs is not known beforehand with certainty,

yet, stochastic information about it is on hand. As the exact values of customer demands are uncertain, the problem can be modeled as the SCVRPSD. Bernardo and Pannek [BP18] deal with the dynamic and stochastic CVRP with stochastic and dynamic demands. For that, they propose an approach that addresses uncertainty by using higher moments calculated via a set of scenarios, permitting the solution to be able to adapt to situations when the true demand is greater than expected without losing structural properties and optimality. Mendoza et al. [MRV16] study the SCVRPSD with duration constraints. In this problem not only the capacity constraints (cf. (2.4)) define the feasibility of a solution but also constraints on the duration of routes. They introduce two strategies to deal with route-duration constraints in the SVRPSD. In the first strategy, a probability of exceeding the maximum duration constraint is assumed to be lower than a given threshold, while in the second, the violations to the duration constraint are penalized in the objective function.

2.4.1.3 STOCHASTIC TRAVEL TIMES

Travel speed is influenced by the traffic jam, road maintenance, and/or weather conditions. This means that travel time brings stochasticity to the VRP. The problem may thus be formulated as the SVRPSTT. The coefficients c_{ij}, which represent transportation costs in (2.1), may as well indicate travel costs, distances and travel times between node i and j [TV14]. These coefficients are then represented by random variables in the SVRPSTT. Real-life examples of this problem are money collection systems in bank branches. Usually, banks dispatch vehicles to their branches to collect cash and transport it to a central office. This operation may be carried out daily by the bank's own vehicles or by a logistics provider. One major issue is the fact that when cash does not return to the central office before a certain time of the day it is credited to the next day and loses one day's interest [LLL93]. Taş et al. [TDWK14] investigate the SVRP with stochastic and time-dependent travel times. They formulate a new model for the problem, one that minimizes the total weighted time which includes not only travel times but also service times. To solve it they applied two metaheuristics. Zhang et al. [ZCZ12] examine the dynamic and stochastic VRP with stochastic travel times and simultaneous pickups and deliveries. The authors propose a new model to

transform the dynamic problem into a static one and construct a metaheuristic to solve it.

2.4.1.4 STOCHASTIC SERVICE TIMES

When stochasticity is present in service times, the problem can be modeled as the SVRPSST. Compared to the VRP with stochastic customers and demands, research on the VRPSST and on the VRPSTT has received less attention [Ord10]. Real-world examples of this problem are faced by maintenance and repair services providers [EDG+18]. In such cases, service requests are generated by the owners of the equipment due to regular maintenance or failure. Each request specifies an estimated service time and a deadline for the start of the service. Service times vary according to different factors such as accessibility at the customer's location, diagnostic of the particular service to perform and complexity of the operation to be carried out [EDG+18]. Thus, in this SVRPSST model a random variable that indicates the duration of the service time at each customer and constraints on the time window are added to the classical formulation (2.1). This situation is studied by Errico et al. [EDG+18] and Souyris et al. [SCOW13]. Both formulated the problem as a SVRPSST with soft time windows, but while Errico et al. [EDG+18] propose a model that designs solutions that are insensitive to the uncertainty in service times, Souyris et al. [SCOW13] develop a method to compute the success probability of the route plans.

2.4.2 DECISION POLICIES

The SVRP can be studied from either a static or dynamic point of view. From a static perspective, the goal of the problem is to design an a priori route plan on which minor alterations will be applied during its execution to cope with the uncertainties. From a dynamic perspective, the problem consists in constructing a route plan in an online manner, informing the vehicles which customer to attend next [NS09, PGGM13a]. That is why the stochastic vehicle routing problem is classified either as static or as dynamic according to the aspect "solution evolution" in the Table 2.1.

   Instead of static and dynamic perspectives, we follow the work of Ritzinger et al. [RPF16] and refer to them as **preprocessed** and **online decisions**, respectively. Hence, if the

stochastic VRP is classified as static it means that preprocessed decisions are made and if the problem is classified as dynamic then online decisions are executed to solve the problem. Preprocessed decisions are decisions computed before the execution of the route plan and define actions applied during the execution of the route plan, e.g., always perform a detour to the depot. These decisions consider all states (e.g. all possible stochastic input realizations) in advance and evaluate each state according to its performance. The evaluation of the states is done before the vehicles start the routes, and enables accurate decision-making based on these values during the plan execution phase. Online decisions are decisions counted as soon as a dynamic event happens. By adopting online decisions, solutions are reoptimized at predefined stages (e.g., as soon as an event occurs) with respect to the current system state and the available stochastic information [BVH04]. To tackle the dynamic and stochastic VRP, Bent and Van Hentenryck [BVH04] propose a Multiple Scenario Approach (MSA). The method starts by initializing a pool of scenarios with realizations of customer demands based on the currently known information. If/when an event occurs, such as the disclosure of a true value of some input or a vehicle breakdown, MSA updates the scenario pool, solves each scenario and selects a route plan. Online decisions are made based on this selected route plan. As new information is revealed, some scenarios might become obsolete and are removed from the pool, leaving space for new ones. Euchi et al. [EYC15] solve a dynamic and stochastic VRP with stochastic pickup and delivery locations by solving one static problem per time interval. Customers are divided into time intervals of predefined length and every time interval represents a static and stochastic VRP. All orders received after a time interval is over are interpreted as being customers that were not serviced before and the re-optimization starts with these customers.

The ability to modify a route plan and redirect a moving vehicle to a new request nearby allows for additional savings. However, it requires real-time knowledge of the vehicle position and being able to communicate quickly with drivers to assign them new destinations [PGGM11]. This ability is limited as a result of the availability of technical support, time of the information disclosure, and the amount of work in computing new solutions. Many practical problems are of high dimension and it is not possible to solve them in appropri-

ate time after the dynamic input is revealed. In addition, modifying the routes too much will lead to the loss of driver familiarity and preparedness, desirable features in practice [Ord10]. In this work, we are, therefore, concerned with the static and stochastic VRP, which means that we only execute preprocessed decisions to solve the problem and that only a few changes are applied to the a priori route plan after the true input values are revealed. For more information on the dynamic and stochastic VRP, readers are referred to [RPF16, PGGM13b, GGLM03, Lar01, Psa95]. In the next sections, we present mathematical models and solution methods used in the static and stochastic (capacitated) vehicle routing problem.

2.4.3 Modeling Techniques

The most common used modeling techniques in the context of the SSVRP are **stochastic programming** and **robust optimization**. In this section, we show how these techniques are applied in the most famous SSCVRP, the SSCVRP with uncertain demands. Apart from the notations already introduced in 2.2, we include for the SSCVRPSD the following definition.

Definition 2.2 (Stochastic Demands)

Suppose that customer demands are known as random variables, $d_i : \Omega_i \to \mathbb{R}_0^+ \forall i \in N$ with sampling space Ω_i, and its exact values are only revealed at runtime.

It is important to highlight that a common and simple modeling approach is to use the same formulation (2.1) but replace d_i by $E[d_i]$, which is the expected demand of a customer. In this way, $m(S)$ is then approximated from $\left\lceil \sum_{i \in N} E[d_i]/C \right\rceil$. Yet, this approach does not consider the impact of uncertainties on the quality and feasibility of the solution. It is thus likely that as the inputs take values different from the nominal ones, constraints may be violated, and the best route plan calculated for the nominal values may no longer be optimal or even feasible [BS04]. SP and RO are described as follows.

2.4.3.1 Stochastic Programming

In this section, we focus on stochastic program models. Stochastic programming was introduced by Dantzig and Ramser [DR59]. Using this technique, uncertainty is formulated in optimization problems by introducing stochastic parameters in the models [BL11]. The random nature of uncertain inputs can cause an feasible route to become infeasible, i.e. as true inputs are revealed, constraints may fail to hold [BS04]. This situation is referred to as a route failure. Considering, for instance, the SSCVRPSD, a route failure may occur any time a customer's demand exceeds the associated vehicle remaining capacity [PGM10]. When a route fails, recourse (also called corrective) actions (decisions) must be implemented. Stochastic programming models are obtained by first defining when the true values of the uncertain inputs are revealed. Based on this, decision variables are determined in stages according to when inputs become known. The a priori decisions are the first-stage decisions, which must be taken initially, before all information is available, and the recourse (corrective) decisions represent the actions made in the second stage and onwards. These decisions point to how route plans are changed as new information becomes available [TV14]. There are two types of stochastic programming models, **two-stage models** and **multi-stage models**. In two-stage models, the variables are partitioned into two sets [DLL93], while in multi-stage models, the decision variables are separated into $t > 2$ sets with t being the number of stages considered in the problem. We do not cover multi-stage SP models in this thesis, since we are concerned with the SSCVRP and these models are usually applied in the dynamic and stochastic problem setting. Two-stage programs are the most studied stochastic program models [Sha]. Once the first and second-stage decisions variables are determined, there exist two general modeling approaches to formulate the SSCVRP using a two-stage SP technique, **chance-constrained program** and **stochastic program with recourse** [OAW16]. The conceptual difference between both approaches lies in the goal of the first stage [PGM10]. CCP aims at designing an a priori route plan of lowest cost while ensuring an upper bound on the probability of a route failure, regardless of the expected cost of the second stage. In turn, in the SPR, the goal is to minimize the cost of the a priori route plan plus the expected cost of the second-stage actions, i.e., the total expected cost

[DLL93].

In the literature on chance-constrainted models, the probability of failure is usually assumed to be given and specified in advance [NC18]. The decision-maker provides a parameter value giving the acceptable probability of failing to meet the constraints [OAW16]. In order to avoid recourse costs, it is usually chosen a very small threshold α for the failure probability. A typical small probability is 5% [BL11]. In SSCVRP with stochastic demands, the constraints that are subject to failure are the capacity constraints. Using a probability of failure of 5% means that a vehicle can collect the total demand of its assigned route with a probability of 95%. In other words, 95 route plans out of 100 are reliable. Given the notations used in section 2.2 and Definition 2.2, we can therefore introduce a chance-constrained model for the SSCVRPSD for a given threshold α

Definition 2.3 (Chance Constrained SSCVRPSD)

$$\min_{y} J(y) := \min \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij} \tag{2.7}$$

$$\text{s.t. } P\left(\sum_{(i,j) \in \delta(S)} x_{ij} \geq 2m(S) \right) \geq 1 - \alpha, \qquad S \subseteq N \backslash \{0\}, S \neq \emptyset, \tag{2.8}$$

$$(2.2), (2.3), (2.5) \text{ and } (2.6).$$

It can be notice that the only difference between this model and Definition 2.1 is that the set of constraints (2.4) is replaced by a set of probabilistic constraints that can be violated with probability α. $m(S)$ is approximated from $\left\lceil \sum_{i \in N} E[d_i]/C \right\rceil$.

Noorizadegan and Chen [NC18] argue that small α may lead to unnecessary extra cost and that choosing the right value is therefore a critical decision. They perform a sensitivity analysis for route reliability level and study its impact on the routing decisions and on the objective values. For that, they consider α as a decision variable ($\alpha = 0.01, 0.05, 0.10,$ $0.15, 0.20, 0.25, 0.30$) and use Monte Carlo sampling to generate scenarios of customer demands, computing the total expected cost (first and second-stage costs), the expected failure cost (second-stage cost) and their standard deviation. The results show that large α (e.g.

0.25 and 0.30) reveals high total expected cost and standard deviations and that very small α (e.g. 0.01) increases total expected cost without improving reliability. The authors conclude the number of routes (vehicles) does not always change as α is varied. This implies it is not necessary to increase the fleet size to reach high degrees of reliability. They state that reliability can be increased by improving customers' assignment.

Dror et al. [DLL93] show that under mild assumptions chance-constrained SSCVRPSD can be reduced to deterministic CVRPs. They also demonstrate that given a set of customers that are served on a single route the probability of failure at the mth customer on this route and no route failure occurred before is $P(\sum_{i=1}^{m-1} d_i \leq Q < \sum_{i=1}^{m} d_i)$, for a number of common probability distribution. Given that d_i are random variables with coefficient of variation not exceeding one, the probability of failure is monotonically increasing in the range of $1, \ldots, m^\star$, where $m^\star = \max_m \{\mu m\} \leq Q, \mu = E[d_i]$. With this result in mind and assuming that the total demand on a route rarely very much exceeds the vehicle capacity, they argue that failures are generally few and often tend to happen towards the end of the route.

According to Morales [Mor06], the main drawback of CCR models is that, although they manage the probability of failure, the failure's location is neglected and its cost is thus not taken into account. Routes of same sequence of customers and failure probability can have quite different total expected costs, depending on the possible failure locations. That is why most of the literature on SSCVRPSD follows the SPR approach instead. In SPR, some feasibility constraints are relaxed and included in the objective function, assuming that violations induced by random events after the implementation of first-stage decisions can be repaired by recourse actions [SC15]. In this modeling approach, the decision-maker must define a recourse policy describing what actions to take in order to repair the solution after a failure [OAW16]. The recourse model frames the a priori route plan as here-and-now decisions that are chosen before the uncertain demands are realized, whereas the recourse decisions are taken once the demands are observed [GWF13]. In SPR models, the first and second-stage costs are optimized, that is, the a priori route plan cost (first-stage cost) plus recourse actions cost (second-stage cost). Considering the notations presented in 2.2 and

Definition 2.2, a SPR for the SSCVRPSD based on Birge and Louveaux [BL11] is represented as follows:

Definition 2.4 (Stochastic Program with Recourse SSCVRPSD)

$$\min_{y} \ J(y) := \min_{a} J(a) + E[Q(a, b)] \tag{2.9}$$

$$\text{s.t. (2.2), (2.3), (2.4), (2.5) and (2.6).}$$

where $Q(a, b)$ is the optimal value of the second-stage problem

$$\min_{\gamma} q(\gamma; a, b) \tag{2.10}$$

$$\text{s.t. } z \in Z(\gamma, b) \tag{2.11}$$

Here a represents the first-stage decisions that must be taken initially, before all information is available. These are routing decisions. The function $J(a)$ evaluates the objective function for the first-stage decisions. The source of uncertainties is represented by b. In general, b and its realizations b are vector-valued. The corrective decisions are represented by γ, which are evaluated using the function $q(\cdot)$ that can be parameterized by a and b. This model highlights that the major difference from a deterministic formulation is the second-stage value function. If that function is given, then a stochastic program is just an ordinary nonlinear program [BL11]. The second-stage cost is problem dependent and is related to the particular choice of possible corrective actions [SKGJR17]. It can thus be said that the corrective action is a modeling choice, resulting in different variants of a SPR model. For the SSCVRP with stochastic demands there are two recourse policies commonly used, **Detour-to-Depot** (DD) and **Preventive Restocking** (PR) [SM09, TCG07].

In DD, the vehicle returns to the depot to load (unload) when its capacity is depleted (or exceeded). Instead of detour-to-depot, Oyola et al. [OAW16], Morales [Mor06] and Hjorring et al. [HH99] call it stockout and classify it into two types, normal and exact. A normal stockout, means the vehicle does not have enough goods to serve the current customer,

so after restocking at the depot, the route is resumed starting at the customer where the route failed. On the other hand, an exact stockout means the residual capacity is precisely equal to the current customer' demand and after restocking, if this customers is not the last customer in the route, the vehicle will resume the trip at the next customer in the route. In other words, this recourse action is executed by visiting the depot after visiting customer i and before visiting customer j by following arcs $(i, 0)$ and $(0, j)$ instead of following the arc (i, j) in the a priori route plan [SOM08]. Oyola et al. [OAW16] show that the most common recourse action in the literature is the detour-to-depot and argue that its widespread use may be related to the fact that it is very simple to understand and to model. Juan et al. [JFG$^+$11] use a set of scenarios

$$S = \{s_j = (d_1^j, d_2^j, d_3^j \ldots, d_n^j) \mid j = 1, \ldots, \overline{sc}\}, \tag{2.12}$$

such that each scenario s_j defines one demand d_i^j for every customer $i \in N$, in order to compute the second-stage cost. The recourse model with detour-to-depot is obtained by introducing an additional binary recourse variable r_i^k to indicate whether the vehicle capacity is depleted (or exceeded) at customer i in scenario k while the a priori route plan is executed. The objective function of the SPR with detour-to-depot becomes then

$$\min_y \ J(y) := \min_a J(a) + \frac{1}{\overline{sc}} \sum_{k=1}^{\overline{sc}} \sum_{j \in S} r_i^k 2c_{i0}. \tag{2.13}$$

In PR, an en route replenishment may be performed before a route fails. It gives the option after each visit of choosing between visiting the next customer in the route or traveling back to the depot to replenish, even if the vehicle capacity has not been reached [OAW16]. The reason is that it may be less costly to travel to the depot to restock from the current customer than to wait for a route failure at a customer further away from the depot [BP18]. Considering the notations introduced in 2.2 and stochastic customer demands d_i that follows a discrete probability distribution $p_{ik} = P(d_i = k), k = 0, 1, 2 \ldots K \leq Q$, Yang et al. [YMB00] propose a stochastic model with preventive replenishment for the SSCVRPSD. For that, they first assume that after the service completion at customer i the vehicle has a

remaining load q and that $J_i(q)$ denotes the total expected cost from node i onwards. The total expected cost of the a priori route plan is thus $J_0(Q)$. If L_i represents the set of all possible loads that a vehicle can have after service completion at customer i, then, $J_i(q)$ for $q \in L_i$ satisfies

$$J_i(q) = \text{minimum}\{J_i^1(q), J_i^2(q)\}, \tag{2.14}$$

where

$$J_i^1(q) = d_{i,i+1} + \sum_{k:k \leq q} J_{i+1}(q - k)p_{i+1,k} + \sum_{k:k > q} [2d_{i+1,0} + J_{i+1}(q + Q - k)]p_{i+1,k}, \tag{2.15}$$

$$J_i^2(q) = d_{i,0} + d_{0,i+1} + \sum_{k=1}^{K} J_{i+1}(Q - k)p_{i+1,k}, \tag{2.16}$$

with the boundary condition $f_n(q) = d_{n,0}$, $q \in L_n$. In this model, $J_i^1(q)$ and $J_i^2(q)$ represent the total expected costs of the two choices at customer i, proceeding directly to the next customer or performing a PR, respectively. Bianchi et al. [BBC+06] argue that the optimal choice is of threshold type. Given the a priori route plan, for each customer $i \in N$ there is an amount threshold h_i such that, if the residual load after serving i is greater than or equal to h_i, then it is better to advance to the next customer on the route, otherwise it is less costly to perform a preventive restocking.

There are, nevertheless, some stochastic models in which the corrective actions do not involve routing decisions. Instead a penalty for late/early arrivals or the extra time cost of the driver can be part of the total expected cost when time windows and stochastic service time are taken into consideration [OAW16, TDWK13]. Chepuri and Homem-de-Mello [Che05] model a SSCVRPSD considering that after a route fails no corrections are applied and unvisited customers will not be attended. According to them, this is a situation faced by many companies while dealing with emergency deliveries when penalties must be paid to the unserved customer. Stewart and Golden [SG83] claim there is a penalty when a constraint fails (e.g. cost of sending another vehicle or customer dissatisfaction) related

to making that constraint feasible. They present a model for the SSCVRPSD that includes a penalty in the first-stage cost for each unit of demand left unsatisfied. Similarly, Laporte et al. [Lap92a] introduce a formulation for the SSVRPSTT that adds an extra unit of time to the a priori route plan total expected time for every time the travel time exceeds the deadline.

2.4.3.2 ROBUST OPTIMIZATION

Robust optimization is a modeling approach in which the uncertainty is not stochastically modeled, but instead deterministically and based on a set [BBC10]. RO first identifies potentially realizable input instances for the problem, without trying to assign probabilities to the different instances, and then looks for a solution that performs well even in the worst case of the identified input data. Rather than trying to protect the solution in some probabilistic way against uncertainty, in RO the decision-maker calculates a solution that is feasible for any realization of the uncertainty in a given set [BBC10, KY97].

There are three important decisions in the application of the robust optimization technique, structure of the data uncertainty, Robust Optimization Criterion (ROC), and problem formulation. Data uncertainty may be structured as a convex set, such as a convex hull, a box, or an ellipsoid [GWF13] or using discrete/continuous scenarios [SCPC15]. In the latter, potentially realizable values are generated to each input of the problem, i.e. data are defined as a set of possible values called scenarios. ROC defines the optimization goal, i.e. the objective of the robust optimization approach. Their use mainly depends on the type of problem. The most used are min-max, min-max regret, min-max relative regret, lexicographical criteria, α-robustness and bw-robustness. Based on the way the uncertain data is modeled and what ROC is chosen, a problem formulation is selected in order to obtain a model able to generate robust solutions. A solution is called robust if it is a solution of minimum value for the ROC, among all feasible solutions. The concept of robustness is discussed in Chapter 3.

Considering the SSCVRPSD, these three decisions are made. First, let the stochastic demands belong to a uncertain set $\mathcal{Q} \subset \mathbb{R}^n_+$, i.e. the uncertainty set $\mathcal{Q}$ contains all possible

demand instances. Second, assume the min-max as the ROC. Finally, third, adopt the notations presented in 2.2 and Definition 2.2. Then, a set of K routes (route plan) y is called robust if $\sum_{i \in y_k} d_i \leq Q$ for all $k \in K$ and $d \in \mathcal{Q}$, that is, if y satisfies the vehicle capacity constraints for all anticipated demand realizations. Let Y be the set of feasible solutions for the problem. This leads to the classical robust min-max model defined as

Definition 2.5 (Robust Min-Max SSCVRPSD)

$$\min_{y \in Y} \left\{ \begin{array}{c} \max_{d \in \mathcal{Q}} J(y) \\ (2.2), (2.3), (2.4), (2.5) \text{ and } (2.6) \end{array} \right\}. \tag{2.17}$$

The robust min-max model seeks a solution that optimizes the worst scenario over all scenarios in $\mathcal{Q}$. One can easily reformulate this model for when the customer demands are modeled by a set of discrete scenarios,

$$S = \{ s_j = (d_1^j, d_2^j, d_3^j \dots, d_n^j) \mid j = 0, \dots, \bar{s} \}, \tag{2.18}$$

such that each scenario j defines one demand d_i^j for every customer $i \in N$. The objective function of the robust min-max model becomes then

$$\min_{y \in Y} \max_{s_j \in S} J(y) \tag{2.19}$$

The ability to obtain efficient robust solution for the robust SSCVRPSD model will depend not only on how the customer demands are modeled, but also on the CVRP formulation used (e.g. two-index compact, Miller-Tucker-Zemlin (MTZ), Set Partitioning, etc.), on the correlations assumed between the uncertainty coefficients [Ord10], and on the ROC. Eufinger et al. [EKB62] assume that the uncertain travel times reside in a set of scenarios, opt for min-max-min as the ROC, and use a MTZ formulation to propose a robust model for the SSCVRPSTT. In this model, k different solutions are calculated, instead of one, such that the worst-case over all scenarios of the respectively best solution in each scenario is optimized. The idea has a practical motivation, when a company has to serve the same customer every

day it can calculate optimal k route plans only once. Then, at the beginning of a working day, when the real scenario is known, the optimal route plan can be selected and executed. Solano et al. [SCPC15] model the uncertain arc costs as discrete scenarios, optimize a lexicographic min-max criteria and use a MTZ formulation to develop a robust model for the SSCVRP with stochastic arc costs. The lexicographic min-max RO criteria performs lexicographic comparison between feasible solutions, it uses not only the worst-cost scenario but also the other scenarios, from the worst to the best, to break ties when two scenarios give the same worst cost. The lexicographic min-max refines min-max criteria, since it selects a unique set of outcomes, but not necessarily a unique solution. Wu et al. [WHB17] also model the uncertain arc costs as discrete scenarios and use a MTZ formulation for developing a robust model for the SSVRP with stochastic arc costs. For the objective of the robust optimization, they propose a ROC that selects a route plan, which performs better than the solution provided by the min-max objective on a majority of scenarios.

2.4.4 SOLUTION METHODS

A solution method, also called algorithm, is a set of operations that are applied in a decision-making problem, so that the problem can be solved. Solution methods are designed according to the problem formulation. For the classical CVRP efficient solution methods have been built. Yet, tackling the static and stochastic problem remains a challenging topic [JFG⁺11] because it is a more complex problem due to the introduced uncertainty [WHB17]. A common practice is to try to reduce the static and stochastic CVRP to its deterministic counterpart (via reformulation, decomposition or etc.) and then make use of efficient algorithms developed for the classical CVRP. A large number of solution methods are available for the static and deterministic CVRP [SOM08]. Providing classification schemes for the solution methods used not only in the CVRP but in any combinatorial optimization problem is a hard task because there are several characteristics and concepts involved in various methods. The particularities among solution methods have become more unclear and many hybrids have been developed. However, following a number of works, such as Oyola et al. [OAW17], we broadly classify solution approaches into two groups, **exact solution meth-**

ods and **heuristics**. Heuristics are also sorted into three classes: **constructive heuristics**, **improvement heuristics** and **metaheuristics**. Rather than presenting a detailed explanation of every available implementation and a comparison among solution methods, in this section, we briefly describe every one of these groups and classes and provide examples of their applications in the context of the SSCVRP.

2.4.4.1 EXACT SOLUTION METHODS

Exact solution methods are algorithms that are capable of solving an optimization problem to optimality, i.e. guarantee the optimal solution is found, if it is given to the method enough time and memory storage. Because any bounded combinatorial optimization problem, such as the VRP, has a finite number of feasible solutions, it would theoretically be possible to enumerate and evaluate all feasible solutions in order to find an optimal one [Win04]. But this finite number usually is very large. A full enumeration of all feasible solutions is therefore time consuming. That is why exact algorithms use clever techniques so that only a fraction of the feasible solution area needs be examined, avoiding full enumeration [Rop06]. Most of the exact solution methods proposed for the SSVRP are based on extensions of the Integer L-Shaped Method or **Branch-and-Price** algorithms [GWF13].

The Integer L-Shaped Method was developed by Laporte and Loveaux [LL93] and has enabled researchers to solve several categories of the SSVRP formulated as a SPR model. It assumes that the recourse function cost $Q(a, \mathbf{b})$ (2.4) can be computed given a feasible route plan y and that a finite lower bound L on $Q(a, \mathbf{b})$ is available. The method works on a so-called current problem (CP) that is obtained by relaxing the stochastic problem. The SSVRP is relaxed by relaxing both the integrality (2.5), (2.6) and the sub-tour elimination (2.4) constraints and by replacing the expected recourse function cost by a lower bound θ. The CP is then modified by dynamically adding constraints to prevent sub-tours and to further improve the relaxation. The steps for the L-Shaped Method [LLH02] are described bellow.

- Step 0. Set the iteration counter t equal to zero and add the bounding constraint $(\theta \geq L)$ in the initial CP. Set the objective value of the best solution found so far

equal to ∞. The only pendent node corresponds to the initial CP.

- Step 1. Select a pendent node from the list. If there is none stop.

- Step 2. Increase the count $t := t + 1$ and solve CP, finding an optimal solution.

- Step 3. Check for any constraints violation and then include at least one violated constraint. A lower bounding functional may also be generated and added. Return to Step 2. Otherwise, if the objective value of the CP is greater than or equal to the objective function of the best solution found so far, the current node is fathomed and the method returns to Step 1.

- Step 4. Branch on a fractional variable if the solution is not integer. Add the corresponding subproblems to the pendent nodes and returns to Step 1.

- Step 5. Compute the total expected cost of the recourse function for the optimal solution of the current problem. Add it to the objective function value. Compare the objective function value of the CP to the objective function value of the best found solution. Set the best value as the optimal objective function value.

- Step 6. Fathom the current node if the lower bound (θ) is greater than or equal to its actual expected recourse cost and return to Step 2. Otherwise, impose the optimality cut and go to Step 2. The optimality cut forces the method to move to a solution different from the CP.

In Laporte and Louveaux [LL98] the L-shaped method was used to solve four different SSCVRP, with stochastic travel times, with stochastic customers, with stochastic demands, and with both stochastic customers and demands. The computational experiments show that the SSCVRP could only be solved to optimality for small size instances. Hjorring and Holt [HH99] apply the same L-shaped method to find a optimal route plan for SSCVRPSD with only one vehicle. However, they use general optimality cuts, instead of the specific optimality cuts used in [LL98]. They argue that if general optimality cuts are generated, it may be possible to reduce the number of feasible solutions (nodes) that must be evaluated and the resulting specific optimality cuts that should be created, while the optimality of the

final solution is still guaranteed. Based on this argument that by using general optimality cuts the L-shaped method does not need to enumerate all integer solutions in-between the lower and upper bound, Chang [Cha05] also uses the general optimality cuts in a L-shaped method to find optimal solutions for a SSCVRP with time windows and stochastic demands.

According to Christiansen and Lysgaards [CL07] the best results obtained by the L-shaped method are on instances with non-constrained vehicle capacities, i.e. instances of small expected customer demands relative to the capacity of the vehicles. As the capacity constraints become more restrictive, the effectiveness of the L-shaped-based methods seems to diminish. On the other hand, branch-and-price solution methods benefit from more capacity constrained VRPs because these tighter constraints decrease the feasible solution space of the column generation subproblem and potentially limit the number of feasible routes that need to be considered [GDG14]. The branch-and-price algorithm is a variant of a branch-and-bound method in which the bounding part is done by using column generation. The idea of branch-and-bound is to construct a proof that a solution is optimal based on successive partitioning of the solution space [PK98]. The "branch" refers to the partitioning process while "bound" means to lower bounds that are used to construct a proof of optimality without exhaustive search. The method maintains a binding solution, i.e. the known feasible solution of lowest cost, and a set of nodes, i.e. a set of linear relaxed problems. Initially the set of nodes contains the subproblems without any branching decisions. A node corresponding to a relaxed problem is solved using column generation [DDS91]. Branch-and-price method is used by Christiansen and Lysgaards [CL07] to solve a SSCVRP with stochastic demands to optimality and by Taş et al. [TGD+14] to optimally solve the SSVRP with soft time windows and stochastic travel times.

2.4.4.2 Heuristics

Due to computational challenges, there are many stochastic problems with no exact solution method known to work for reasonable sized problems. Since real instances are much larger, this creates the need for heuristics [OAW17]. Differently to exact algorithms, heuristics do not guarantee to find an optimal solution for the optimization problem, instead they

aim at finding a feasible solution of reasonable quality in a shorter computational time [Rop06]. Moreover, because the model parameters of the decision-making problem usually are only an estimation of the real data, the optimal solution becomes one of no great concern [MRS12]. Several heuristics have been developed for the deterministic and for the stochastic VRP [GRT$^+$16, SCPC15]. In this work we categorize them into three types described bellow.

Constructive heuristics build a solution from scratch. At each iteration, one solution component is added to the solution according to a set of rules defined beforehand. They are usually called greedy because the solution component that is added is the one that achieves the maximal myopic benefit [SC15]. Two constructive heuristics applied in the VRP are the Clarke and Wright (C&W) method and the Sweep method.

C&W [CW64], also called savings algorithm, is the most used constructive heuristic for the VRP. The reason is its simplicity and the ease with which it can be adapted to handle variations of the VRP [SG83]. Many variants of this solution method have been proposed and applied to different classes of the VRP [Rop06]. The algorithm begins with a solution formed by a set of single routes to every customer $(0-i-0)$. Cost savings $S_{ij} = c_{0i}+c_{0j}-c_{ij}$ can be obtained by satisfying the demands of customers i and j using one vehicle. These savings are sorted in decreasing order. C&W then merges the routes (connecting customers i and j) that provide a better impact on the total cost $J(y)$ (2.1), i.e. the highest saving S_{ij}, without violating the capacity constraints. The procedure continues until no further merges are possible. Dror and Trudeau [DT86] use a modified savings algorithms to solve the SSCVRP with stochastic customer demands. Mendoza et al. [MCG$^+$11] propose a C&W algorithm to tackle a generalization of the SSVRPSD, called multicompartment CVRPSD (MC-CVRPSD). The only difference between this problem and the SSCVRPSD is that in the latter, each customer orders different products that must be transported by independent vehicle compartments.

The Sweep method was proposed by Gillett and Miller [GM74]. In this method, customers' locations are represented in a polar coordinate system (θ_i, ρ_i), where θ_i is the angle and ρ_i is the ray length [Lap92b] with the depot as (0,0). The algorithm begins by ranking

the customers in increasing order of their θ_i. It then sweeps the customers into clusters either clockwise or counterclockwise. For that, it starts from the first customer in the rank and adds this customer to a cluster. Customers are swept to a cluster as long as the cluster can be served by a single vehicle. After the capacity constraint is reached a new vehicle is selected and the remaining customers are swept into a new cluster [Lap09]. When all customers were assigned to a cluster, the algorithms finishes by calculating a route for each cluster in order to create a route plan. The Sweep method is used by Goodson et al. [GOT12] to generate a initial candidate route plans for the SSCVRP with stochastic demands.

Constructive heuristics are usually followed by an **improvement heuristic** [CGH+05]. Improvement heuristics, also referred as local search heuristics, examine a neighbourhood of a feasible solution in order to find an improving solution. Most classical improvement heuristic work in a descent mode until a local optimum is reached [CGH+05]. These methods take a initial feasible solution as input and then try to improve it exploring its neighbourhood by applying successive small changes to it. At every iteration, the heuristic keeps a current solution and may modify it based on the evaluation of the effect of changing the solution. When one of the changes leads to a better solution, this solution becomes the current solution. This process is repeated [Rop06] until no improving solutions can be found in the neighbourhood [SCPC15].

One of the best known local search heuristics is the K-Opt Local Search proposed by Lin [Lin65]. In this heuristic, the operator k defines how many arcs in the current route plan are exchanged to possibly create a improved solution. An exchange is either inter-route when it occurs between a pair of routes or intra-route when it occurs between a pair of customers. In the 2-opt local search with intra-route improvement, two arcs are swapped in only one route. Let y be a route plan for the VRP. The neighbourhood of y, defined by $N(y)$, contains all route plans created by swapping two arcs in y. Two arcs, $(i, i+1)$ and $(j, j+1)$, are replaced by two other arcs, (i, j) and $(i+1, j+1)$, and the reversal of path $p(j, i+1)$ [EYC15]. If this change brings a solution of improved objective function value, the current route plan is substituted and the local search continues. The k-opt local search is used by Erera et al. [ESU09] and Marinakis et al. [MMS15]. Erera et al. [ESU09] apply intra-route

and extra-route exchanges with $k = 1$ for improving the solutions obtained for the SSVRP with Time Windows (SSVRPTW) with both stochastic customers and demands. Marinakis et al. [MMS15] adopt an 1-opt intra-route local search heuristic to enhance solutions for the SSCVRPSD.

Another commonly used improvement heuristic is the Large Neighbourhood Search (LNS) method first presented by Shaw [Sha98]. The LNS consists in building and exploring the neighbourhoods of a given solution. At each iteration, it first removes u customers from a route (usually called destroying part) and then reinserts them (called repairing part) in different positions into the route plan in order to find a better route plan [WHB17]. Several rules can be used for choosing the customers to be removed and how to reinsert them [Rop06]. An crucial decision in this method is how many customers (u) are removed and reinserted into the route plan, i.e. the degree of destruction. When only a few customers are extracted, the method explores small neighbourhoods. On the other hand, if a large number of customers are deleted of the route plan, the repairing phase stands for repeated optimizations from zero. Wu et al. [WHB17] apply a LSN-based procedure to improve solutions obtain for the SSVRP with stochastic travel time. Lei et al. [LLG11] develop an extension of the LNS for the SSCVRPTW with both stochastic demands and time windows.

Improvement heuristics may not only be employed in order to reinforce constructive heuristics but also as intensification components in **metaheuristics** [SCPC15]. A metaheuristic is a problem-independent solution method, often nature-inspired, that supplies a set of rules (or strategies) to develop heuristic algorithms [SS09, BDGG09]. Different to classical improvement heuristics, metaheuristics incorporate mechanisms to continue the exploration of the search space after a local optimum is found [CLSV07]. In order to escape local optima, metaheuristics allow for intermediate deteriorated and even infeasible solutions [CGH$^+$05]. A efficient metaheuristic can provide good solutions in reasonable computation times [CLSV07]. That is why optimization solution methods have converged to metaheuristics [CGSA14].

Good surveys on metaheuristics for the CVRP are provided by Gendreau et al. [Gen08], Golden et al. [GWKC98] and Toth and Vigo [TV14]. Metaheuristics for the classical CVRP,

and similarly for the SSCVRP, can be generally divided into **population-based** and **local search** metaheuristics. Population-based evolve a population of solutions which may be combined together in the hope of generating better ones. One example of this category is Genetic Algorithms [TV14]. Genetic Algorithms (GA) are a family of solution methods inspired by the theory of evolution by means of natural selection. The framework of this metaheuristic was proposed by Holland [Hol92]. In this metaheuristic, solutions are represented by chromosomes, also called individuals. The GA begins with an initial population of chromosomes and then improves it in a way that the goodness (or fitness) value of the individuals in the population increases. The idea is that individuals that are more adapted to the environment, i.e. higher "fitness", have a better change to live [Whi94]. At each iteration a new generation of individuals is created by means of crossover and simple mutation. First, two parent chromosomes are selected and two offspring from these parents are created using a crossover operator [AMM16]. A random mutation to each offspring is then applied. The weaker individuals are deleted from the population and replaced by new offsprings [CGH$^+$05], improving the overall fitness of the population. Applications of GA in the SSVRP can be found in [AMM16, SS09, AT06]

On the other hand, local search metaheuristics, like improvement heuristics, explore the solution space by moving at each iteration from a solution to another solution in its neighbourhood. This category include Simulated Annealing (SA). The use of SA as a solution method in optimization problems was introduced by Kirkpatrick et al. [KGV83]. Simulated annealing is based on the thermodynamics, in the process of physical annealing with solids. To grow a crystal, a row of materials is heated and then cool down until the crystal structure is frozen in. When this process is done very slowly the crystalline solid reaches its most regular possible crystal lattice configuration, i.e. its minimum lattice energy state, and is, therefore, free of crystal defects [NJ10]. At each iteration, a neighbour solution is generate and its objective function value is compared to the objective function value of the current solution. Improved solutions are always accepted, while non-improved solutions are accepted in an attempt to escape a local optimum [BBC$^+$04]. The probability of accepting non-improved solutions depends on a parameter, called temperature, which starts with high

values and it is gradually decreased at each iteration [MF98]. The metaheuristic stops until some terminal condition is satisfied. SSVRPs are solved via SA in [Goo15, GOT12, GG08].

2.5 Conclusions

The proceeding chapter examined the capacitated vehicle routing problem, more precisely its stochastic variant, in order to outline the decision-making problem studied in this thesis, the static and stochastic CVRP with uncertain customer demands. A new classification for the classical VRP is proposed based on the aspect "uncertain information" and "solution evolution". The stochastic CVRP was classified according to four aspects: source of stochasticity, decision policy, modeling technique, and solution method, enabling us to taper it down to the SSCVRPSD. There is a enormous amount of literature on each one of these four aspects and researchers will continue to try to improve what it has already been proposed, not only for science but also for resembling better what the real SSCVRP faced by the companies look like.

Stochastic Vehicle Routing Problem
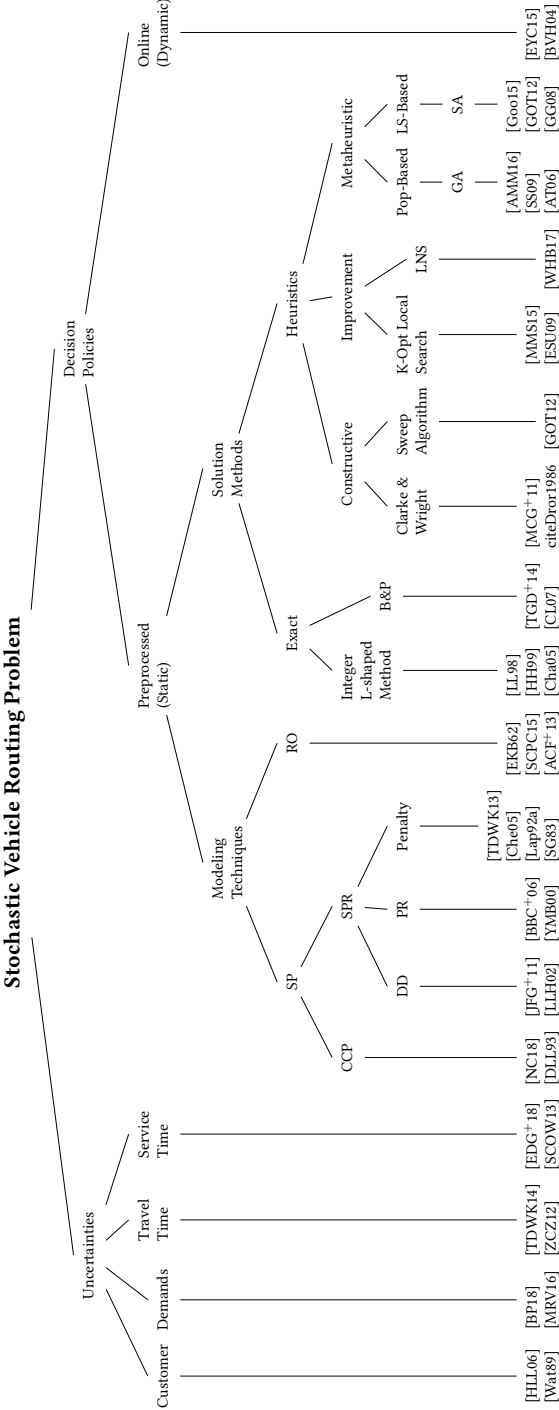
Fig. 2.1: Papers Dealing with the Stochastic Vehicle Routing Problem

3 ROBUSTNESS IN THE CAPACITATED VEHICLE ROUTING PROBLEM WITH STOCHASTIC DEMANDS

3.1 INTRODUCTION

Uncertainties affect a wide range of decisions decision-makers have to make. As presented in Chapter 2, uncertainty in customer demands, customers' locations, service times as well as in travel times complicate the task of a logistics manager in planning a route plan. The SVRP is thus more complicated to handle than its deterministic counterpart. In this way, the stochastic problem raises concepts that the deterministic problem does not, such as robustness. In this chapter, we first present the concept of robustness and then introduce approaches found on the literature for achieving robustness in the decision-making problem.

3.2 DEFINITION OF ROBUSTNESS AND ITS PRICE

As described in Section 2.4.3, the SSCVRP problem can be modeled as a deterministic CVRP. By doing this, one instance of the input data is fed to a mathematical model, which minimizes the transportation cost in order to calculate the "optimal" route plan. In the selected instance, the stochastic input is set equal to the most likely estimator of its realization in the future, i.e., the expected value (nominal values) of each customer's demand [BS04]. The main drawback of this strategy is its inability to notice the existence of plausible instances other than the one used to calculate the optimal solution. For these plausible instances, the solution calculated using the nominal values might be suboptimal or even infeasible,

since several constraints may be violated [MRS12]. For decision-makers that have to live with the consequences of the decision, suboptimality or infeasibility may be unacceptable, even though that decision is the "optimal" for the "most likely" future scenario [KY97]. This situation leads to the goal of designing solutions that are less affected by data uncertainty, i.e. robust solutions [SC15].

The definition of robustness is highly dependent on the decision-maker involved [SS09]. What most decision-makers refer to as a robust solution is a solution resisting as much as possible perturbations in the uncertain inputs [SCPC15]. Robustness in optimization problems can be related with feasibility when the decision-maker wants to guarantee that the solution is feasible, or can be related with cost when the decision-maker wants to secure that the objective function value does not change.

Mulvey et al. [MVZ95] differentiate two terms, *solution robustness* and *model robustness*. Solution robustness corresponds to optimality meanwhile model robustness to feasibility. The term solution robust appears when a solution remains "close" to optimal for any realization of the uncertainty and the term model robust appears when a solution remains "almost" feasible for any realization of the uncertainty. According to them, the notions of "close" and "almost" are made precise through the choice of norms. Kouvelis and Yu [KY97] also adopt these definitions of *optimality robustness* (solution robust) and *feasibility robustness* (model robust). Sörensen and Sevaux [SS09] use the word *robustness* to refer to feasibility robustness and similarly to [MVZ95] and [KY97] assume that a solution is said to be robust when has a high performance in most or all realization of the uncertain inputs. Bernardo and Pannek [BP18] acknowledge robustness similarly to [MVZ95] and [KY97] but instead of solution robust and model robust they use the terms optimality and *robustness*, respectively. To address the notion of "almost" (feasible), Liebchen et al. [LLMS09] introduce the term *recoverable robustness*. According to the authors, a recoverable robust solution is a solution that is "almost" feasible for any realization of the uncertain inputs and therefore needs few recoveries when the real inputs are revealed. Instead of recoverable robustness, Ben-Tal et al. [BTGGN04], Dhamdhere et al. [DGRS05] and Thiele et al. [TTE09] adopt the names *adaptive robustness*, *demand-robustness* and *two-stage robustness*,

respectively.

Another definition of robustness, which is in the end of the spectrum, is the so-called *strict robustness* introduced by Soyster [Soy73]. A solution is said to be strictly robust when it is feasible for any realization of the uncertain inputs. A drawback of such a solution is its overly conservativeness in the respect that to find a strictly robust solution we need to forgo too much of optimality in order to guarantee feasibility [BS04]. To protect the feasibility of any solution against fluctuations on the stochastic inputs, we need to accept a suboptimal solution [SOM08], i.e., a solution that has worse objective function value. Feasibility robustness has thus a cost associated with it, which is called *Price of Robustness*, introduced by Bertsimas and Sim [BS04]. Mulvey et al. [MVZ95] argue that a solution will hardly remain both optimal and feasible for all realization of the uncertainty. In fact, there is a trade-off between optimality and feasibility robustness [SW15]. The higher the degree of feasibility robustness the higher the price needed to pay, and therefore the worse the objective function value [BP18]. Several studies have been proposed to decrease the *Price of Robustness* and control the degree of robustness [LLP12]. For instance, Mulvey et al. [MVZ95] propose a decision model, in which the objective function is a utility function that embodies the expected solution cost (optimality) and a variability in the mean value (robustness). The second term variability is multiplied by a goal programming weight ω to be used to derive a spectrum of answers that trades off solution for model robustness.

In this thesis we assume that a solution is called strict robust when it is able to endure variations on the demands without the need of recourse actions, i.e., a solution that is completely insensitive to changes in the customer demands defined in a possible demand range. While a recoverable robust solution is one that is relatively insensitive to fluctuation on the demands, that is, little additional cost is incurred.

3.3 APPROACHES TO ACHIEVE ROBUSTNESS

There are mainly two approaches used to achieve robustness in decision-making problems under uncertainties, stochastic programming and robust optimization [SC15]. But their concept of robustness is different. For SPR models, robustness means less corrective actions

applied in the a priori solution when the real inputs are revealed. It allows a solution to violate the constraints affected by uncertainties [BTN98]. In SP, the term robustness thus denotes *recoverable robustness*. On the other hand, RO abstains from second-stage actions [LLMS09], and tries to find a solution that is feasible for any realization of the uncertainty set [ACF⁺13]. Therefore, RO aims at strictly robust solutions.

Stochastic programming is usually applied when uncertainty can be described by known distributions [SCPC15], meanwhile RO is proposed to handle cases where the probability distributions are hard to justify or estimate [AJ15]. Apart from assuming that the probability distribution modeling the demands is available, stochastic programming suffers from another drawback. It is affected by the dimension of the problem, which impacts its computational tractability [GWF13]. Birge and Louveaux [BL11] say that the complexity of SP models increases proportionally to the number of possible realizations of the stochastic inputs, which in turn increases exponentially with the number of stochastic inputs. Robust optimization avoids these weaknesses. First, because this approach handles the uncertainty without making assumptions on probability distributions. It rather assumes that the uncertainty belongs to a deterministic set [LLP12] and that all uncertain realizations are equiprobable [EKB62]. Second, because RO usually simplifies the stochastic problem to a version that is not more difficult to solve than the deterministic CVRP. However, it is not always easy or even tractable to analytically determine the worst-case performance of a given solution [SS09]. Moreover, robust optimization draws strictly robust solutions and these solutions are often too conservative [MPB05]. The solution that has the best worst-case performance (RO) will generally be more conservative than the one that has the best average-case performance (SP) [SS09], being necessary to pay a higher *Price of Robustness*.

Methods based on stochastic programming and robust optimization are available to attain robustness in the SSCVRPSD [SCPC15]. SP uses the probability distribution of customers' demands to compose recoverable robust route plans and permit failures caused by fluctuations on the stochastic demands to happen. It attempts to reduce the amount of corrective actions (detour-to-depot or preventive restocking) necessary to handle these route failures. RO adopts the range of the customers' demands, in which they can fluctuate [LLMS09], to

design a set of scenarios for the demands in order to calculate strictly robust route plans that do not need any corrective actions. We presented the mathematical models for the SSCVRP with stochastic demands used in both stochastic programming and robust optimization approaches in Section 2.4.3.1 and Section 2.4.3.2, respectively. Nevertheless, apart from SP and RO, other ways of achieving robustness in the SSCVRSD (be that strict or recoverable robustness) can be encountered on the literature. It is important to highlight that despite the ability of these approaches of producing robust route plans yet not all studies that apply them in the SSCVRPSD cite the term robustness. In the next section, we present studies on approaches that are not only able to attain robustness but also acknowledge the concept in the context of the SSCVRP with uncertain demands. Figure 3.1 exhibits the position of these studies based on the approaches used.

3.3.1 Robust Optimization Method

When applying robust optimization in the SSCVRPSD the goal is to find a solution that is insensitive (immunized) to the uncertain demands, as it is a route plan that minimizes the worst case demand scenario [Ord10]. Therefore, as mentioned before, all studies reviewed in this section refer to *strict robustness* as they mention robustness. The works of Sungur et al. [SOM08], Lee et al. [LLP12], Gounaris et al. [GWF13], Sun and Wang [SW15], and Gounaris et al. [GRT+16] deal with uncertainties in the demands via robust optimization. They are reviewed as follows.

Sungur et al. [SOM08] were the first to propose a RO formulation for the capacitated vehicle routing problem with uncertain demands. The authors adopt the MTZ formulation and assume that the customer demand belong to three uncertainty sets (convex hull, box and ellipsoidal). By doing that, they show that the problem (RVRP) is an instance of the capacitated VRP and it can therefore be solved through efficient exact algorithms present in the literature. For comparison purposes, the proposed formulation is contrasted with a chance constrained model and a stochastic model with recourse. The robust solutions are also compared with solutions obtained from a uniform and a non-uniform strategy of distributing the excess capacity among all the vehicles. For that, they use a dataset with

random, clustered and modified from literature instances and two performance measures, ratio k and ratio δ. The ratio k measures the relative extra cost of the robust solution with respect to the cost of the deterministic solution, which is the referred as the *Price of Robustness*. In turn, the ratio δ is the relative unsatisfied demand for the deterministic solution when it faces its worst-case scenario for the demands. The results show that all SPR, CCR and RVRP models result in no unmet demand in all cases. For small instances, the SPR and RVRP models design solutions of same cost, but when uncertainty becomes higher the recourse model is more efficient. Nevertheless, the authors argue that recourse models are more difficult to solve because of their large problem size and require specialized algorithms. The constraint model can be more or less efficient than the proposed robust formulation depending on the problem parameters and degrees of uncertainty. Finally, the robust solution approach seems superior to simple strategies of distributing the excess capacity among all vehicles.

Lee et al. [LLP12] address the static and stochastic CVRP with deadlines. They assume the problem is under not only demand but also travel time uncertainties. Robustness is achieved be making the solution feasible for any demand and travel time uncertainty sets (strict robustness). The solution approach has three parts: column generation subproblem, Desrochers' labelling, and branching scheme. Uncertainties are encapsulated in the column generation subproblem. They consider two types of uncertainty sets with adjustable parameters (Γ for travel times and Λ for demands) for the possible realizations of demand and travel time. This means that a budget of uncertainty is defined by limiting the sum of deviations of the uncertain inputs to their nominal values. The subproblem is then solved using a robust shortest path problem with recourse constraints. Computational experiments are performed on two datasets from the literature. The costs of the robust solutions and deterministic solutions are compared to estimate the percentage increments in the optimal values due to the introduction of robustness of the solution, i.e. *Price of Robustness*. The performance of the robust solutions obtained via the proposed algorithm is compared with deterministic solutions using Monte Carlo simulation. The results show that a more robust solution can be designed with only a small penalty in the objective value. Besides,

the authors also propose some guidelines to adapt the parameters to control the robustness of the solution in different real life situations.

Gounaris et al. [GWF13] propose four robust counterparts of deterministic capacitated VRP formulations (two-index vehicle flow, MTZ, reformulated robust MTZ, commodity flow, and vehicle assignment) and applied a branch-and-cut algorithm to solve them to optimality. The demands are assumed to be supported on a polyhedron, aiming to avoid overly conservative solutions obtained when all demands reach their worst-case values at the same time. The authors use two broad classes of demand supports (budge and factor) and partition the customers into four geographic quadrants. For the first demand support, customers demands are allowed to diverge by at most $\alpha\%$ from their nominal values and the cumulative demand in each quadrant may not exceed its nominal value by more than $\beta\%$. Similarly, for the second demand support the customers' demands may deviate by at most $\alpha\%$ from their nominal values and the cumulative demand in each quadrant may not exceed its nominal value by more than $\beta\%$ as well, but demands are modeled as a convex combination of different factors that may be interpreted as quadrant demands. The performance of the solutions (objective function) obtained for each formulation are compared using instances from the literature.

Sun and Wang [SW15] provide a mixed-integer programming model (E-SDROA formulation) and a solution method to solve the SSCVRP with two sources of uncertainty, demands and transportation costs. The authors suppose the coexistence of failure and successful scenarios. A failure scenario occurs when the real demands are higher than predicted and the capacity of the vehicle is exceeded at runtime, otherwise a scenario is called successful. The formulation trades off the expected value of the transportation cost in all failure scenarios and its variation under conditions of the coexistence of failure and successful scenarios. The first objective is multiplied by a weight ω that can reflect some matters of concern to decision-makers at the depot, such as bidding ($\omega < 1$) and capital budget situations ($\omega > 1$). Similarly, the second objective, which is the potential risk with respect to a solution caused by sources of uncertainty, is multiplied by a weight μ that denotes the level of concern about exceeding ω. In this way, the second term protects against potential scenarios that can in-

cur extremely high costs and therefore represents the robustness measure. Both demands and transportation costs are modeled as uncertainty sets and two nonnegative integer parameters, Γ and Λ are introduced for controlling the degree of uncertainty. Instances from the literature are used for comparing the non robust solutions designed with the deterministic version of the problem ($\Gamma = 0$, $\Lambda = 0$) and the E-SDROA robust solutions. Significant increments in the robustness of the solutions are achieved without much loss in optimality. The solution method is also compared with two algorithms and it is shown that it provides better solutions in terms of cost.

Gounaris et al. [GRT$^+$16] present an Adaptive Memory Programming (AMP) metaheuristic to address the SSCVRPSD. The idea underlying AMP is to exploit a set of long-term memories for the iterative construction of new route plans. This means that the method keeps track of "elite components" of the solutions computed during the search procedure and accounts them as building blocks for restarting and intensifying the search. Similarly to Gounaris et al. [GWF13], they use two classes of uncertainty sets, budget and factor instead of the generic polyhedral uncertainty set. Using designed instances based on the literature, it is observed that including uncertainty does not significantly deteriorate the performance of the metaheuristic. The proposed solution framework can help to improve the performance of exact methods by comparing the performance of solutions designed via a branch-and-cut method [GWF13] with solutions obtained by taking the best solution calculated via the AMP heuristic and feeding it as an initial solution to the branch-and-cut method. In this way, they are able to obtain improved best-known solutions and to solve three benchmark instances to certified optimality. Moreover, by comparing the costs of solutions obtained with the deterministic CVRP model with AMP solutions, it can be seen that robust solutions are more expensive, and the bigger the uncertainty set the higher the price.

3.3.2 STOCHASTIC PROGRAMMING

As mentioned before, SP with recourse models are also able to produce robustness (*recoverable robustness*) against fluctuation on the demands in the SSCVRPSD. Since a SPR model for

the SSCVRP (see 2.4) aims at minimizing both the cost of the a priori route plan and the cost of the corrective actions needed to adjust the initial solution to the real demands [DLL93], this implies that route plans designed via this approach are in nature robust. Chance constrained models might produce robust solutions as well. It is possible to develop chance constrained models that minimize transportation cost with a certain confidence level, that is guaranteeing that demands are fullfilled with some very high probability [Ord10]. Sungur et al. [SOM08] and Ordóñez [Ord10] show that a chance constrained and a recourse models for the SSCVRPSD can be very similar to robust optimization models via reformulations. The works of Erera et al. [EMS10] and Sorensen and Sevaux [SS09] are studies that present applications of stochastic program models in the SSCVRPSD and address (recoverable) robustness.

Erera et al. [EMS10] study the SSCVRP with stochastic demands and duration constraints, called SSCVRPSD-DC. In this problem, apart from handling uncertain demands, each route must be feasible for all demand realizations. The goal is then to design a route plan subject to soft capacity constraints and hard constraints on route duration at minimum total travel time. The problem is modeled as a SPR, where a detour-to-depot is defined as the corrective action. Apart from the first and second-stage travel times, the objective function includes a penalty term for using more than m vehicles, in case they are necessary. By defining a high penalty value, the use of more than m vehicles is prevented, and the number of vehicles therefore increases only when it is no longer feasible to satisfy the route duration constraints with m routes. The solution method relies on solving an "adversarial" optimization problem, which defines a demand realization that maximizes the actual total duration of an a priori route, i.e. worst-case demand realization in terms of route duration. In this way, recoverable robustness is reached. Computational experiments are performed in order to assess the effect of including duration constraints in the SSCVRSD. For that, they compare the results obtained via constrained and unconstrained version of the problem. The results show that enforcing route duration limits affects the structure of the route plans and may forge the necessity of extra routes. A small increment in the total expected travel time of more than 7% was observed. However, in some cases, as the fleet size in-

creased, the total expected travel time decreased and this robustness achieved comes with a small *Price of Robustness* around 2%.

Sorensen and Sevaux [SS09], trying to partially overcome the curse of dimensionality in stochastic programming, propose a modified Memetic Algorithm with Population Management (MA|PM) that uses a Sample Average Approximation (SAA) method. The idea is to sample the stochastic input generating a set of sample scenarios in order to estimate the objective function value. The authors apply the proposed solution method in the SSCVRP with stochastic demands and cost and in the VRP with stochastic customers. The method starts by generating a set of solutions that are both diverse and have a high quality. For each designed route plan, a robustness/flexibility evaluation function (recoverable robustness) value is estimated by repeatedly applying the solution to a sample of the stochastic input and calculating the corresponding objective function value. The objective function value is thus a sum of the expected total cost and a penalty cost. Two penalty functions are used, one due to overtime costs payable to the drivers (if total cost of a given plan is larger than a threshold) and the other due to loss of goodwill by the customers whose demand are not to satisfied (if the total demand served in a given route is higher than the vehicle capacity). The algorithms then chooses the solution that has the best robustness/flexibility evaluation function value. To assess the robustness of solutions drawn by the MA|PM, the authors calculate the standard deviation of each solution. The computational experiments demonstrate that MA|PM solutions are considerably more robust than the solutions designed using the deterministic approach.

3.3.3 Capacity Allocation Strategy

A natural strategy to cope with uncertain demands is to save vehicle capacity in order to be able to handle situations when the real demand is greater than the expected demand [SOM08]. In this kind of strategy, only a percentage of the vehicle capacity is used in the design of a priori route plan. The remaining capacity works as a buffer to deal with fluctuations on the customers' demands that might happen during the execution of the a priori route plan [JFG+11]. Two studies that utilize this strategy as a way to produce

robustness were found. They are described as follows.

Robustness addressed by Juan et al. [JFG$^+$11] is the recoverable robustness. They propose a hybrid solution approach which combines simulation and heuristics for solving SS-CVRPSD. Apart from saving a certain amount of vehicle capacity, called safety stock, while designing the routes there is other idea behind the approach: to transform the issue of solving a given SSCVRP into a new issue of solving several deterministic CVRPs. For solving every deterministic problems, only a percentage k of the vehicle capacity in considered. The percentage k is increased up to 1 and for each k a solution is obtained. The performance of every solution under demand fluctuation is evaluated by using Monte Carlo simulation and the output is a short list of solutions with their corresponding properties, providing the decision-maker a set of alternative solutions. Such properties are second-stage cost $Q(y)$, called by the authors expected variable cost, and expected final cost $J(y)$ (see Definition 2.4). To calculate the second-stage the authors adopt detour-to-depot as the corrective action. Other performance measures used in the paper are number of routes, first-stage cost, called base cost, and estimation of the reliability. Computational experiments show that higher values of k calculate route plans with less routes and lower first-stage cost, but with lower reliability. It means that such solutions are less robust against demand fluctuations, i.e., they need more corrective actions to cope with the real demands. These solutions present, therefore, higher expected second-stage costs, increasing the expected final costs. The authors say that among the alternatives solutions a good candidate for most decision-makers would be the solution obtained with $k = 0.96$, since it balances the trade-offs reliability and cost minimization.

Juan et al. [JFJ$^+$13] propose the same hybrid solution approach, but they study the role of parallel and distributed computing system in the context of the SSCVRP with stochastic demands. For that, parallelization techniques are used at two different parts of the approach. First, a parallel-execution environment is adopt for designing the set of k instances. Second, they use several concurrent threads sharing a common memory for solving each instance.

3.3.4 COMBINED STRATEGIES

Ordóñez et al. [Ord10] present modeling alternatives for the capacitated vehicle routing problem with different (combined or separated) sources of uncertainty (demands, travel times, cost and customers). The authors present three conditions where the robust CVRP reduces to a problem similar to the deterministic VRP. First, it is assumed that there is no correlation between uncertain demands, travel times and travel costs. The resulting model can be computed b solving $m+1$ versions of the deterministic CVRP. Secondly, it is inferred that there is only uncertainty on the customers' demands and that uncertainty experienced by each vehicle has an overall bound but is independent from one vehicle to another. The unlikely situation in which all customers serviced by a vehicle present the largest possible demand at the same time is thus excluded. Thirdly, it is presumed that there is correlated uncertainty on the same route and a global bound on the uncertainty at all customers. For both the second and third conditions, they show that the resulting robust counterpart is a deterministic model with a additional number of variables and constraints and argue that a column generation method usually used to solve the deterministic CVRP could be adapted to solve the resulted problem. Solutions obtained using the model proposed by Sungur et al. [SOM08] and randomly generated instances are compared with deterministic solutions, solutions designed via stochastic models (SPR and CCP), and solutions obtained considering capacity management strategies (uniformly and nonuniformily). The performance of the solutions are assessed by looking at unmet demands, costs and *Price of Robustness*. Computational experiments indicate that it is difficult to conclude which formulation is preferable and further conclusion should be made based on the application. They finish the paper applying robust optimization to calculate the a-priori route plan and adjusting it when uncertainty is realized to two routing applications: routing in large-scale bioterrorism emergencies and courier delivery problem.

3.3.5 OTHER FRAMEWORKS

Moghaddam et al. [MRS12] present a new Particle Swarm Optimization (PSO) metaheuristic to solve the determinisitc capacitated VRP. In this solution approach every particle is

represented by an array of real number and the particle moves in a multidimensional space to find the optima/near optimal position. PSO are usually designed to solve continuous optimization problems. That is why the authors propose a decoding method to apply PSO in the CVRP. In order to improve the quality of the solution obtained with PSO metaheuristic 4 local search algorithms, 2-Optimal Local Search (2-OPT), Exchange 1-1, Variable Neighbourhood Search (VNS) and Iterated Greedy (IG) are applied between the iterations of the metaheuristic. The authors apply the proposed solution approach in instances from the literature and compare the solutions with those from literature. Computational experiments show that PSO provides solutions much closer to the optimal solutions. After that, uncertainty demands are considered in the problem by assuming that their probability distribution is unknown. They assume that the demands belong to an interval $[d_i - \varepsilon \cdot d_i, d_i + \varepsilon \cdot d_i]$, where ε indicates the perturbation percentage of the demands from the nominal value d_i. To solve the stochastic CVRP, the PSO objective function is modified by adding an index ψ to compared the balancing of unused capacities in different solutions. This index increases when all routes are adjusted and have enough buffer capacities to deal with fluctuation in the demands. The metaheuristic then looks for solutions with higher ψ, achieving robustness. The performance of the modified PSO is evaluated by comparing the unmet demands percentages for the deterministic and robust route plans and determining the *Price of Robustness*. The PSO robust solutions are also contrasted with exact robust optimum solutions shown by Sungur et al. [SOM08]. Computational results demonstrate PSO designs route plans with smaller costs, nonetheless such solutions do not meet al. l uncertain demands.

3.4 Conclusions

In this chapter, the approaches for dealing with uncertainties in the decision-making problem were introduced, more precisely approaches for coping with fluctuations on the demands in the SSCVRPSD. General definitions of the concept robustness, a crucial attribute that rises when solving uncertain decision-making problems, were discussed together with the meaning of its price (*Price of Robustness*). A review of studies that cover these two topics, approaches for dealing with uncertain demands in the SSCVRPSD and robustness, was

Approaches for Dealing with Uncertain Demands in SSCVRP
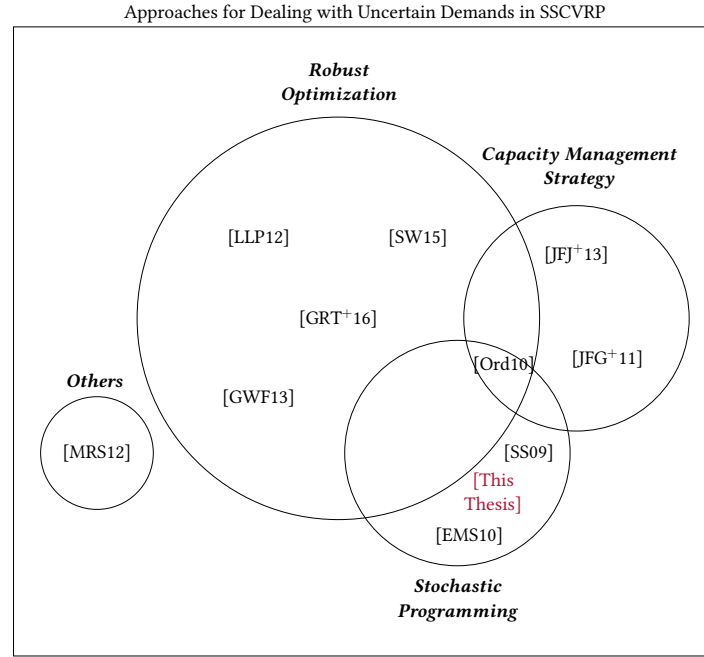
Fig. 3.1: Positions of the Works Based on the Approach Used to Achieve Robustness

presented. It can be seen that although the interpretation of robustness (*strict robustness*, or *recoverable robustness*, etc.) is dependent on the decision-maker involved, not so many approaches deliver flexibility to the decision-maker to define the safety level she/he desires, i.e. the level of robustness. The mathematical formulation proposed in this thesis acknowledges the need for such flexibility by adding a weight ω that reflects the decision-maker's preference in the objective function. In this way, decision-makers can obtain route plans that can be more or less robust against small changes in the demands. Thus, as we use the term robustness in the next chapters, we refer to *recoverable robustness*. Our proposed decision model is similar to a two-stage SP models, where a priori routing decisions are made in the first-stage, while in the second, corrective actions are applied. That is why we placed our approach for dealing with stochastic demands and achieving robustness in the *Stochastic Programming* set from Figure 3.1. Nevertheless, we do not minimize the expected value of the cost of corrective actions (second-stage transportation cost).

4 The Robust Multi-Objective Capacitated Vehicle Routing Problem

4.1 Introduction

The decision-making problem, namely the static and stochastic CVRPSD, was addressed in a way that is different from the methods presented in Chapter 3. The SSCVRPSD was formulated as a two-stage stochastic program with recourse, where uncertain customer demands were modeled as discrete samples. Nonetheless, this model includes a parameter of choice for decision-makers that combines the two conflicting objectives, optimality and robustness, into a scalar one which enables the trade-off between them. In this way, decision-makers are free to determine the amount of robustness she/he is willing to have. Based on the proposed formulation, called MAD formulation, we designed the RoMO solution approach. Both formulation and solution approach are presented in this chapter.

4.2 Mean Absolute Deviation Formulation

One of the goals of this thesis is to develop a mathematical decision model for the SSCVRPSD, one that gives a decision-maker the ability to choose between minimal transportation cost (optimality) and safety against demand uncertainties (robustness). In this way, she/he is able to design route plans of different degrees of robustness. In this thesis, a route plan is said to be robust when it is insensitive to a range of possible demand conditions, i.e. little additional transportation cost is incurred as demands fluctuate during execution phase.

When the probability distributions of the uncertain demands are known, this problem can be modeled as a two-stage stochastic programming with recourse (Definition 2.4). In this model, some feasibility constraints are relaxed and included in the objective function, assuming that violations on the capacity constraints induced by uncertain demands after the implementation of first stage decisions can be repaired by recourse actions [SCPC15]. Therefore, the transportation cost $J(y)$ (expected transportation cost) is split into two costs. Let the objective function (2.9) be changed to

$$\min_{y} \ J(y) := \min_{y} \ J_0(y) + Q(y). \tag{4.1}$$

The first term of the objective function is the transportation cost of the deterministic CVRP formulation (2.1), i.e., the cost of the a-priori route plan calculated using the stochastic knowledge, and represents the first-stage decisions, whereas the second term stands for the second-stage decisions, i.e. the expected transportation cost of corrective actions applied during execution phase. Unfortunately, we can evaluate (4.1) only a posteriori, that is, upon completion when all real demands are revealed, or via stochastic analysis. To partially overcome this, methods that use some form of Monte Carlo sampling of the stochastic parameters can be used [SS09]. Like so, a set of $\overline{s}$ samples in which a sample specifies a demand for each customer $i \in N$ is defined as

$$Sp = \{sp_j = (d_1^j, d_2^j, d_3^j \ldots, d_n^j) \mid j = 0, \ldots, \overline{s}\}. \tag{4.2}$$

A set of scenarios is correspondingly described as

$$S = \{s_j = (d_1^j, d_2^j, d_3^j \ldots, d_n^j, N, A) \mid j = 0, \ldots, \overline{s}\}. \tag{4.3}$$

All the scenarios have the same set of nodes N and set of arcs A. The distance c_{ij} between each pair of nodes is thus identical for all scenarios. Consequently, each scenario is a deterministic instance of the SSCVRPSD. s_0 is called nominal scenario, within which demands are equal to their expected values, i.e $d_i^0 = E[d_i]$. $J_0(y, s_j)$ is the deterministic objective

function value of solution y when applied to the scenario s_j. Then,

$$J_0(y) = \{J_0(y, s_1), J_0(y, s_2), J_0(y, s_3), \ldots, J_0(y, s_{\bar{s}})\}. \tag{4.4}$$

is a vector of first-stage transportation costs for solution y considering all scenarios. According to Sorensen and Sevaux [SS09], a typical measure for robustness is the sample estimator of the standard deviation of the first-stage transportation costs vector

$$\sigma^2(y) = \frac{1}{\bar{s} - 1} \sum_{j=1}^{\bar{s}} (J_0(y, s_j) - \widehat{J_0}(y))^2, \tag{4.5}$$

where

$$\widehat{J_0}(y) = \frac{1}{\bar{s}} \sum_{j=1}^{\bar{s}} J_0(y, s_j) \tag{4.6}$$

is the sample estimator of the mean of the first-stage transportation costs vector. Mulvey et al. [MVZ95] argue that the mean sample estimator measures optimality, whereas the standard deviation is a measure of robustness, and that we could therefore create a utility function that embodies a trade-off between optimality and robustness. This function

$$J_{MV}(y) = \widehat{J_0}(y) + \omega\sigma^2 \qquad\qquad \omega \in [0, \infty[\tag{4.7}$$

is called Mean-Variance (MV) objective function. The goal programming weight ω, which combines the two objectives into a scalar one, is used to derive a spectrum of answers that trade off optimality and robustness. Given that $\widehat{J_0} = J_0(y, s_0)$, i.e. the mean of the vector of first-stage transportation costs equals to the deterministic objective function value of solution y when applied to the nominal scenario. The mean-variance objective function then becomes

$$J_{MV}(y) = J_0(y, s_0) + \omega\sigma^2 \qquad\qquad \omega \in [0, \infty[\tag{4.8}$$

and (4.5) becomes

$$\sigma^2(y) = \frac{1}{\bar{s} - 1} \sum_{j=1}^{\bar{s}} [J_0(y, s_j) - J(y, s_0)]^2. \tag{4.9}$$

Equivalently to the vector of first-stage transportation cost, we can calculate the second-stage transportation costs vector

$$Q(y) = \{Q(y, s_1), Q(y, s_2), Q(y, s_3), \ldots, Q(y, s_{\bar{s}})\}. \tag{4.10}$$

and introduce $Q(y, s_0)$ as the second-stage transportation cost of solution y when applied to the nominal scenario. The mean absolute deviation of the second-stage transportation costs vector is the average of the absolute deviations from the central point $Q(y, s_0)$, i.e.

$$MAD(Q(y, s_j)) = \frac{1}{\bar{s}} \sum_{j=1}^{\bar{s}} [Q(y, s_j) - Q(y, s_0)]. \tag{4.11}$$

Based on the MV objective function and on the MAD($Q(y)$), we introduce an objective function that tries to anticipate the second-stage transportation costs and trades off optimality and robustness as well. This objective is represented by

$$J_{MAD}(y) := J_0(y, s_0) + \omega \frac{1}{\bar{s}} \sum_{j=1}^{\bar{s}} [Q(y, s_j) - Q(y, s_0)]. \tag{4.12}$$

Similarly to (4.8), in this multi-objective function, the first term expresses optimality and the second robustness. The weight ω represents the parameter of choice for logistics managers to modify the importance of the two aspects in the cost function. The only difference between (4.12) and (4.8) is that instead of the standard deviation of the first-stage transportation costs vector, we use the mean absolute deviation of the second-stage transportation costs vector. Both are measures of variability, which can represent a robustness measure. Nevertheless, the mean absolute deviation utilizes absolute values, whereas the standard deviation uses squares. The latter avoids the issue of negative differences between vector values and the mean and thus disables the objective function to notice the incremental and/or reductional trend on the second-stage transportation costs ($\{Q(y, s_1), Q(y, s_2), Q(y, s_3), \ldots, Q(y, s_{\bar{s}})\}$)

from the central point $(Q(y, s_0))$ among the $\bar{s}$ scenarios. This comportment circumvents the issue of designing solutions of too high degrees of robustness. Hence, despite input changes, the plan is still structurally optimal, i.e. the number of routes as well as the route sequences remain unchanged. In other words, MV objective function may identify route plans that are too robust, and which could be considered unreasonable by some decision-makers. MAD objective function avoids this.

In both MV and MAD objectives, for $\omega = 0$ we solve the classical linear programming formulation of the problem (see Definition 2.1). The solution obtained with the linear programming formulation is very sensitive to the demand changes, whereas the route plan obtained with higher ω is much less sensitive [MVZ95, BP18].

In (4.12), the second term can be interpreted as the steepness of the cost functional with respect to input changes. Comparing undisturbed optimal solutions $y^\star$ and $y^\star_{mad}$ for $J(y)$ and $J_{mad}(y)$, respectively (see Figure 4.1), we know that $J(y^\star_{mad}) \geq J(y^\star)$, and typically this inequality is strict, i.e., a robust solution corresponds to increased transportation cost (*Price of Robustness*). Yet, we expect that the solution $y^\star$ of (2.1) has to be modified to $\tilde{y}^\star$. Our aim is not to develop a systematic way of designing $J_{mad}(y)$ such that the corresponding solution $y^\star_{mad}$ improves the performance measures, i.e. $J(\tilde{y}^\star) > J(y^\star)$.

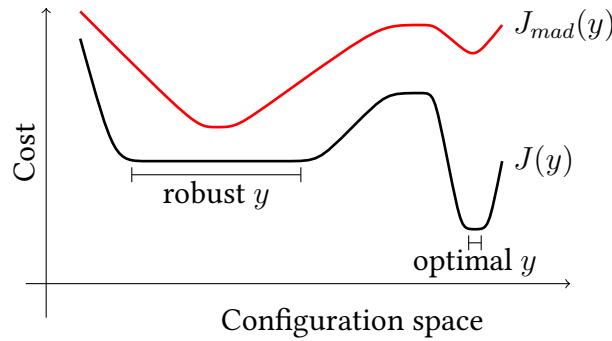**Fig. 4.1:** Exemplary of Cost Development Regarding Solution y.

The computed solution will typically exhibit higher total expected transportation costs than the one in Definition 2.1. Upon implementation, however, our numerical results, cf. Chapter 5, indicate that the second-stage transportation cost will be lower. Thus, these two aspects, robustness and optimality, represent trade-offs for routing solutions and must be

balance in accordance with the goals of the company.

Given the notations presented in 2.2, the Definition 2.2, and let S be the set of scenarios (4.3) and $Q(y, s_j)$ be the second-stage transportation cost of solution y when applied to the scenario j. We can finally introduce the mean absolute deviation model for the SSCVRPSD

Definition 4.1 (MAD SSCVRPSD)

$$\min_y \ J_{mad}(y) := \min_y J_0(y, s_0) + \omega \frac{1}{\bar{s}} \sum_{j=1}^{\bar{s}} [Q(y, s_j) - Q(y, s_0)] \qquad \omega \in [0, \infty[\qquad (4.13)$$

s.t. (2.2), (2.3), (2.4),(2.5) and (2.6).

Note that we do not characterize a certain tolerable bound on the disturbances, which would lead to a worst case estimate (see Section3.2). Instead, we let the optimization mechanism decide, which (modified) minimum is robust in a structural sense. For this reason, we will not state explicite bounds on tolerable disturbances, and if disturbances are too large, they will be handled by re-planning or extra tours.

4.3 Solution Approach

Based on the mathematical decision model introduced in the previous section, we develop a solution method to solve the decision-making problem. Since the objective function of the MAD formulation is a multi-objective function, we call the solution method RoMO. The proposed approach includes four stages, *Inputting*, *Data Handling*, *Initial Setting*, and *Solving*. All steps performed in each stage are shown in 4.3.

4.3.1 Stage 1: Inputting

In the first stage, all necessary parameters for the approach are inputed, namely ω and historical data on the uncertain customer demands. As mentioned before, the parameter of choice ω should be chosen according to the risk level that the decision-maker is willing to take. Higher values of parameter ω produce route plans of higher degrees of robustness.

These solutions are less affected by changes in the demands and therefore present lower corrective actions cost. In contrast, lower values of ω create route plans that are less robust.

4.3.2 Stage 2: Data Handling

The historical data on the stochastic demands is handled in the second stage. A Probability Distribution Function (PDF) is fitted to every customer demand d_i. We assume that each customer's demand follows an independent probability distribution. Because of the nature of the customers' demands, we made few particular assumptions on the type of distributions that can be fitted to this input. First, only discrete probability distributions should be used for fitting, since demand values are discrete. Second, as demands cannot be negative, only non-negative distributions such as Poisson is acceptable. After fitting a probability distribution for each stochastic demand, their corresponding parameters and the mean are estimated. For instance, if a uniform PDF is chosen, i.e. $d_i \sim U(a; b)$, the parameters a and b are estimated together with the mean of the distribution.

4.3.3 Stage 3: Initial Setting

In this stage, the PDFs fitted to every customer's demand in the previous stage is used to generate the set of samples Sp (4.2) and the set of scenarios S (4.3). This is done by means of Monte Carlo sampling. As mentioned before, each sample contains a potential state of the uncertain demand and each scenario is thus a static and deterministic instance of the SSCVRPSD. s_0 is the nominal scenario, where each demand d_i^0 is equal to the expected value of the fitted distribution. We generate new scenarios instead of using the existing customer demand scenarios (historical data) because in some situations using historical data as a scenario may be impractical. For example, a new company may not have enough data for generating a number $\bar{s}$ of scenarios.

After that, a so-called robust scenario is determined by using the set of scenarios S and

via equation

$$d_i^R = d_i^0 + \omega \frac{1}{\overline{s}} \sum_{j=1}^{\overline{s}} [d_i^j - d_i^0] \qquad \forall i \in N. \tag{4.14}$$

Every customer's demand in the robust scenario d_i^R is calculated by a linear combination of S with the weight ω, which increases the deviation from the expected value, allowing to create worse-case scenarios. Hence, it is possible to decide how robust a solutions can be. The robust scenario

$$s_R = \{d_1^R, d_2^R, d_3^R \ldots, d_n^R\}, \tag{4.15}$$

which is a deterministic instance of the SSCVRPSD, is then solved in the last and next stage in order to calculate the final solution y_{mad} which may be more or less robust against uncertainties on the demands.

Note that we chose to calculate this robust scenario and solve it instead of calculating $\overline{s}$ solutions, each designed for a different scenario $j \in S$, computing their second-stage transportation cost, and finding the MAD of the second-stage transportation costs vector. The reason is that this MAD computes transportation cost and it does not indicate any structural property. In other words, it only shows how much cost should be added to the first-stage transportation cost $J_0(y)$ so that the solution y can become more robust and not how the structure of the solution, i.e. sequence of customers and number of routes, should be changed in order to achieve robustness.

4.3.4 Stage 4: Solving

The robust instance s_R designed in the previous stage is solved in the last stage. Since s_R is a deterministic instance of the SSCVRPSD, we are able to make use of the efficient well established solution methods to solve it. Moreover, the aim of this thesis is not to propose a new solution method for the decision-making problem. For solving the problem, we employ three heuristics, a constructive, a improvement, and a metaheuristic.

The constructive heuristic Clarke and Wright savings is applied for calculating an initial

route plan. The pseudocode of the method is presented in Algorithm 1. After that, this initial solution is improved by a local search heuristic. The local search used in this thesis is the 2-opt with intra-route improvements, that is, the method performs swaps between all combination of customers at positions u and v, within all routes calculated by the C&W algorithm in order to find a route plan of better transportation cost. Let p be a route in the solution y_{cw} and $J_0(p)$ its transportation cost and $p[k]$ and $p[m]$ represent the customer indexes k and m within route p. Algorithm 2 shows the 2-opt LS pseudocode. At each iteration, the change on the route cost $J_0(p)$ that results from swapping $p[k]$ and $p[m]$ in route p is computed (8). If the swap improves the transportation cost, then the exchange is executed. Finally, the solution obtained via 2-opt LS is further improved by simulated annealing metaheuristic (Algorithm 3). Apart from the definition of the neighbourhood of a solution, there are another four aspects to be chosen before implementing SA: initial temperature, cooling ratio, termination condition, and halting criteria.

Similarly to [GRT$^+$16], [GT10], and [TK02], we use neighborhood structures based on customer exchange moves, namely the 2-OPT LS with intra-route improvements (Algorithm 2) and 1-0 and 1-1 exchange move. The 1-0 exchange move injects a customer from its original route and inserts it after another in a different route and produces the neighbourhood $N_1(y)$. The 1-1 exchange move works similar to 2-OPT LS, but instead of swapping two customers in the same route it swaps two customers in different routes. It constructs the neighbourhood $N_2(y)$. The 2-OPT LS builds the neighbourhood $N_3(y)$. Figure 4.2 shows the difference between the three local search operators [TQZB15]. The selection of the operator at each iteration of the simulated annealing is random (21).

The other four parameters formed what it is called the cooling schedule. An effective cooling schedule is a important aspect in the performance of the algorithm and a number of studies, such as [GEE$^+$15, CF99, NA98], have been devoted to this topic. Our goal is not to contribute to this question, and we therefore adopt the cooling schedule that was proposed by Osman [Osm93] and is commonly used on the literature. For specifying the parameters on the cooling schedule we need to perform a "test cycle". The test cycle is a search over the neighbourhood $N(S)$ (designed by using the 1-1 operator) of the solution provided by the

2-OPT LS method without performing the exchanges. By executing a test cycle, we obtain the largest Δ_{max} and smallest Δ_{min} change in the objective function values, and an estimate of the total number of feasible exchanges N_{feas}. After that, we set the initial temperature $T_s = \Delta_{max}$; the final temperature $T_f = \Delta_{min}$; the cooling ratio $\alpha = nxN_{fea}$, which is the decrement rule for updating the temperature T_k after each iteration k; the termination condition $L = 3$, which is the total number of temperature resets to be performed after the best solution was found.

SA starts from the solution calculated by the 2-OPT LS. At each iteration, a neighbour y_n is taken from the neighborhood of the current solution y_c. The transportation cost of this neighbour $J_0(y_n)$ is compared with that of the best route plan $J_0(y_b)$ found so far, so that if an improvement is achieved the neighbour becomes the current route plan and a new best solution is found. Otherwise the neighbour with worse transportation cost is accepted as the current solution with a small probability $e^{\frac{-\delta}{T_k}}$. The current temperature T_k is then updated and a new iteration begins. This is done until the stopping criteria is met, i.e. L resets were performed since the best solution y_b was found. The solution reported at the end of the SA is robust against small changes in the input customer demands.

Algorithm 1 Clarke and Wright Savings Algorithm

INPUT: s_R
OUTPUT: y_{cw}

1: Design single routes $(0 - i - 0)$, for each $i \in N \backslash \{0\}$
2: Compute the savings for merging customers i and j given by $S_{ij} = c_{0i} + c_{0j} - c_{ij}$, for all $i, j \in N \backslash \{0\}$ and $i \neq j$
3: **while** No savings can be achieved **do**
4:     Sort the savings in descending order
5:     Starting at the top of S_{ij}, merge the two routes that produce the highest savings, such that
6:         The customers i and j are not in the same route
7:         Neither i and j are internal customers, i.e. they are the first or the last customer in a route
8:         The total demand in a route does not exceed the vehicle capacity
9: **end while**

Algorithm 2 2-Opt Local Search Algorithm

INPUT: y_{cw}
OUTPUT: y_{2opt}

1: **for** All routes in y_{cw} **do**
2:     $best_{cost} \leftarrow J_0(p)$
3:     **for** $k = 1$ to $k \leq \text{size}(y_{cw} - 1)$ **do**
4:         **for** $m \leftarrow (k+1)$ to $m \leq \text{size}(y_{cw})$ **do**
5:             $new_{cost} \leftarrow J_0(p) + c_{p[u-1]p[v]} + c_{p[u]p[v+1]} - c_{p[u-1]p[u]} - c_{p[v]p[v+1]}$
6:             **if** $new_{cost} \leq best_{cost}$ **then**
7:                 $best_{cost} \leftarrow new_{cost}$
8:                 $\text{swap}(p[k], p[m])$
9:             **end if**
10:         **end for**
11:         $m \leftarrow m + 1$
12:     **end for**
13:     $k \leftarrow k + 1$
14: **end for**

(a) 1-0 Exchange   **(b)** 1-1 Exchange   **(c)** 2-Opt
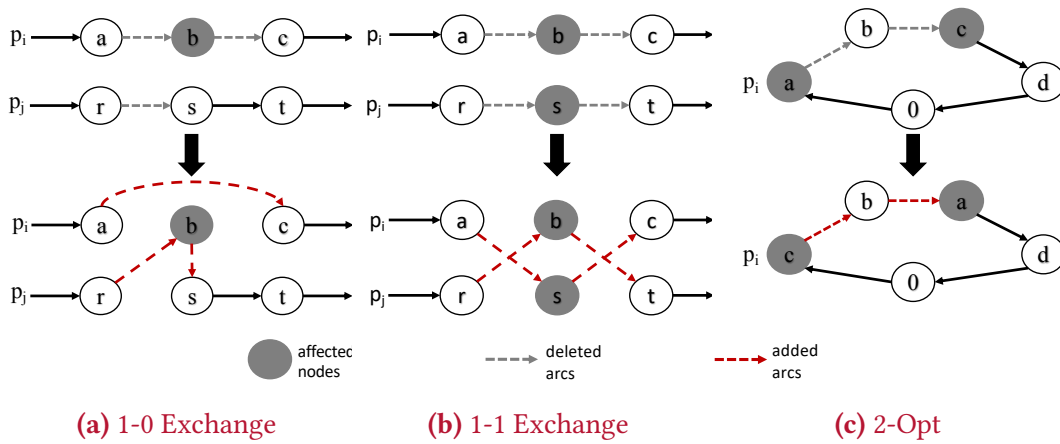
Fig. 4.2: Local Search Operators

Algorithm 3 Simulated Annealing Algorithm

INPUT: y_{2opt}
OUTPUT: y_{mad}

1: $y_c \leftarrow y_{2opt}$
2: Initialize T_{min}, T_{max}, α and L
3: $k \leftarrow 0$
4: **while** $i \neq L$ **do**
5:    **for** $T_k > T_{min}$ **do**
6:       $j \leftarrow U[1,3]$
7:       Generate a solution y_n in the neighbourhood $N_j(y_c)$
8:       Compute $\Delta = J_0(y_n) - J_0(y_c)$
9:       **if** $\Delta \leq 0$ and $e^{\frac{-\Delta}{T_k}} \geq$ random[0,1] **then**
10:         $i \leftarrow i + 1$
11:         $y_c \leftarrow y_n$
12:       **else if** $\Delta > 0$ **then**
13:         $i \leftarrow 0$
14:         $y_c \leftarrow y_n$
15:         $y_b \leftarrow y_c$
16:       **end if**
17:       $T_k = \frac{T_k}{(1+\beta_k T_k)}$, where $\beta_k = \frac{T_s - T_f}{(\alpha + n\sqrt{k})T_s T_f}$
18:       $k \leftarrow k + 1$
19:    **end for**
20: **end while**
21: $y_{mad} \leftarrow y_b$

RoMO
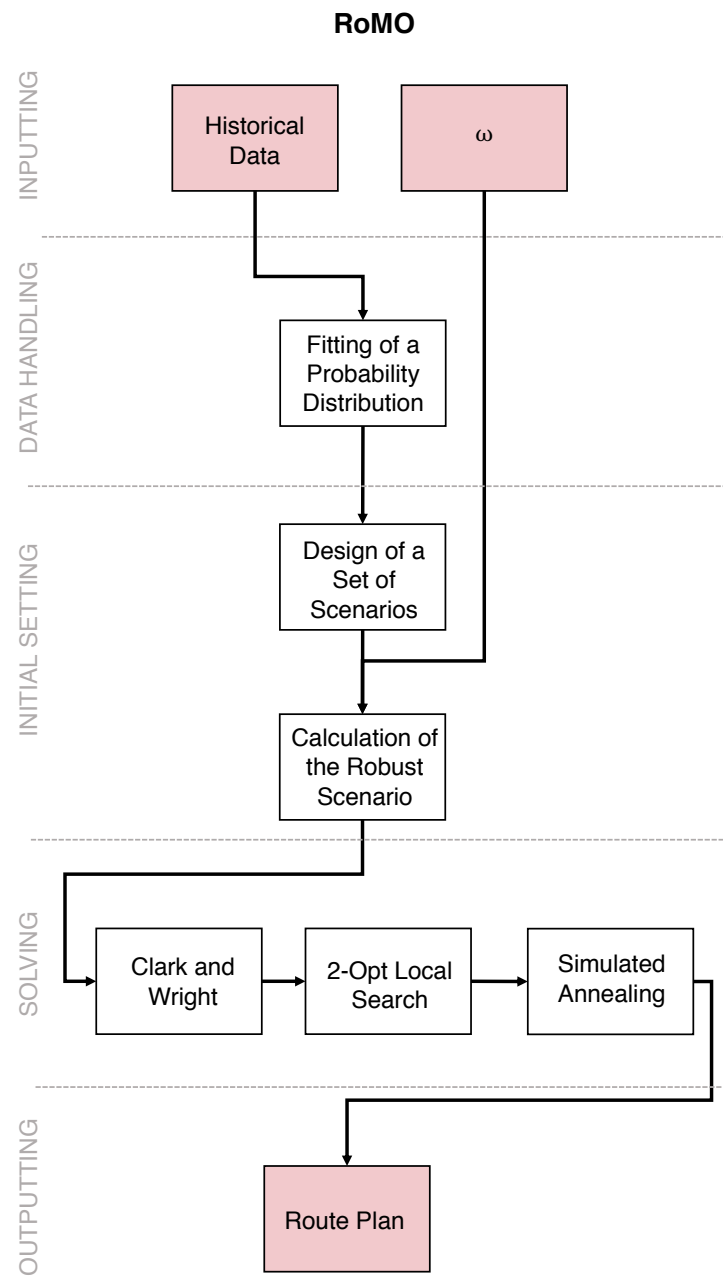
Fig. 4.3: Robust Multi-Objective Solution Approach

4.4 CONCLUSIONS

This chapter treated the static and stochastic CVRP with stochastic demands as a two-stage SPR, where a detour-to-depot is defined as the corrective action which is applied when a failure occurs during the execution of the a priori route plan. Uncertain customers' demands are modeled as a set of discrete samples based on their probability distribution and via Monte Carlo sampling. Each sample defines a deterministic instance of the problem. This allows us to make use of efficient heuristics for the classical CVRP that are well stablished on the literature to solve the instance. In contrast to what have been presented on the literature, the proposed MAD formulation permits deciding between optimality and robustness. The RoMO solution method computes a priori robust route plans that allow for small changes in demands without changing solution structure and losing optimality.

5 COMPUTATIONAL EXPERIMENTS

5.1 INTRODUCTION

This chapter is divided into three parts. First, we present the settings used in the computational experiments, namely the benchmark dataset and the performance measures. Second, using the instances of the benchmark dataset and considering the performance measures, RoMO's performance is compared with the performance of two frameworks to solve the decision-making problem. The first strategy is defined as solving the linear programming formulation (2.1) adopting the expected values of the demands (see Section 2.4.3) as demand input. The other method is a simulation-based solution approach. Similarly to RoMO, this approach addresses demand uncertainty and robustness in the decision-making problem. Finally, in the last part, we analyse the tradeoff between optimality and robustness.

5.2 BENCHMARK DATASET

In the literature on the static and deterministic CVRP, there are famous benchmarks commonly used to evaluate the performance of the solution methods. Yet, for its stochastic counterpart, there is no commonly used benchmark [BBC$^+$06]. Some papers use datasets generated based on the famous deterministic dataset, others develop their own. Here, we follow the latter approach.

When a logistics company uses the SSCVRPD with stochastic demand for designing its operational planning system, it must compile VRP data, such as number of customers (n),

| Instance | Index | Coordinates | | Demands | | |
		x	y	PDF	Data	Mean
1	0	38	5	$U(40, 60)$	$\{42, 45, \ldots, 58\}$	50
	1	84	24		$\{51, 56, \ldots, 46\}$	
	2	56	168		$\{40, 49, \ldots, 60\}$	
	3	190	69		$\{49, 41, \ldots, 57\}$	
	4	84	140		$\{53, 50, \ldots, 58\}$	
	5	76	141		$\{50, 49, \ldots, 58\}$	
	6	103	108		$\{52, 47, \ldots, 46\}$	
	7	178	144		$\{42, 45, \ldots, 56\}$	
	8	83	20		$\{60, 59, \ldots, 48\}$	
	9	116	188		$\{42, 58, \ldots, 44\}$	
	10	19	5		$\{54, 58, \ldots, 51\}$	
	11	130	18		$\{47, 41, \ldots, 53\}$	
	12	26	169		$\{49, 45, \ldots, 51\}$	
	13	25	64		$\{60, 43, \ldots, 41\}$	
	14	72	71		$\{49, 40, \ldots, 48\}$	
	15	113	57		$\{53, 45, \ldots, 59\}$	
	16	138	38		$\{42, 47, \ldots, 43\}$	
	17	186	19		$\{48, 51, \ldots, 58\}$	
	18	185	95		$\{59, 41, \ldots, 48\}$	
	19	145	112		$\{52, 55, \ldots, 60\}$	

Tab. 5.1: Illustration of an Instance (Instance 1)

locations of the depot and the customers (N), transportation cost (c_{ij}), demands (d_i and $i \in N \backslash \{0\}$), and vehicle capacity (C). Thus, these are the inputs that have to be included in the benchmark dataset.

Our dataset contains a set of six instances that are different regarding number of customers and their location and demands. The instances consist of complete graphs of $N =$ 20, 40, 60, 80, 100 and 120 nodes. Considering that the graph is symmetric, a number of $|A| = n(n-1)/2$ arcs was generated for each instance. The customers were randomly distributed in a square, that is, the coordinates x and y of each customer are randomly chosen in $[0, 200]$. The transportation costs (c_{ij}) were assumed to be equal to the Euclidean distance between $i, j \in N$ and rounded to the nearest integer, i.e. $c_{ij} = \sqrt{(i_x - j_x)^2 + (i_y - j_y)^2}$. For all instances and $i \in N$, we considered the demands to be uniformly distributed $d_i \sim U(30, 70)$. For the representation of the historical data on the stochastic demands we generated 100 values of demand for each customer. We adopted that each customer requests an amount of a specific good per day and that demand data from the last 100 working days was available for every customer. Table 5.1 displays one of the created instances.

5.3 Performance Measures

For comparison purposes, we developed seven performance measures, probability of route failure, reliability of the route plan, expected and real number of routes, expected and real transportation cost, and *Price of Robustness*. The performance measures *Price of Robustness*, expected number of routes, and expected transportation cost are characteristics inherited by a route plan. On the other hand, we needed to use Monte Carlo simulation to estimate the probability of route failure, reliability of a route plan, real number of routes, and real transportation cost. All the performance measures are described in the following.

5.3.1 Probability of Route Failure

Considering that a subset $S_j = \{i_1, i_2, \ldots, i_s\} \subseteq N$ of customers is served in a route p_j, i.e. $p_j = (i_0, i_1, \ldots, i_s, i_{s+1})$ and $p_j[0] = p_j[s+1] = 0$, a route failure occurs when the capacity of a vehicle is exceeded while it executes its route p_j, i.e.

$$\sum_{i \in S_j} d_i > C. \tag{5.1}$$

If a route plan y contains a number k of routes, each served by one of the vehicles in the fleet $K = \{1, 2, ..., k\}$, the number of routes that suffer a failure in this route is

$$\eta := \sharp \left\{ j \in K \mid \sum_{i \in S_j} d_i > C \right\}. \tag{5.2}$$

The probability of route failure in this route plan is then

$$P_{route}(\text{failure}) := \frac{\eta}{k}. \tag{5.3}$$

To estimate the probability of route failure, we used the probability distributions that model the demands and Monte Carlo Simulation with $M = 1000$, i.e.

$$P_{route}(\text{failure}) := \frac{\sum_{m=1}^{M} \eta(m)}{M \cdot \sharp k}. \tag{5.4}$$

5.3.2 PLAN RELIABILITY

The reliability of a route plan is defined as the probability that the plan did not suffer a failure. This means that none of the k routes in the route plan failed. Reliability was estimated using the probability distributions that model the demands and Monte Carlo simulation with $M = 1000$ trials. Adopting (5.1), we define the indicator function

$$\chi(m) = \begin{cases} 1, & \text{if (5.1) holds for some } j \in K \\ 0, & \text{else.} \end{cases}$$

This allows us to approximate the reliability of a route plan y via

$$R(y) := 1 - \left(\frac{\sum\limits_{m=1}^{M} \chi(m)}{M} \right). \tag{5.5}$$

5.3.3 EXPECTED AND REAL NUMBER OF ROUTES

The number of routes that a solution calculated via any method that uses preprocessed decisions, such as RoMO, corresponds to what we call the expected number of routes. However, as the problem is stochastic and the proposed solution approach is deterministic, we only know the real number of routes when all customers were attended, i.e. when all stochastic inputs were revealed. Therefore, when one simulates the real demand, the real number of routes can be computed. With the simulated demands we are able to deduct how many times a failure occurs, and thus how often recourse actions need to be applied, revealing the real number of routes. In this thesis, we assume that when a failure occurs and a *detour-to-depot* is applied, then an extra route appears (a route starts and ends at the depot). Thus, the real number of routes is equal to the planned plus extra number of routes.

5.3.4 Expected and Simulated Route Plan Transportation Cost

Similarly to the real number of routes, the real transportation cost is only known when all vehicles finish serving the customers on their routes. By using the probability distributions that model the demands and Monte Carlo simulation we are able to deduct how many times a failure occurs and a *detour-to-depot* is applied, revealing the real transportation costs. If a route failure occurs (5.1) at position $\bar{k}$, the second-stage transportation cost $Q(y)$ is the sum of distances traveled during the detours to the depot, i.e.

$$Q(y) := 2 \sum_{r \in \bar{k}} c_{r0}, \tag{5.6}$$

and the real transportation cost is the sum of the planned transportation cost and recourse transportation distance.

5.3.5 *Price of Robustness*

As explained before (see Section 3.2), the *Price of Robustness* is the price one needs to pay in order to allow for certain deviations within the stochastic variables. It is defined as the additional cost we incur if we apply a robust solution approach, such as RoMO, instead of solving the CVRP as defined in Definition 2.1. Hence, if y is a minimizer for the CVRP according to Problem 2.1 and y_{mad} is a minimizer of Problem 4.1, then the *Price of Robustness* is given by

$$Price := J_0(y) - J_0(y_{mad}). \tag{5.7}$$

Note that the calculation of this price requires solving two instances of CVRP, one defined by the nominal scenario s_0 (4.3) and the other by the robust scenario s_R (see Section 4.3.3).

5.4 Comparison between Approaches

After developing the benchmark dataset and the performance measures, we applied the proposed solution approach on the dataset using a total of $\overline{s}$ = 40 scenarios. RoMO solved every instance using different values of ω ($\omega \in \{0, 1, 5, 10\}$). Thus, four final solutions were calculated per instance, that is, one route plan per chosen ω. These four solutions are different in the matter of level of robustness. RoMO solutions are then compared with solutions obtained via the other two methods. It is important to highlight that in this section we do not want either to define a set of values for ω or an upper bound on it. We want to analyse how solutions designed for different ω perform.

Note that the first-stage of the proposed solution approach is to fit a probability distribution in the customer demand data. To render the approach realistic, we included the fitting for the developed benchmark dataset. For that, we assumed that we do not know the PDF used to generated the instances. After the fitting, we obtained approximated PDF for all customer demands, which are similar to the one used to generate the benchmark.

5.4.1 RoMO versus Deterministic Approach

As described in Section 2.4.3 and discussed in Section 3.2, the decision-making problem can be modeled as a deterministic CVRP. By doing this, one instance of the input data is fed to the linear programming model 2.1. In this instance, the uncertain demands are assumed to be equal to their expected values (nominal values). We call this the Deterministic Approach (DA) because it does not consider any stochastic information that is available about the uncertain demands.

To implement DA, we set $\omega = 0$ and used the MAD formulation. This means that RoMO solutions designed for $\omega = 0$ are solutions for the classical linear programming formulation. Therefore, as we compared the performance of solutions calculated for higher ω with of solutions for $\omega = 0$, we contrasted RoMO solutions with DA solutions.

From our results given in Figure 5.1 one can notice that for all instances as we increased ω less route failures occurred. In turn, the *Plan Reliability* grew for higher ω in Instances 1, 2 and 3, but remained unchanged in Instances 4, 5 and 6. Route plans of same reliability

may have very different *Probability of Route Failure*. For instance, in Instance 4, the route plan obtained using $\omega = 1$ and $\omega = 10$ had the same reliability, but, for $\omega = 1$ route failures occurred in 46% of the routes and for $\omega = 10$ this amount decreased to 23%. A higher reliability of a route plan came associated with a price, as mentioned before, the *Price of Robustness*. For all instances a growth in ω caused an increment on the *Price of Robustness*. But the prices paid for robust solutions were no higher than 11% of the planned transportation costs of the DA solutions.
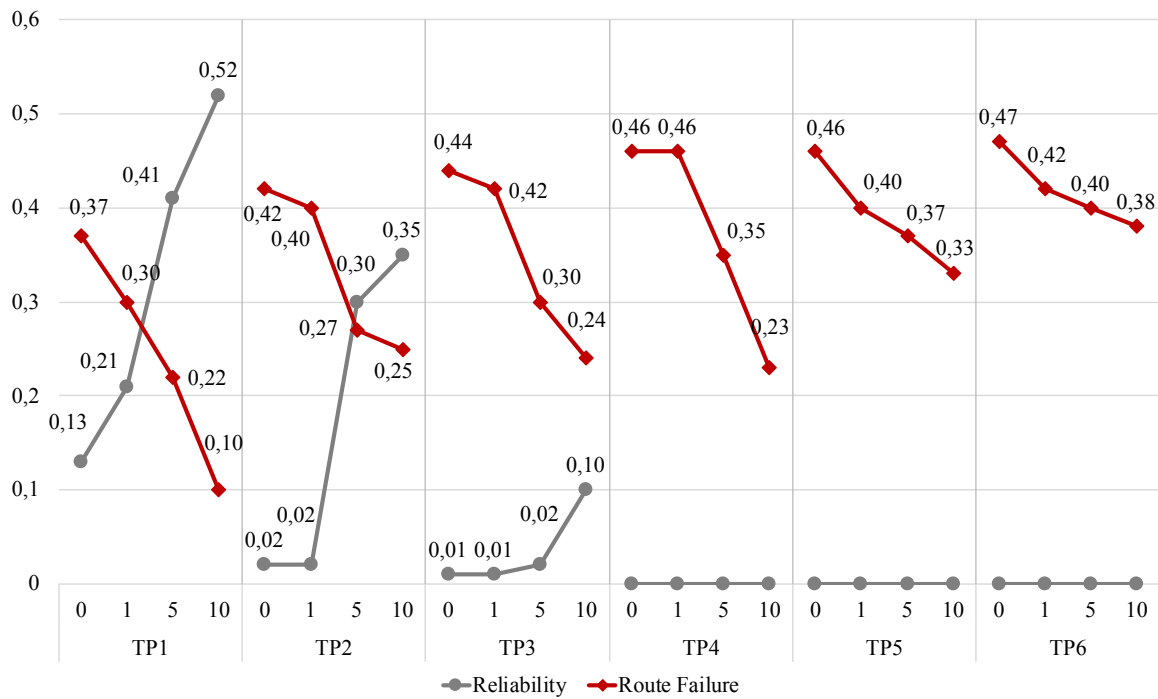
Fig. 5.1: Behaviour of the Performance Measures Route Plan Reliability and Probability of Route Failure among the Instances

The *Real Number of Routes* was higher than the *Planned Number of Routes* for all route plans. This means that detours to depot were applied in all route plans for all instances to meet the real demands. For example, for Instance 1 and $\omega = 8$, the route plan was composed of five routes, see Figure 5.2. Nevertheless, as we simulated the real values for the demands, the route plan became one of six routes, see Figure 5.3. Hence, one route had failed, and therefore more routes were required to attend the same clients. Customer 12 was previously included in the planned route {0-6-5-1-12-0}, but when the real demands were simulated the total demand of this route was higher than expected. Thus, a vehicle needed to attend customer 12 in only one route. Nevertheless, the costs of such detours, i.e.

second-stage transportation costs, were different among the solutions. The DA solutions presented the worst second-stage costs. The second-stage cost declined from $\omega = 0$ to $\omega = 5$ and rose again for $\omega = 10$ in all instances. So did the *Real Transportation Cost* in Instances 3, 4, 5 and 6.

We can then infer that for all instances the higher the ω the better the respective route plan handled changes in the demands. RoMO solutions compared to DA solutions needed less corrective actions to cope with the simulated (real) demands, and they therefore presented smaller second-stage transportation costs.
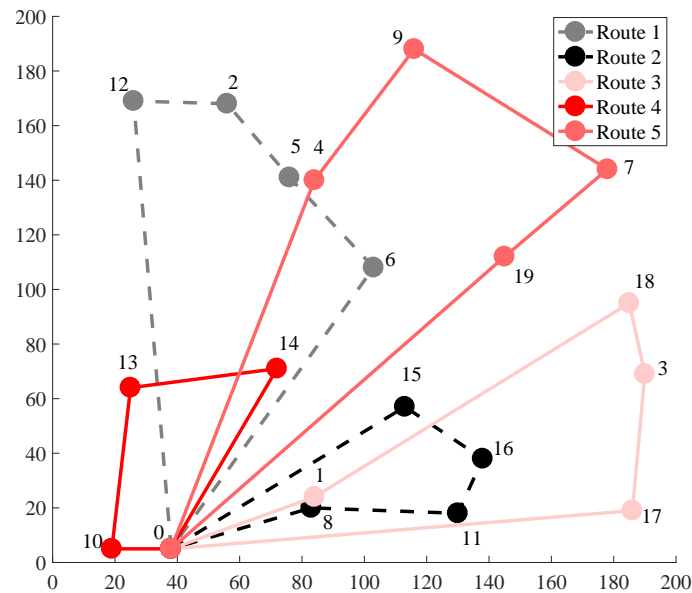

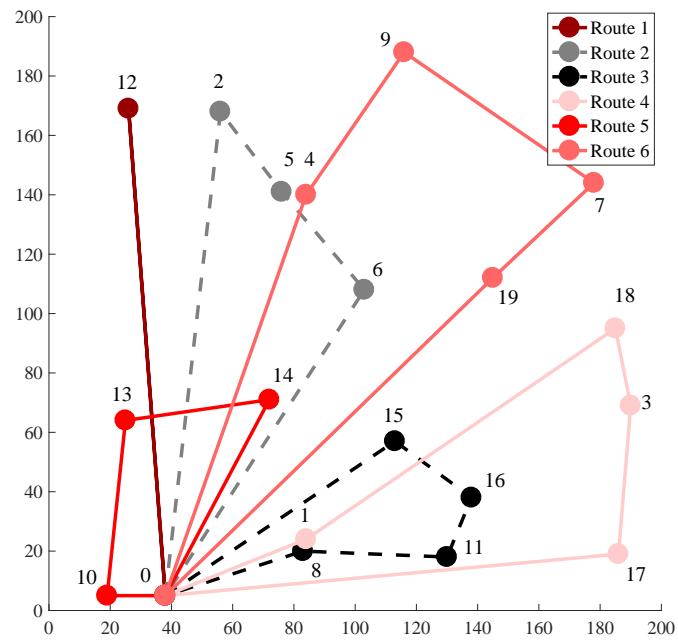**Fig. 5.2:** Route Plan without Route Failure

Fig. 5.3: Route Plan after Corrective Actions Were Applied

Instance	n	ω	Planned Number of Routes	Planned Transportation Cost	*Price of Robustness*	Real Number of Routes	Second-Stage Transportation Cost	Real Transportation Cost
1	20	0	4	1270	-	6	632	1902
		1	4	1300	30 (0,02)	5	160	1460
		5	4	1397	127 (0,10)	5	163	1560
		10	4	1412	142 (0,11)	5	297	1709
2	40	0	7	1780	-	10	972	2752
		1	7	1840	60 (0,03)	10	260	2100
		5	7	1962	182 (0,10)	10	300	2262
		10	7	1932	152 (0,08)	13	622	2554
3	60	0	10	2488	-	15	1289	3677
		1	11	2638	150 (0,06)	14	681	3319
		5	12	2644	156 (0,06)	15	761	3285
		10	12	2720	232 (0,09)	15	746	3466
4	80	0	14	4249	-	21	2941	7090
		1	14	4342	193 (0,04)	20	1079	5421
		5	14	4287	138 (0,03)	20	858	5145
		10	14	4748	399 (0,09)	20	1841	6389
5	100	0	17	3626	-	25	2124	5750
		1	18	3752	126 (0,03)	24	1355	5107
		5	18	3735	109 (0,03)	24	1100	4835
		10	18	3851	225 (0,06)	24	1166	5017
6	120	0	20	4653	-	30	2610	7263
		1	22	4799	146 (0,03)	29	1094	5893
		5	22	4885	232 (0,04)	28	569	5454
		10	23	4834	181 (0,03)	29	1305	6139

Tab. 5.2: Comparison Between RoMO Solutions and DA Solutions

5.4.2 RoMO VERSUS RoSi

Since the DA is the deterministic way to deal with the SSCVRPSD and thus does not consider the impact of uncertainties on the quality and feasibility of the solution, we created a solution approach, called Robust Simulation-Based (RoSi) approach, that has similar goals as RoMO approach. It addresses uncertain demands and it is able to produce more or less robust route plans according to a parameter of choice for decision-makers (ω). The idea behind RoSi is to transform the issue of solving complex stochastic capacitated VRP into a new issue which consists of solving a limited set of deterministic CVRPs. The main framework of RoSi is described in Algorithm and the specific details are explained in the following.

Algorithm 4 Robust Simulation-Based Approach Pseudo Code

INPUT: $n, \omega, \overline{s}$

OUTPUT: y^{RoSi}

1: **for** $i = 1$ to n **do**

2:     input d_i^0

3: **end for**

4: Save scenario s_0 in the set of scenarios S

5: **for** $j = 1$ to b **do**

6:     **for** $i = 1$ to n **do**

7:         $d_i^j \leftarrow$ random $[d_i^0, (1 + \frac{\omega}{10})d_i^0]$

8:         Save scenario s_j in the set of scenarios S

9:     **end for**

10: **end for**

11: $i \leftarrow$ random$[1, \overline{s}]$

12: $s_b \leftarrow s_i$

13: Calculate initial solution y_0 by solving scenario base s_b using C&W

14: Improve initial solution y_0 by using 2-opt local search

15: Save solution y_0 in the set of feasible solutions Y

16: $y_c \leftarrow y_0$

17: Initialize T_{min}, T_{max}, α and L
18: $k \leftarrow 0$
19: **while** $i \neq L$ **do**
20:     **for** $T_k > T_{min}$ **do**
21:         $j \leftarrow U[1,3]$
22:         Generate a solution y_n in the neighbourhood $N_j(y_c)$
23:         Compute $\Delta = J_0(y_n) - J_0(y_c)$
24:         **if** $\Delta \leq 0$ and $e^{\frac{-\Delta}{T_k}} \geq$ random[0,1] **then**
25:            $i \leftarrow i + 1$
26:            $y_c \leftarrow y_n$
27:            Save y_c in Y
28:         **else if** $\Delta > 0$ **then**
29:            $i \leftarrow 0$
30:            $y_c \leftarrow y_n$
31:            $y_b \leftarrow y_c$
32:            Save $y_c \in Y$
33:         **end if**
34:         $T_k = \frac{T_k}{(1+\beta_k T_k)}$, where $\beta_k = \frac{T_s - T_f}{(\alpha + n\sqrt{k})T_s T_f}$
35:         $k \leftarrow k + 1$
36:     **end for**
37: **end while**
38: Compute F
39: **for** $j = 1$ to b **do**
40:     **for** $i = 0$ to $(F - 1)$ **do**
41:         Compute $J^j(y_i)$
42:         Save $J^j(y_i)$ in $J(y_i)$
43:     **end for**
44: **end for**
45: **for** $i = 0$ to $(F - 1)$ **do**
46:     Compute mean $E[J(y_i)]$ and variance $V[J(y_i)]$
47: **end for**
48: $y^{RoSi} \leftarrow$ solution with minimum $V[J(y_i)]$

First, the number of customers n, the degree of robustness ω and the number of scenarios $\bar{s}$ to be used in the approach are defined. Following, the average values for demands are computed (line 1). Different from RoMO, the statistical distribution for the uncertain demands do not need to be known in the RoSi. This means that the decision-maker may use any prediction for the average value. Equivalently to RoMO, a set of scenarios S (see (4.3)) is generated, such that each scenario is a deterministic instance of the stochastic problem (line 4). But here each demand d_i^j is drawn at random in $[d_i^0, (1+\frac{\omega}{10})d_i^0]$, where ω represents a deviation from the nominal scenario. Therefore, scenarios designed with different ω correspond to different growing levels of fluctuation. For instance, for $\omega = 10$ the customer demand are doubled in the worst case. After that, the scenario base s_b is selected out of the set S (line 12). The scenario base is then used to calculate the initial solution (y_0) for the problem. Since s_b is a deterministic instance of the SCVRP we are able to make use of the efficient well established metaheuristics to solve it. We use the same heuristics used in RoMO (see Section 4.3.4). An initial solution is calculated via C&W and improved by a 2opt-local search. After that, SA in applied to further improve it. At each SA iteration one new solution y_n is generated in the neighbourhood of the current solution y_c and the temperature T decreases. Whether y_n is accepted as the current solution or not depends on a acceptance probability (line 24). Every time y_n is accepted, the new current solution becomes a element of a set of feasible solutions Y. This is done until the stopping criterion is met (L iterations without improvement). The number of feasible solutions F is then computed. In the next step, we use the set of feasible solutions Y and of scenarios S to estimate the real transportation cost $J(y_i)$ for each route plan $i \in Y$ due to possible failures (line 39). This is done by using Monte Carlo simulation. The real transportation cost of solution y_i for the scenario j, represented by $J^j(y_i)$, is the transportation cost of the plan y_i when applied to scenario j. That is, $Q(y_i)$ is the cost of *detours-to-depot* applied to route plan y_i for attending the demands specified by scenario j when a route fails. The vector of real transportation costs for solution y_i is

$$J(y_i) = (J^1(y_i), J^2(y_i), J^3(y_i), \ldots, J^{\bar{s}}(y_i)). \tag{5.8}$$

Each vector of costs has a sample of $\overline{s}$ observation costs, an estimation for its expected value $E[J(y_i)]$ and variance $V[J(y_i)]$ are thus calculated (line 46) via sample estimators

$$\widehat{J}(y_i) = \frac{1}{\overline{s}} \sum_{j=1}^{\overline{s}} J^j(y_i), \tag{5.9}$$

$$\sigma^2(y_i) = \frac{1}{\overline{s}-1} \sum_{j=1}^{\overline{s}} (J^j(y_i) - \widehat{J}(y))^2, \tag{5.10}$$

respectively. Finally, the route plan y^{RoSi}, which is the solution y_i with minimum variance, is selected out of the set of feasible solutions Y as the final solution (line48). Indeed, the solution approach optimizes the expected value of the transportation cost, and the solution, therefore, may be sensitive to the uncertainty outcome. That is the reason why we select the solution with minimum variance. The variance represents an estimation of the second stage cost Q (2.9) for solution y_i, so a solution that has the lowest variance among all F solutions in the set of feasible solutions can be said to be the solution that handles best the fluctuations on demands reproduced by the set of scenarios S, i.e. the most robust route plan among all route plans in the set Y.

Figure 5.4 shows the phases included in both RoMO and RoSi approaches. All phases are sorted in 6 steps: *Inputting*, *Data Handling*, *Initial Setting*, *Solving*, *Evaluation*, and *Outputting*. In *Inputting*, all necessary parameters for the approaches are provided, the difference between the approaches in this step is the fact that the RoMO utilizes historical data on demands and the RoSi does not. As mentioned before, in the RoSi the decision-maker may include only a prediction of the expected value for the demands. Following, only RoMO executes data handling when a probability distribution is fit to the historical data. In the third step, the main difference between the approaches is how the deterministic instances of the SSCVRPSD, called scenario base in RoSi and robust scenario in RoMO, are designed. After designing the deterministic instances, they are solved in the next step. Both RoSi and RoMO approaches make use of the same heuristics to solve the scenario base and the robust scenario, respectively. RoMO calculates only one solution for the robust scenario and RoSi

calculates several solutions for the scenario base, creating a set of feasible solutions. That is why the next step is carried only by RoSi when the set of feasible solutions is evaluated by using Monte Carlo simulation and a single solution is selected out of the set. In the last step, RoMO reveals the single solution calculated in the *Solving* step and RoSi delivers the single solution selected in the *Evaluation* step. The final solutions of both RoSi and RoMO are a route plan that can be more or less robust depending on the chosen ω. In both approaches, when $\omega = 0$ we are solving the classical linear programming formulation of the problem (see Definition 2.1).

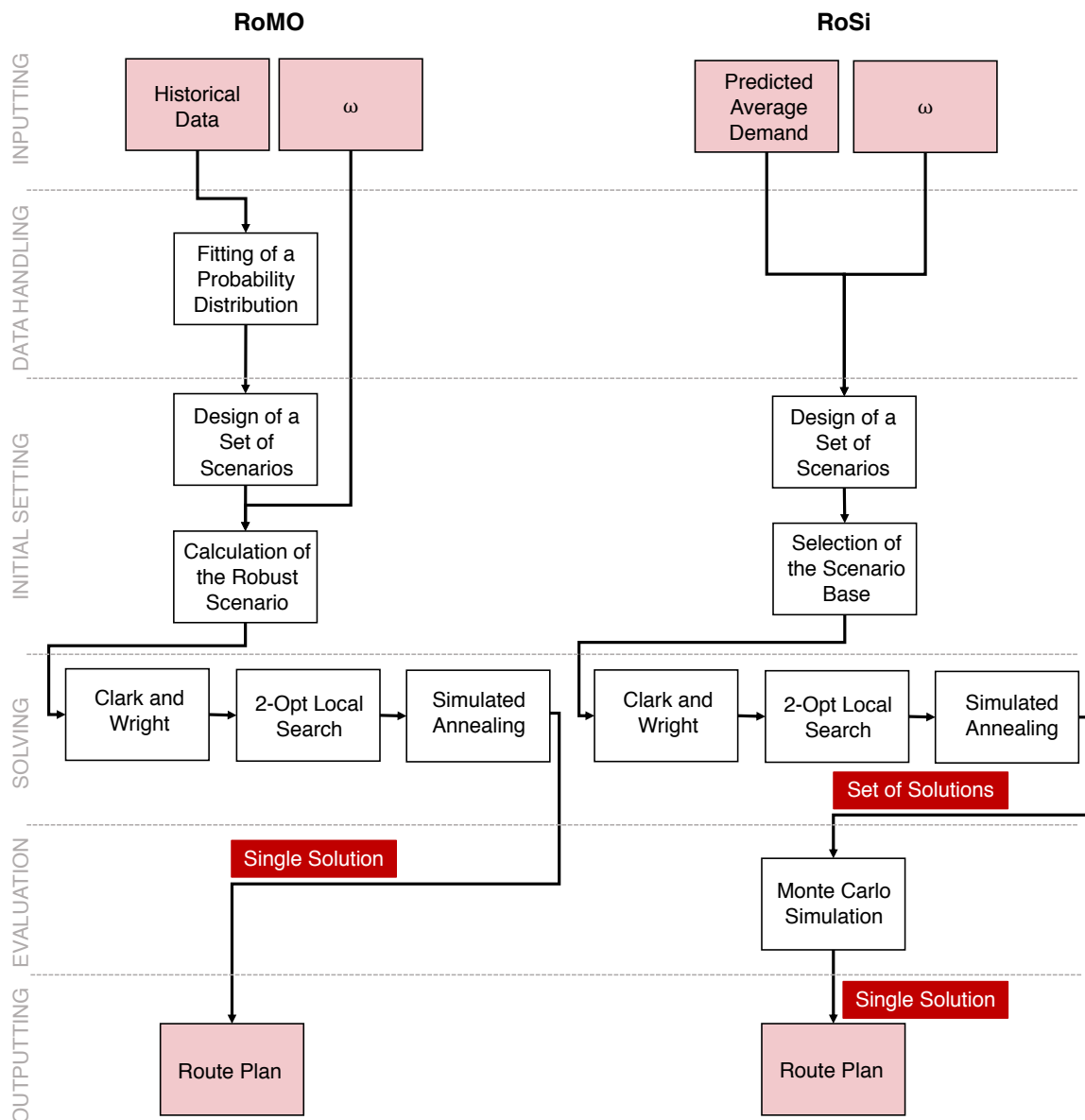**Fig. 5.4:** Proposed Solution Method and Simulation-Based Approach Flowcharts

5.4.2.1 SIMULATION EXPERIMENTS

For comparison purposes, for RoSi we designed a set of $\overline{s} = 40$ discrete scenarios, where all demands were randomly drawn in $[40, 60]$ with average value equal to 50. We used $\omega \in \{0, 1, 5, 10\}$, computing four solutions of different degree of robustness for every instance as well.

Based on the probability distribution of the demands assumed in RoMO and the interval used to draw the demands in RoSi it was possible to measure the upper bound on the number of routes for every instance. As we adopt the worst case demand for a given set of discrete scenarios (or a probability distribution) the upper bound on the number of routes represents the number of routes that should be enough to attend all the customers. The worst case scenario happens when all demands are equal to the highest value in the set of scenarios (or in the distribution). The worst case means 60 for both approaches, and in this situation only up to $300/60 = 5$ customers can be served per route. Therefore, for example, for Instance 1, which has 20 customers, the upper bound on the number of routes is 4. The upper bound on the number of routes as well as the comparison between RoMO and RoSi solutions considering *Planned Number of Routes, Planned Transportation Costs, Price of Robustness* and computational time is shown in Table 5.3.

Since we wanted to compare the performance of RoMO and RoSi solutions when fluctuation on demands arise, we simulated then twice the real demands via Monte Carlo simulation. We adopted first the same probability distribution used in RoMO, i.e. $pd_1 : d_i \sim U(40; 60)$ and then a more spread one, i.e. $pd_2 : d_i \sim U(20; 80)$. The latter means that the simulated demands vary more than in the first. Table 5.4 compares the performance of both approaches using pd_1 and pd_2 by considering the *Real Number of Routes* and *Real Transportation Cost.*

5.4.2.2 COMPARISON ANALYSIS

When $\omega = 0$ both RoMO and RoSi approaches solve the classical linear programming formulation of the problem. That is why the solutions calculated by both approaches per instance for such ω are in most instances alike. For the smallest instance, they designed the

Instance	n	Upper Bound Routes	ω	Planned Number of Routes		Planned Transportation Cost		Price of Robustness		Computational Time (s)	
				RoMO	RoSi	RoMO	RoSi	RoMO	RoSi	RoMO	RoSi
1	20	4	0	4	4	1270	1270	-	-	9111	8976
			1	4	4	1300	1471	30	201	9089	8999
			5	4	4	1397	1514	127	244	9696	9545
			10	4	6	1412	1876	142	606	10931	10790
2	40	8	0	7	7	1941	1780	-	-	20735	20509
			1	7	8	1840	2136	60	195	21235	20902
			5	7	9	1962	2311	182	370	22150	21999
			10	7	12	1932	2749	152	808	22964	22756
3	60	12	0	10	10	2488	2525	-	-	38498	38234
			1	11	12	2638	2847	150	322	40065	39877
			5	12	14	2644	3108	156	583	44526	44167
			10	12	17	2720	3605	232	1080	51300	51001
4	80	16	0	14	14	4249	4867	-	-	51935	51667
			1	14	16	4342	5467	193	600	57359	57100
			5	14	19	4287	5971	138	1104	58172	57890
			10	14	21	4748	6780	399	1913	62485	62200
5	100	20	0	17	17	3626	3788	-	-	73170	72865
			1	18	20	3752	4219	126	431	77310	76965
			5	18	23	3735	4696	109	908	77828	77435
			10	18	28	3851	5373	225	1585	86120	85786
6	120	24	0	20	20	4653	4974	-	-	101424	100856
			1	22	24	4799	5487	146	693	114966	114501
			5	22	27	4885	5829	232	1035	115619	115402
			10	23	33	4834	6921	181	2127	121385	120876

Tab. 5.3: Comparison Between Planned RoMO and RoSi Solutions

Instance	n	ω	pd_1 Real Number of Routes RoMO	RoSi	Real Transportation Cost RoMO	RoSi	pd_2 Real Number of Routes RoMO	RoSi	Real Transportation Cost RoMO	RoSi
1	20	0	6	6	1902	1902	6	6	2767	2767
		1	5	4	1460	1471	5	5	2484	2217
		5	5	4	1560	1514	5	7	2390	2191
		10	5	6	1709	1876	5	7	2710	2141
2	40	0	10	10	2752	2908	10	10	4136	4202
		1	10	8	2100	2136	10	9	3808	3875
		5	10	9	2262	2311	10	10	4009	3265
		10	13	12	2554	2749	10	13	4481	2963
3	60	0	15	15	3677	3986	15	15	6146	6272
		1	14	12	3319	2847	14	14	5539	4832
		5	15	14	3285	3108	15	15	5287	4505
		10	15	17	3466	3605	15	18	5489	4092
4	80	0	21	21	7090	7386	21	21	10996	11126
		1	20	16	5421	5467	21	18	9806	10601
		5	20	19	5145	5971	21	21	9837	9815
		10	21	21	6389	6780	21	22	11005	7565
5	100	0	25	25	5750	6030	25	25	8817	8950
		1	24	20	5107	4219	25	22	8639	7476
		5	24	23	4835	4696	25	24	8365	6514
		10	24	28	5017	5373	24	29	8620	6181
6	120	0	30	30	7263	7849	30	30	12895	12400
		1	29	24	5893	5487	30	27	11784	10556
		5	28	27	5454	5829	29	29	11334	9051
		10	29	33	6139	6921	30	34	12023	7969

Tab. 5.4: Comparison Between Real RoMo and RoSi Solutions Adopting Different Demand Distributions

same route plan, computing equal planned number of routes and transportation cost, and real number of routes and transportation cost. When $\omega \in \{1, 5, 10\}$ for all instances (except Instance 1) the Robust Simulation-Based approach calculated solutions with more routes and higher transportation costs compared to the proposed approach. It can be seen that for RoSi the higher the ω, the higher the planned number of routes and the transportation cost are, and that solutions designed for $\omega \in \{5, 10\}$ have more routes than the upper bound on the number of routes. These two situations do not occur in the RoMO and affect the *Price of Robustness*. The RoSi approach makes up route plans that exhibit higher *Price of Robustness* because it requires more routes to attend the same amount of customers. This indicates that this approach tries to service fewer customers in the same route in order to be safer against a worse-case scenario.

This safety does its job in some situations. For example, considering pd_1, for all instances and $\omega \in \{1, 5\}$, RoSi solutions required fewer routes to deal with the real demands. In fact, for all ω, RoSi solutions presented same planned and real number of routes and transportation cost. This suggests that the solutions designed by the Robust Simulation-Based approach did not suffer any failure when the real demands were simulated, i.e. no recourse actions were necessary. In contrast to that, all the solutions drawn by the proposed approach needed extra routes. However, for the other omega ($\omega = 10$) RoSi solutions did show a higher planned (and real) number of routes and transportation cost compared to the real number of routes and transportation cost of RoMO solutions, see Figure 5.5a and Figure 5.6a. This indicates that a higher level of robustness might not be desirable, especially when a bad scenario does not become reality. Nevertheless, considering a worse-case scenario, namely pd_2, a higher degree of robustness might pay off. This is shown by Figure 5.5b and Figure 5.6b. For $\omega \in \{1, 5, 10\}$ and all instances RoSi solutions not only required fewer extra routes and shorter second-stage transportation cost to deal with the real demands, but also displayed smaller real (planned plus second-stage) transportation cost compared to RoMO solutions.

It can be seen that for the more spread probability distribution (pd_2), RoSi solutions designed for $\omega = 10$ showed more routes compared to RoMO solutions and still had a smaller

real transportation cost. This situation arises because fewer routes failed in one RoSi route plan than in one RoMO route plan. For example, for Instance 5 ($\omega = 10$) 28 routes were drawn by RoSi approach to attend 100 customers (see Table 5.3), however one extra route was necessary when the real demands were simulated (29 real routes). It means one route in the route plan failed. On the other hand, 18 routes were designed by the RoMO to serve the same 100 customers, but required 24 to cope with the real demands, i.e. 6 routes failed in the route plan. Therefore, more *detours-to-depot* were performed and higher second-stage transportation cost incurred (see Figure 5.6b).

In fact, Figure 5.5b and Figure 5.6b show that for RoSi solutions a growth in ω causes a decrease in the number of extra routes and in the second-stage transportation cost. In other words, the higher the ω the higher the level of robustness is. However, for RoMO solutions this was not the case. Although RoMO solutions for $\omega \in \{1, 5, 10\}$ required fewer extra routes and lower recourse distance compared to solutions for $\omega = 0$. The most robust RoMO solutions among all RoMO solutions, that is solutions that required fewer extra routes and lower second-stage transportation cost, occurred either for $\omega = 1$ or $\omega = 5$. As we compared the real transportation cost when pd_1 the most robust RoMO solutions ($\omega = 1$ or $\omega = 5$) also computed the best real transportation cost among all (RoMO and RoSi) solutions. However, when pd_2 the solution with best real transportation cost was calculated by RoSi (see Table 5.3).

Finally, pondering the computational time it can be noticed that the proposed solution method performs better than the Robust Simulation-Based approach. But the differences were not so significant. The longer computational time required by RoSi happens because every solution that was generated at each SA iteration is used for computing its vector of cost (39-44) by means of Monte Carlo simulation in the evaluation step (see Figure 5.4).

5.5 Tradeoff Analysis

Maximization of the optimality and increment of the robustness are two conflicting objectives. In the MAD formulation (4.1), optimality is represented by the first term of the objective function, i.e. planned transportation cost and robustness is expressed by the sec-
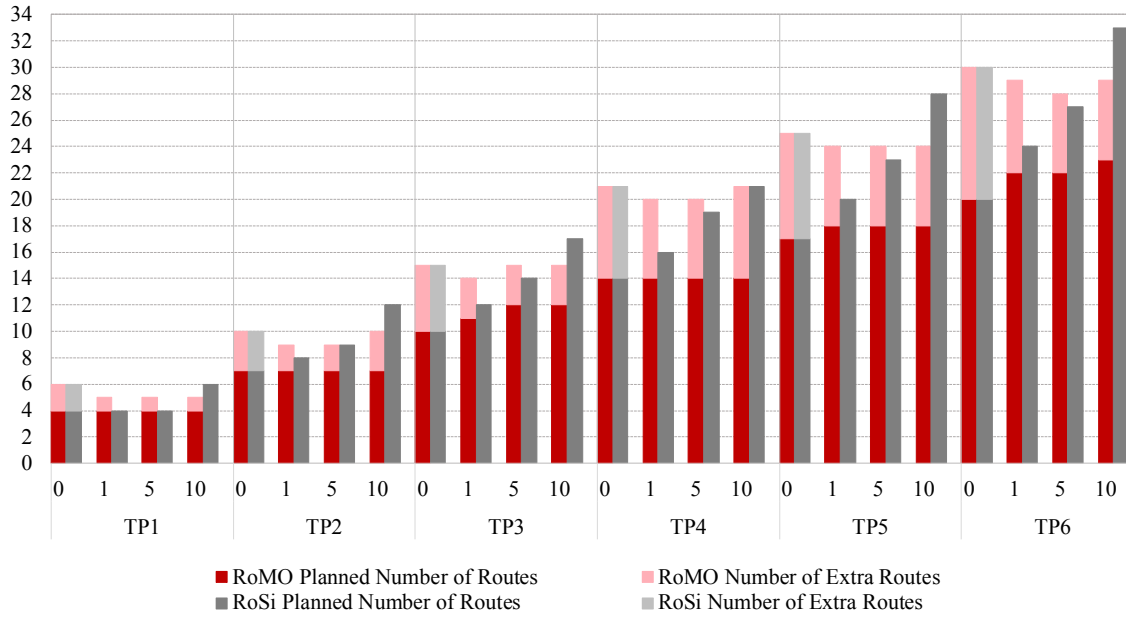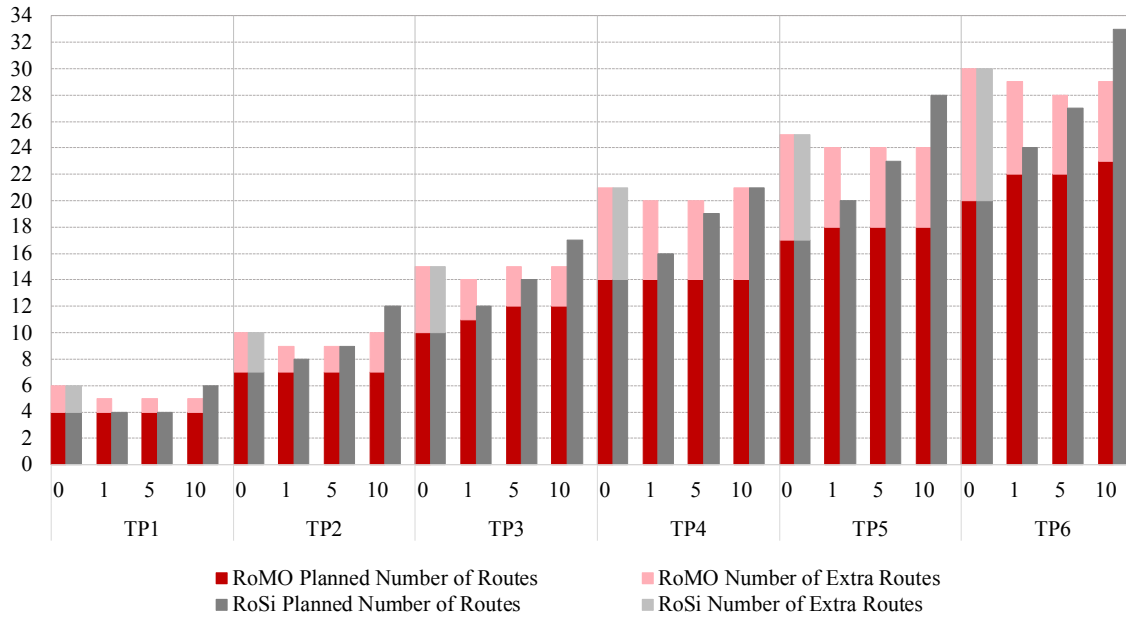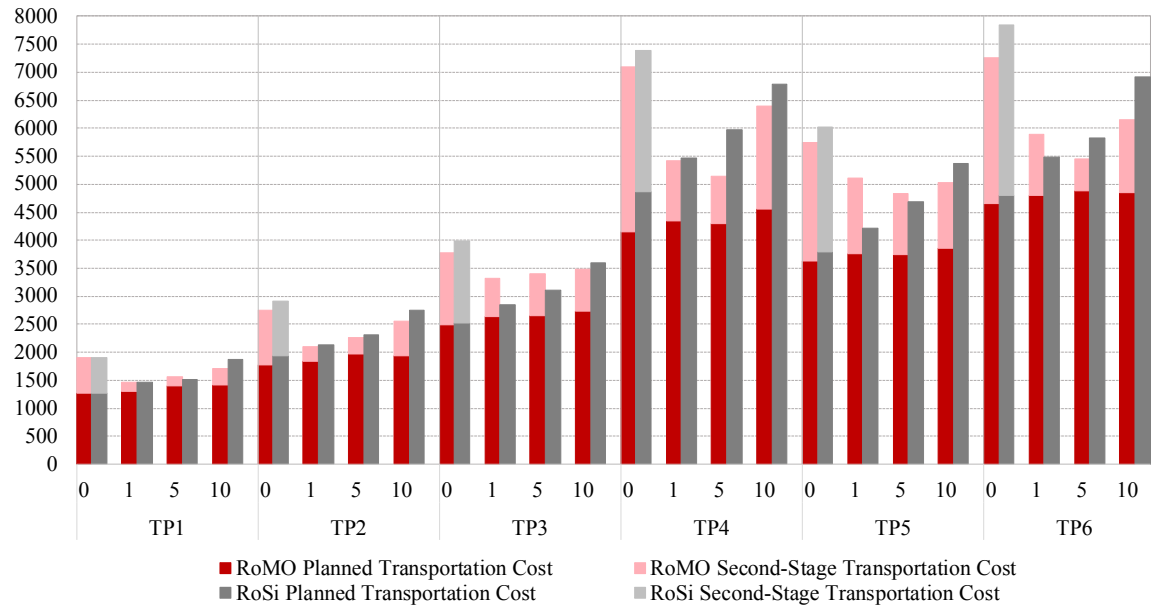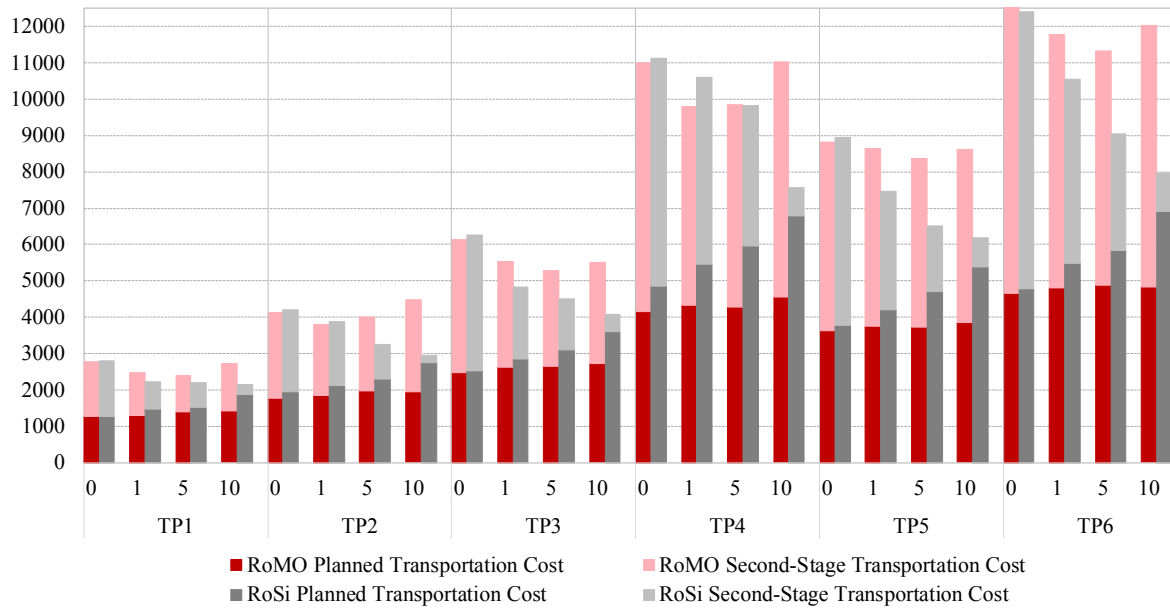
(a) pd_1

(b) pd_2

Fig. 5.5: Planned and Real Number of Routes per Instance for Both RoMO and RoSi Solutions

(a) pd_1

(b) pd_2

Fig. 5.6: Planned and Real Transportation Costs per Instance for Both RoMO and RoSi Solutions

ond one, i.e. second-stage transportation cost. This means that the planned transportation cost of any non-dominated solution in the Pareto front set can only be improved by deteriorating the second-stage transportation costs. Hence, a trade-off exists between these two objectives. The planned and second-stage transportation costs of the solutions designed for $\omega \in \{0, 1, 5, 10\}$ and for each instance can illustrate this trade-off. Figure 5.7 shows the trade-off between optimality and robustness as the route plans become more robust with respect to variations on the demands.

As predicted, we observe that as ω grew from 0 to 1, the planned transportation cost grew and the second-stage transportation cost declined in all instances. The same happened as ω increased from 1 to 5 in instances 1, 2, and 6. In instances 1, 3, 4, and 5, the first-stage transportation cost augmented when ω rose from 5 to 10, but the second-stage transportation cost did not lessen. This shows that, as indicated before, a higher level of robustness might not pay off.

Since a trade-off is observed between planned and second-stage transportation cost, the crucial decision is to find a balance for these two costs. As mentioned before, the MAD formulation takes the configuration of a multi-objective optimization problem, more precisely a bi-objective optimization problem, where the first objective is the minimization of the first-stage transportation cost, and the second is the minimization of the second-stage transportation cost. In the previous section, we did not regard the problem as a multi-objective optimization problem and this is not the aim of this thesis. But in order to find a solution that calculates the best trade-off between optimality and robustness, we exploit some concepts of such problems.

A common procedure to multi-objective optimization problems is to combine the individual objective functions into a single one via weighted sum method. Using this method involves selecting scalar weights W_i for each individual objective function, such that

$$\sum_{i=1}^{l} W_i = 1, \tag{5.11}$$

(a) Instance 1

(b) Instance 2

(c) Instance 3

(d) Instance 4

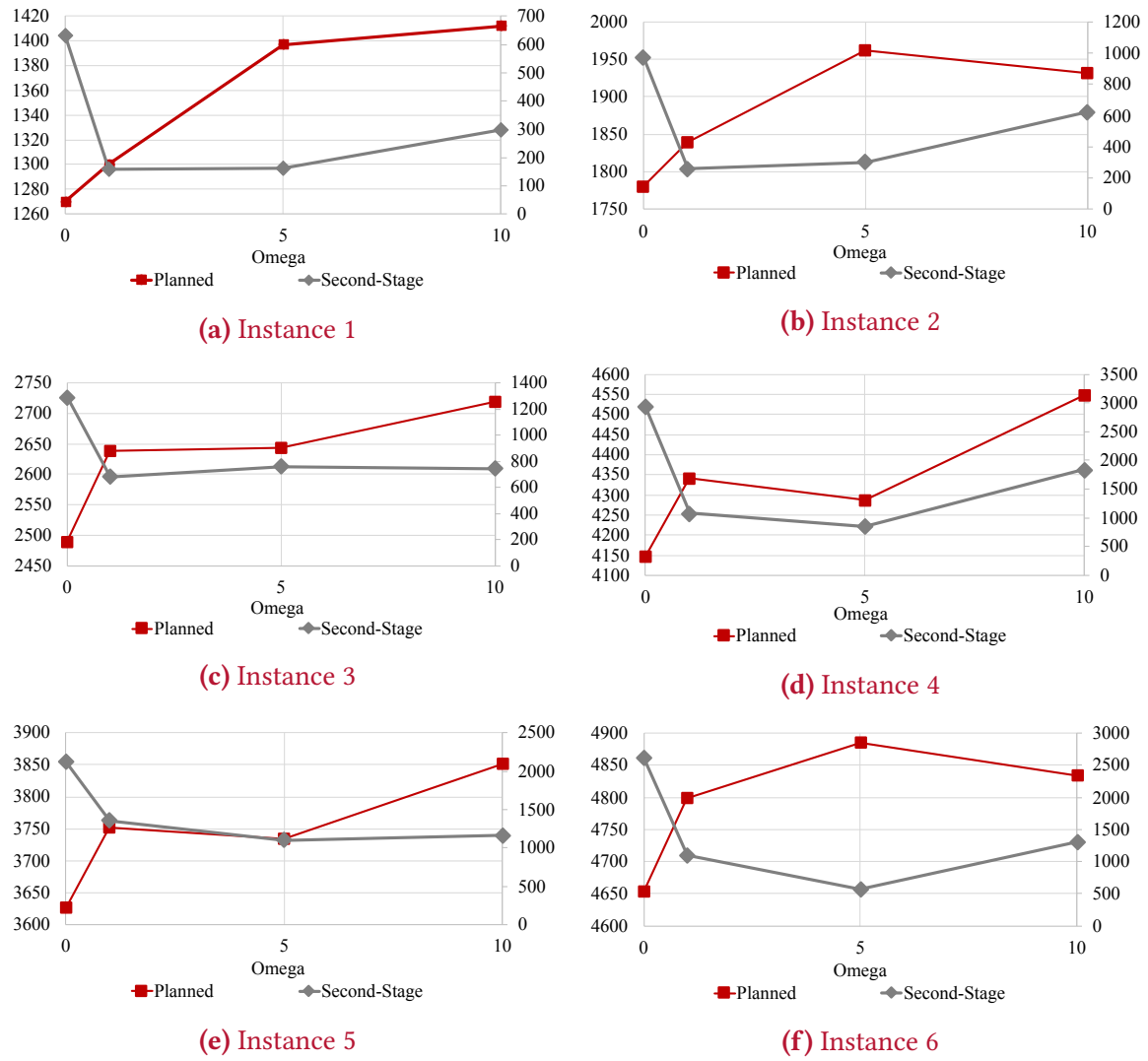**(e)** Instance 5

(f) Instance 6

Fig. 5.7: Trade off between Planned Transportation Cost and Second-Stage Transportation Cost per Instance

in order to form the objective function

$$U = \sum_{i=1}^{l} W_i F_i(x), \qquad (5.12)$$

where l is the number of objectives in a given problem [MA10]. The weights characterize the decision-maker's preferences on the conflicting objectives. Based on that, we adjusted the objective function of the MAD formulation (Definition 4.1). The interval defined for the decision-maker's parameter of choice ω was changed to $\omega \in [0, 1]$. The parametrized objective function is then

$$\min_{y} J_{mad}(y) := \min_{y} \omega J_0(y, s_0) + (1 - \omega)\frac{1}{\bar{s}}\sum_{j=1}^{\bar{s}}[Q(y, s_j) - Q(y, s_0)] \quad \omega \in [0, 1]. \quad (5.13)$$

To solve the parametrized MAD formulation, the weight ω was discretized into a finite number of values, computing 19 different weighting combinations $[W_1 = 0.05, W_2 = 0.95]$, $[W_1 = 0.1, W_2 = 0.9], \dots, [W_1 = 0.95, W_2 = 0.05]$, where W_1 prioritizes optimality and W_2 robustness. For each of these combinations, we solved the six instances by applying the same heuristics described in the solving step (see Section 4.3.4). Although in here, SA searched for a solution y that minimized the parametrized MAD objective function (5.13), instead of the first-stage cost $J_0(y)$. For each weight vector a single solution y was obtained. For each one of the 19 solutions calculated per instance a planned and a second-stage transportation costs were computed. As it was done before, we also simulated the real transportation cost of each one of the 19 solutions by means of Monte Carlo simulation and using the probability distributions that govern the demands. Figure 5.8 presents planned, second and real transportation costs of each instance for the 19 weight combinations.

From Figure 5.8 it can be seen that as ω grew, and W_2 consequently decreased, the second-stage transportation cost rose. Since we were favoring optimality, route plans selected for such higher ω were solutions of lower first-stage transportation cost. Thus, it was expected that the first-stage transportation cost declined as ω increased. Nonetheless, these solutions needed more recourse actions during the second stage. We also observed that in all instances the route plan with the best planned transportation cost among the 19 solutions
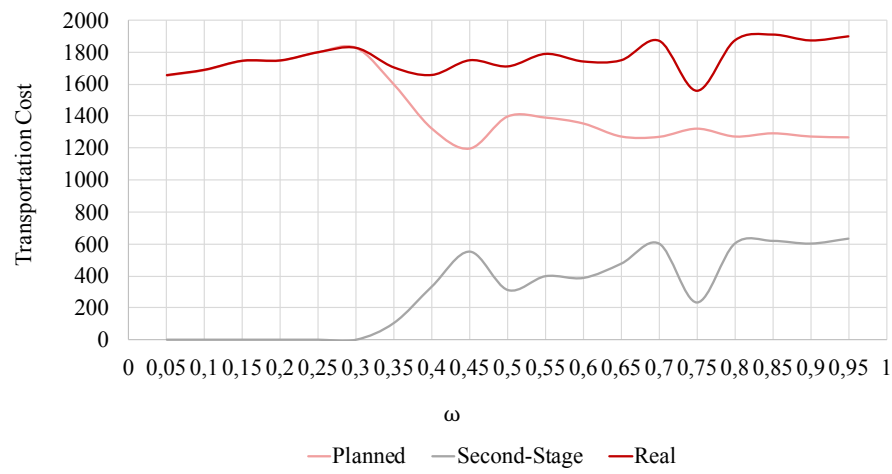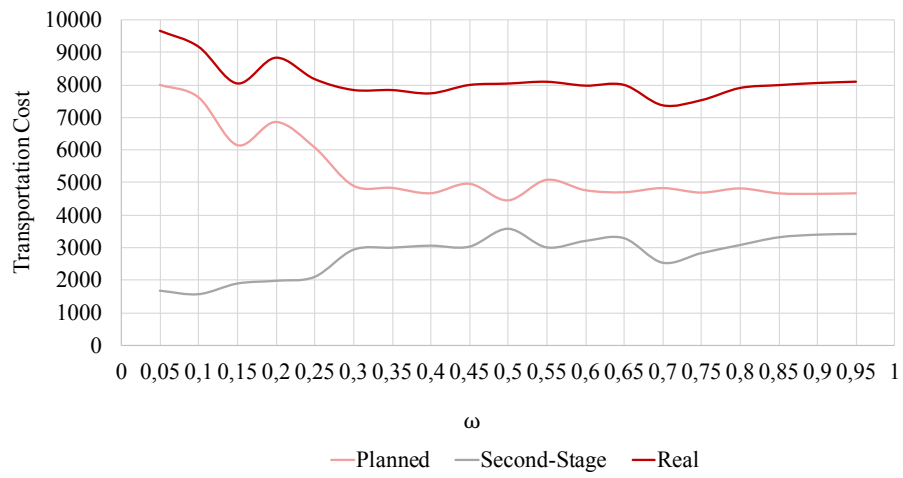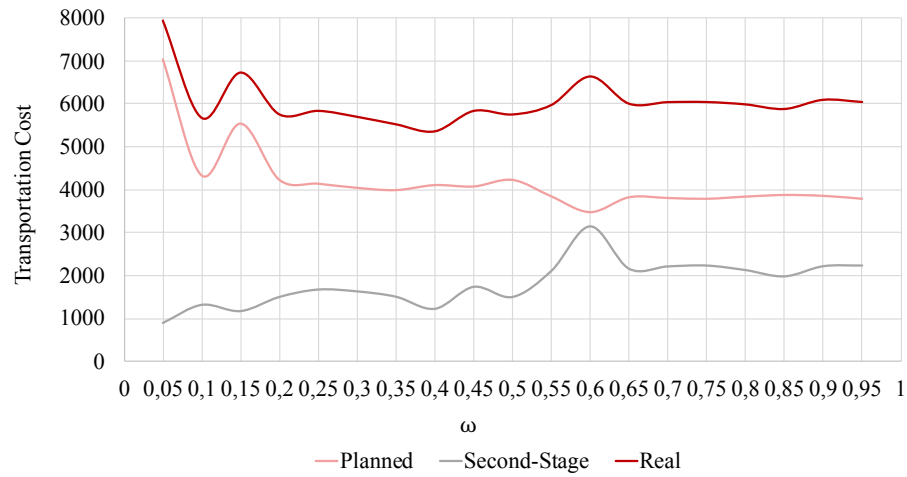
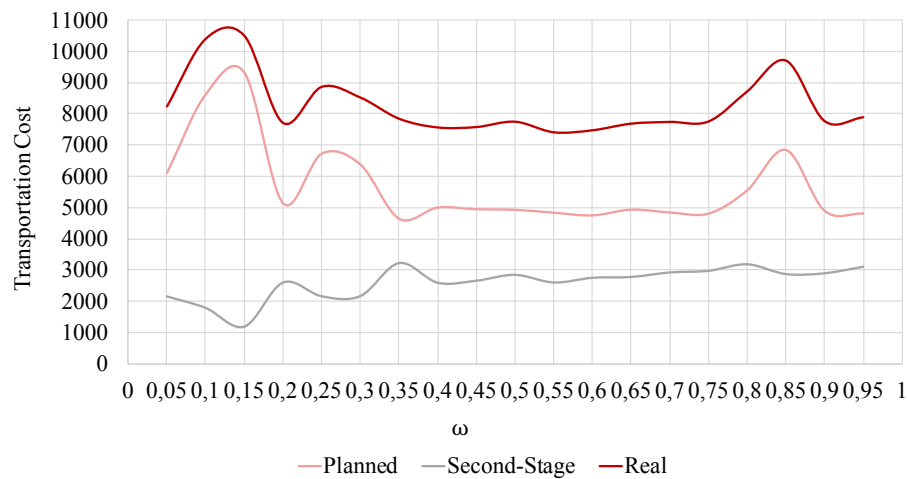**(a)** Instance 1
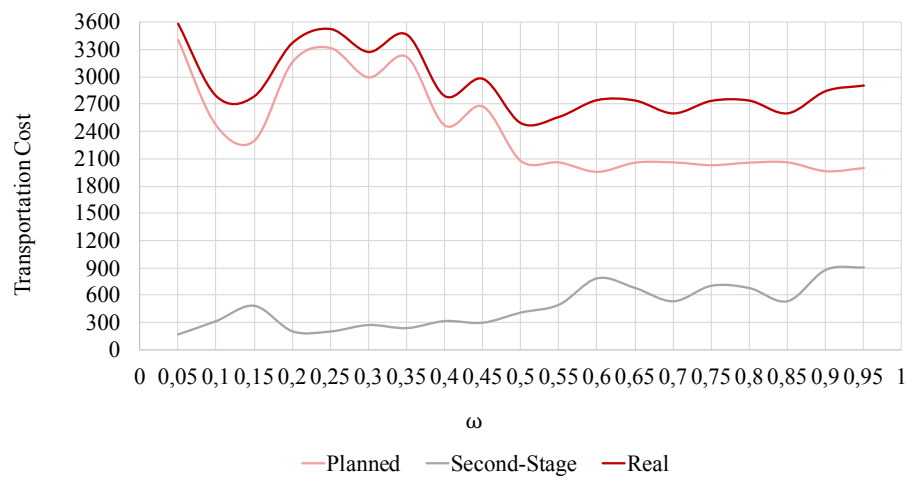
(b) Instance 2
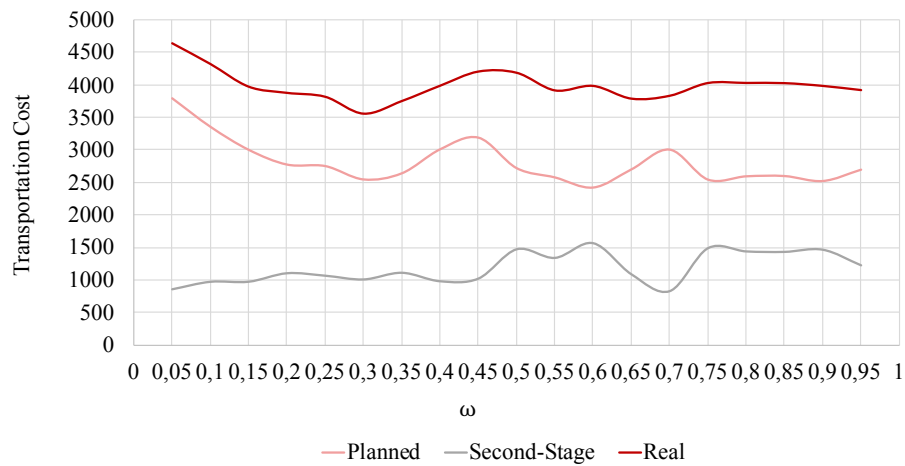
(c) Instance 3

(d) Instance 4

(e) Instance 5

(f) Instance 6

Fig. 5.8: Planned, Second-Stage, and Real Transportation Cost for Parametrized ω

was the one with the highest recourse transportation cost. This route plan turned then into a solution with high real transportation cost. Moreover, the ω that designed the best solution for the SSCVRPSD, that is, the route plan with the smallest real transportation cost, was different in each instance. For example, the best route plan for Instance 1 was calculated when $\omega = 0.75$ and for Instance 3 when $\omega = 0.30$.

5.6 Conclusions

In this chapter, we present the application of the RoMO solution approach in a set of developed instances and the comparison between RoMO's performance and those of two approaches. One of the approaches (DA) treated the decision-making problem as deterministic and it thus does not take into account the impact of uncertainties on the quality and feasibility of the solutions. The other (RoSi) was designed with the same goal of RoMO and is therefore also capable of calculating solutions that are not overly conservative. For comparison purposes, we developed seven performance measures. The *Price of Robustness*, expected number of routes, and expected transportation cost were built-in features of any route plan. The remaining performance measures were estimated by means of Monte Carlo simulation and using the probability distribution that models the demands. The comparison between RoMO and DA solutions showed that the proposed approach provided significant improvements over DA. It is evident that RoMO designed robust route plans. That is, as ω increased the *Plan Reliability* grew and the *Probability of Route Failure*, *Price of Robustness* and *Real Transportation Cost* diminished. The robust solutions were not associated with an high *Price of Robustness*. The prices payed for robust route plans were no more than 11% of the first-stage transportation cost in all instances. This comparison points out that robust route plans, i.e. solutions that were less sensitive to uncertainty than the linear programming solution were possible at very little cost. The comparison between RoMO and RoSi solutions demonstrated that for RoSi approach the higher the weight ω the higher the robustness of solutions designed for such ω. But increment on ω did not mean proportional addition to robustness for the RoMO. In some cases, a higher level of robustness offered by RoSi approach was dispensable. When we assumed customer demands pd_1, RoSi solutions

designed for $\omega = 1$ showed the best real distance. This means that RoSi solutions designed for higher $\omega \in \{5, 10\}$ were conservative, i.e., presented a higher level of robustness without being necessary, though when demands were modelled with the more spread distribution (pd_2) this robustness payed off. For this reason, when pd_1 RoMO solutions performed better than RoSi solutions considering real distances, in turn when customer demands were modelled by the more spread probability distribution (pd_2) RoSi solutions performed better. Moreover, in this chapter, we showed that there exists a trade-off between robust and optimality in the context of the decision-making problem. The existence of this trade-off was intuitive, but the proposed formulation provided a means of quantifying the trade-off and determining a good decision for varied levels of ω. The parametrized MAD formulation was able to design solutions that calculate the best trade-off between first and second-stage transportation costs, i.e. to calculate a route plan of lowest real transportation cost.

6 General Conclusions and Future Research Directions

The decision-making problem under uncertainty addressed in this thesis was the static and stochastic capacitated vehicle routing problem with uncertain demands. In this problem, the routing decisions are separated into two groups, first-stage and second-stage decisions. The first-stage decisions are made before the true values of the stochastic demands are revealed, while the second-stage decisions (corrective/recourse actions) are made after the actual values become known and intend to recover the first-stage decisions that are no longer feasible. In the context of the SSCVRPSD, corrective actions are applied if/when a route failure occurs, that is, if/when the true total demand on a route is higher than the vehicle capacity. The most commonly used recourse actions are *detour-to-depot* and *preventive restocking*. In the former, the vehicle returns to the depot to load when its capacity is depleted. Since it is preferable that the vehicle capacity is depleted at a customer located near the depot, an en route replenishment, called *preventive restocking*, may be performed at strategic nodes along the route before a route fails. Thus, these corrective actions represent extra transportation costs. The goal of the SSCVRPSD is then to find a robust a priori route plan of minimum transportation cost, i.e., a priori solution that allows for small changes in demands without changing the solution structure and losing optimality. Protection against fluctuations in the demands (robustness) typically comes at a price (*Price of Robustness*). It is therefore decisive to weigh the possible decrement on the objective function value involved in implementing a robust a priori route plan against the likely reduction of flexibility in adopting the "optimal" route plan.

For that, the problem was formulated based on a mean absolute deviation objective func-

tion, in which the first goal is the minimization of the expected value of the planned (first-stage) transportation cost and the second is the minimization of the mean absolute deviation of the second-stage (corrective actions) transportation cost. The first objective is a measure for optimality while the second for robustness. In this work, we assume that if/when a route failure occurs, the *detour-to-depot* is applied. In the MAD model, the variability term is multiplied by a parameter ω to be chosen by a decision-maker and used to obtain a spectrum of route plans that can be more or less robust. In this way, the MAD mathematical formulation incorporates a measure of the decision-maker's level of risk aversion in the objective function. We also propose a solution method based on the MAD decision model. Since in this solution method the SSCVRPSD is reduced to its deterministic counterpart, the solution approach can make use of efficient and well-established heuristics for the classical CVRP. So, solving the decision-making problem using the RoMO solution approach is no more difficult than solving a single deterministic instance of the CVRP while satisfying all uncertain demands.

To examine the efficiency of the proposed solution approach, we developed a benchmark data set with six instances, seven performance measures and compared its performance with those of two solution approaches, Deterministic Approach and Robust Simulation-Based solution approach. DA is the most commonly used approach to deal with a decision-making problem under uncertainty, thought it does not take into account the effect of the uncertain inputs on the feasibility and optimality of the solutions. In this approach, the values of the stochastic inputs are assumed to be equal to their expected values so that the linear programming formulation of the problem can be solved. On the other hand, RoSi is a solution approach with similar goals as RoMO. In this stage, because we wanted to compare solutions of different degree of robustness, we selected different values of $\omega \in \{0, 1, 5, 10\}$. Each instance includes a number of customers, locations of the depot and the customers, transportation cost, demands, and vehicle capacity. The performance measures were probability of route failure, reliability of the route plan, expected and real number of routes, expected and real transportation cost, and *Price of Robustness*. The performance measures *Price of Robustness*, expected number of routes, and expected transportation cost are char-

acteristics inherited by a route plan. For estimating the other performance measures we used Monte Carlo simulation together with the probability distributions that model the uncertain demands. The computational experiments show that RoMO provided significant improvements over DA. It is evident that the proposed method provided route plans that are more robust than the solutions for the classical linear programming formulation. In all instances, solutions designed for $\omega = 0$ presented not only the lowest plan reliability but also the highest probability of route failure and worst real transportation cost. Considering that the real transportation cost it is the true transportation cost that companies are subjected to for attending their customers, one concludes that in uncertain situations is better to hold a certain level of robustness than otherwise. The efficiency of RoMO solutions while dealing with fluctuation in the demands was achieved at a small cost, i.e. the *Price of Robustness* was lower than 11% of the transportation cost of the respective a priori route plan in all instances. This means that RoMO solutions are not overly conservative and therefore would not be regarded as unnecessary by decision-makers. Compared with RoSi, which is a solution method to design solutions of different degree of robustness based on a decision-maker's parameter of choice ω, we noticed that RoSi solutions can be conservative for higher ω when the "normal" scenario occurred. A "normal" scenario (pd_1) refers to when the probability distributions used to simulate the demands are assumed to be equal to the fitted PDF. In this case, these solutions presented a higher level of robustness than necessary. Nevertheless, when the "worse" scenario (pd_2) occurred, i.e., probability distribution with larger spread was used to simulate the demands, this robustness payed off. In this scenario, RoSi solutions were less affected by the fluctuation in the demands compared to RoMO solutions, requiring fewer corrective actions (extra routes). For pd_1, RoMO solutions designed with $\omega = 1$ or $\omega = 5$ were the ones with the best real transportation cost. In contrast, for pd_2, RoSi solutions calculated with $\omega = 10$ were the route plans with the best real transportation cost. One may then conclude that the choice between RoMO or RoSi solution approaches depends on the degree of uncertainty present in the demands.

After comparing RoMO solutions with both DA and RoSi solutions for different degrees of robustness, i.e. $\omega \in \{0, 1, 5, 10\}$, we discretized the MAD formulation and changed ω

to $\omega \in [0, 1]$. 19 weighting combinations $[W_1 = 0.05, W_2 = 0.95]$, $[W_1 = 0.1, W_2 = 0.9], \ldots, [W_1 = 0.95, W_2 = 0.05]$ in which W_1 prioritized optimality and W_2 robustness were used. In this manner, the parametrized decision model provided a way of quantifying the trade-off between the two conflicting objectives, first (optimality) and second-stage (robustness) transportation cost, and calculating the route plan that best trades off optimality and robustness. For each instance of the decision-making problem, RoMO designed 19 route plans, one per weighting combination. The first-stage transportation cost of each of these solutions is a characteristic inherited by the route plan, but the second-stage transportation cost was simulated using Monte Carlo Simulation and the distribution that models the demands. The solution that balanced best the trade-off between optimality and robustness was the one that calculated the lowest real transportation cost (planned plus second-stage transportation costs). Which weighting combination provided this solution depends on the settings of the problem and thus changes from instance to instance.

In this thesis, the mean absolute deviation formulation was used to deal with the SS-CVRPSD, however, in may be applied to other combinatorial optimization problems under uncertainty with appropriate modifications. In other real-world problems, it may be unclear when robust solutions are preferable to the solutions for the deterministic optimization approach, since this implicates balancing minimization of costs and protection. Whether the additional protection justifies the incurred extra cost may depend on the situation. Nevertheless, once the traded off solutions are available, a decision-maker can easily make a decision on which solution is to be adopted to cope with the stochastic inputs based on the settings of the company. The MAD formulation and RoMO solution approach can thus provide a basis for improved decision-making in other real-world applications where the decision-making problem is under uncertainty. We highlight that we defined a solution to be robust if it is less affected by fluctuation of the stochastic inputs and therefore needs just few corrective actions (detours-to-depot) during the second stage (recoverable robustness). Although we believe that in the context of the SSCVRPSD corrections are possible to be implement during the execution of the a priori route plan, for different real-world problems this may not be the case.

Though the proposed formulation and solution approach comes out with advantages, they still show some limitations. First, one needs to hold historical data about the uncertain demands in order to be able to fit a probability distribution to it. Second, these probability distributions were assumed to be known or possible to estimate. In practice, however, it can be difficult to precisely and accurately estimate them. It may require a long period of time for collection of statistics of real data on the customer demands. Third, despite the fact that companies usually have a fixed number of vehicles that are used to service their customers, we did not fix the amount of vehicles that could be used in the decision-making problem. Since each vehicle performs only a single route, one of the outputs of the decision model is the size of the fleet of vehicles. Thus, an unlimited amount of routes could be performed to attend the customers and unlimited amount of extra routes could be executed to deal with the fluctuation in the demands. Moreover, the cost on the use of an extra vehicle, such as hiring a new driver, was not considered.

One avenue for future research is the extension of the MAD formulation in order to cope with other sources of uncertainties, such as stochastic travel times. It could also be interesting to adopt the mean absolute deviation objective function as the robust optimization criteria in the robust optimization framework and to compare the results provided by the RoMO solution approach against solutions obtained by the robust optimization framework. Another interesting comparison is the difference between the performance of RoMO which is a static approach and the performance of a dynamic solution method, in which the route plan is designed in an ongoing fashion.

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